

SOLUTIONS & ANSWERS FOR AIEEE-2011 VERSION – Q

PART – A – PHYSICS

1. Ans: Wave moving in $-x$ direction with speed

$$\sqrt{\frac{b}{a}}$$

Sol: $y(x, t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$
This is of the form $y(x, t) = f(x + vt)$, where

$$v = \frac{\sqrt{b}}{\sqrt{a}} \text{ travels in negative } x \text{ direction.}$$

2. Ans: 0.052 cm

Sol: $LC = \frac{1}{100} = 0.01 \text{ mm}$

$$\begin{aligned} \text{Reading} &= \text{PSR} \times \text{pitch} + \text{CSR} \times \text{LC} \\ &= 0 + 52 \times 0.01 \\ &= 0.52 \text{ mm} \\ &= 0.052 \text{ cm} \end{aligned}$$

3. Ans: $\frac{2}{3}g$

Sol: $mg - T = ma$

$$TR = \frac{mR^2}{2} \cdot a$$

$$\Rightarrow mg = \frac{3}{2}ma$$

$$\Rightarrow a = \frac{2}{3}g$$

4. Ans: $0.4\pi \text{ mJ}$

Sol: $E = T \cdot 8\pi(r_2^2 - r_1^2)$
 $= 8\pi T \left(\frac{25}{10^4} - \frac{9}{10^4} \right)$
 $= 8 \times 16 \times \pi \times 0.03 \times 10^{-4}$
 $= 0.4\pi \text{ mJ}$

5. Ans: First increases and then decreases.

Sol: Angular momentum is conserved.
I decreases ω increases then I increases
 ω decreases.

6. Ans: $\frac{\pi}{3} \text{ rad}$

Sol: $x_1 = A \sin \omega t$
 $x_2 = x_0 + A \sin(\omega t + \phi)$
 $d = x_2 - x_1 = x_0 + A [\sin(\omega t + \phi) - \sin \omega t]$

$$\therefore d = x_0 - 2A \cos\left(\omega t + \frac{\phi}{2}\right) \sin\left(\frac{\phi}{2}\right) \quad \text{---(i)}$$

$$d \text{ is maximum when } \cos\left(\omega t + \frac{\phi}{2}\right) = -1$$

Given $d_{(\text{max})} = x_0 + A$

$$\therefore \text{(i)} \Rightarrow x_0 + A = x_0 - 2A \times -1 \cdot \sin\frac{\phi}{2}$$

$$= x_0 + 2A \sin\frac{\phi}{2}$$

$$\Rightarrow \frac{1}{2} = \sin\frac{\phi}{2} \text{ or } \frac{\phi}{2} = \frac{\pi}{6} \text{ rad}$$

$$\Rightarrow \phi = \frac{\pi}{3} \text{ rad}$$

7. Ans: $\frac{-9Gm}{r}$

Sol: $\frac{Gm}{x^2} = \frac{G4m}{(r-x)^2}$

$$\frac{(r-x)^2}{x^2} = 4$$

$$r-x = 2x$$

$$x = \frac{r}{3}$$

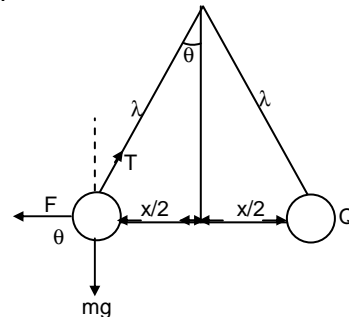
$$V = \frac{-Gm}{r} - \frac{G4m}{2r}$$

$$= -\frac{Gm}{r} (3+6)$$

$$= \frac{-9Gm}{r}$$

8. Ans: $x^{-1/2} \propto v$

Sol:



$$\tan \theta = \frac{x}{2\lambda} \quad \text{--- (i)}$$

$$T \sin \theta = F = \frac{kQ^2}{x^2}$$

$$T \cos \theta = mg$$

$$\therefore \tan\theta = \frac{F}{mg} = \frac{kQ^2}{x^2mg} \quad \text{--- (ii)}$$

$$\text{From (i) and (ii) } \frac{kQ^2}{x^2mg} = \frac{x}{2\lambda}$$

$$\Rightarrow Q^2 \propto x^3 \quad \text{--- (iii)}$$

$$\therefore 2Q \frac{dQ}{dt} \propto 3x^2 \frac{dx}{dt}$$

$$\Rightarrow Q \frac{dQ}{dt} \propto x^2 v \left(\Theta \frac{dx}{dt} = v \right)$$

$$\Rightarrow x^{3/2} \propto x^2 v$$

$$\left(\Theta Q \propto x^{3/2} \text{ and } \frac{dQ}{dt} = \text{constant} \right)$$

$$\Rightarrow x^{-1/2} \propto v$$

9. Ans: 0.15 mV

$$\begin{aligned} \text{Sol: } \varepsilon &= B\lambda v \\ &= 5 \times 10^{-5} \times 2 \times 1.50 \\ &= 0.15 \text{ mV} \end{aligned}$$

10. Ans: 2 s

$$\text{Sol: } \frac{dv}{dt} = -2.5\sqrt{v}$$

$$\frac{dv}{\sqrt{v}} = -2.5 dt$$

$$\Rightarrow -2.5t = \left[2\sqrt{v} \right]_{6.25}^0$$

$$t = \frac{2\sqrt{6.25}}{2.5}$$

$$= \frac{2 \times 2.5}{2.5} = 2$$

11. Ans: $\frac{\pi}{4}\sqrt{LC}$

$$\text{Sol: } q' = q_0 \cos\omega t$$

$$E = \frac{q_0^2}{2C}$$

$$\frac{E}{2} = \frac{1}{2} \frac{q_0^2}{2C}$$

$$\text{i.e. } q' = \frac{q_0}{\sqrt{2}}$$

$$\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$$

$$\Rightarrow \omega t = \frac{\pi}{4}$$

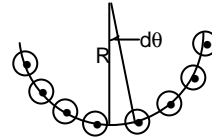
$$t = \frac{\pi}{4}\sqrt{LC}$$

12. Ans: 45°

Sol: $\mu_1[\hat{N} \times K_1] = \mu_2[\hat{N} \times K_2]$. But plane of separation need to be XY.

13. Ans: $\frac{\mu_0 I}{\pi^2 R}$

$$\text{Sol: } B = \frac{I}{\pi R} R d\theta \frac{\mu_0}{2\pi R} \sin\theta$$



$$= \frac{\mu_0 I}{2\pi^2 R} \int_0^{\pi/2} \sin\theta d\theta$$

$$= \frac{\mu_0 I}{\pi^2 R}$$

14. Ans: $\frac{1}{2} \frac{Mv^2(\gamma-1)}{R}$

Sol: Volume is constant

$$C_v = \frac{R}{(\gamma-1)}$$

$$KE = \frac{1}{2} Mv^2$$

$$\Delta Q = nC_v \Delta\theta = 1 \times C_v \Delta\theta$$

$$\therefore \Delta\theta = \frac{KE}{C_v} = \frac{1}{2} \frac{Mv^2(\gamma-1)}{R}$$

15. Ans: $\left(\frac{M+m}{M}\right)^{1/2}$

$$\text{Sol: } Mv_1 = (M+m)v_2$$

$$\frac{v_1}{v_2} = \frac{M+m}{M}$$

$$\frac{1}{2}(M+m)v_2^2 = \frac{1}{2}KA_2^2$$

$$\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$$

$$\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$$

$$\Rightarrow \frac{A_1^2}{A_2^2} = \frac{M}{M+m} \left(\frac{M+m}{M}\right)^2$$

$$= \frac{M+m}{M}$$

$$\therefore \frac{A_1}{A_2} = \left(\frac{M+m}{M}\right)^{1/2}$$

16. Ans: $3.6 \times 10^{-3} \text{ m}$

$$\text{Sol: } P_0 + \frac{1}{2}\rho v_1^2 + \rho gh$$

$$= P_0 + \frac{1}{2}\rho v_2^2$$

$$\begin{aligned} \Rightarrow 2gh &= (v_2^2 - v_1^2) \\ \Rightarrow 2gh + v_1^2 &= v_2^2; \\ v_1 &= 0.4 \text{ m s}^{-1}, h_2 = 0.2 \text{ m} \\ \Rightarrow v_2 &= 2.0396 \text{ m s}^{-1} \end{aligned}$$

$$A_1 v_1 = A_2 v_2 \Rightarrow d_2^2 = \frac{d_1^2 v_1}{v_2}$$

$$\begin{aligned} \Rightarrow d_2 &= d_1 \sqrt{\frac{v_1}{v_2}} \\ &= 8 \times 10^{-3} \times \sqrt{\frac{0.4}{2.0396}} \\ &\approx 3.6 \times 10^{-3} \text{ m} \end{aligned}$$

17. Ans: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement - 1.

Sol: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement - 1.

18. Ans: $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

Sol: $P_1 V = n_1 K T_1$
 $P_2 V = n_2 K T_2$
 $P_3 V = n_3 K T_3$

$$\begin{aligned} \frac{1}{2} m v^2 &= \frac{3}{2} K T_1 \times n_1 + \frac{3}{2} K T_2 n_2 + \frac{3}{2} K T_3 n_3 \\ &= \frac{3}{2} K (n_1 + n_2 + n_3) T \\ T &= \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3} \end{aligned}$$

19. Ans: more than 3 but less than 6.

Sol: $\tau = Fr = 40t - 10t^2$
 $\alpha = \frac{\tau}{I} = 4t - t^2$

$$\frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = 2t^2 - \frac{t^3}{3}$$

(\ominus At $t = 0$, $\omega = 0$)
 At $t = 6$ s. ω again become zero

$$\omega = \frac{d\theta}{dt} = 2t^3 - \frac{t^3}{3} \Rightarrow \theta = \frac{2t^3}{3} - \frac{t^4}{12}$$

$\therefore \theta$ in 6 s = $(144 - 108) = 36$ rad
 $\Rightarrow N = \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.72$ rotation.

20. Ans: $2.7 \times 10^6 \Omega$

Sol: $V = V_0(1 - e^{-t/RC})$
 $120 = 200(1 - e^{-t/RC})$
 $e^{-t/RC} = \frac{2}{5}$
 $e^{t/RC} = 2.5$

$$\frac{t}{RC} = 0.4 \times 2.5 \times 2.303$$

$$\Rightarrow R = 2.7 \times 10^6 \Omega$$

21. Ans: 372 K and 310 K

Sol: $1 - \frac{T_2}{T_1} = \frac{1}{6}$
 $1 - \frac{T_2 - 62}{T_1} = \frac{1}{3}$
 $\frac{T_2}{T_1} = \frac{5}{6}$

$$\frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\frac{T_2}{T_2 - 62} = \frac{5}{4}$$

$$4T_2 = 5T_2 - 310$$

$$T_2 = 310 \text{ K}$$

$$\Rightarrow T_1 = 372 \text{ K}$$

22. Ans: Increases by 0.2%

Sol: $R \propto \lambda^2$
 $R' \propto \lambda'^2$
 $\propto (1.001)^2 \lambda^2$
 $\frac{\Delta R}{R} = 0.002$
 $\therefore 0.002 \times 100 = 0.2\%$

23. Ans: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

Sol: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

24. Ans: $\frac{1}{15^2} \times 15 = \frac{1}{15} \text{ m s}^{-1}$

Sol: $\frac{1}{v} + \frac{1}{-2.8} = \frac{1}{0.2}$
 $\Rightarrow \frac{1}{v} = \frac{15}{2.8}$
 $v = \frac{2.8}{15}$
 $\frac{v}{u} = \frac{1}{15}$
 $\frac{v^2}{u^2} = \frac{1}{15^2}$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2}$$

$$\left| \frac{dv}{dt} \right| = \frac{v^2}{u^2} \cdot \frac{du}{dt}$$

$$= \frac{1}{15^2} \times 15 = \frac{1}{15} \text{ m s}^{-1}$$

25. Ans: 108.8 eV

Sol: $\frac{13.6 Z^2}{n^2} = 13.6 \times 9 \left[1 - \frac{1}{9} \right]$

$$= 13.6 \times 9 \times \frac{8}{9}$$

$$= 108.8 \text{ eV}$$

26. Ans: $-6 \epsilon_0 a$

Sol: $V = ar^2 + b$

$$E = -\frac{dV}{dr} = -2ar$$

$$4\pi r^2 \cdot E = \frac{Q}{\epsilon_0}$$

$$Q = -4\pi r^2 \cdot 2ar \cdot \epsilon_0$$

$$\rho = \frac{-8\pi ar^3 \epsilon_0}{\frac{4}{3}\pi r^3}$$

$$= -6 \epsilon_0 a$$

27. Ans: $\frac{\pi v^4}{g^2}$

Sol: $R_{\max} = \frac{v^2}{g}$

$$\text{Area} = \pi (R_{\max})^2$$

$$= \frac{\pi v^4}{g^2}$$

28. Ans: 8.4 kJ

Sol: $\Delta U = mC\Delta T$

$$= 4184 \times 20 \times 0.1$$

$$= 8.4 \text{ kJ}$$

29. Ans: 20 min

Sol: $N = \frac{N_0}{2^{t/T_{1/2}}}$

$$\frac{N_0}{3} = \frac{N_0}{2^{t_2/20}} \Rightarrow t_2 = 20 \frac{\log 3}{\log 2}$$

$$N_0 \frac{2}{3} = \frac{N_0}{2^{t_1/20}} \Rightarrow t_1 = \frac{20(\log 3 - \log 2)}{\log 2}$$

$$t_2 - t_1 = \frac{20}{\log 2} (\log 3 - \log 3 + \log 2)$$

$$= 20 \text{ min}$$

30. Ans: Statement – 1 is false, Statement-2 is true.

Sol: If $v \Rightarrow 2v$,
 $V_0' > 2V_0$, well known result
 \Rightarrow Statement 1 is wrong.
Statement 2 is true.

Part – B – Mathematics

31. Ans: Statement-1 is true, Statement-2 is false.

Sol: P is (-2, -2) and Q (-1, 2) since R bisect $\angle POQ$, $PR \perp RQ = OP : OQ$

$$= \sqrt{4+4} : \sqrt{1+4} = \sqrt{8} : \sqrt{5}$$

\therefore Statement 1 is true
But statement 2 is false.

32. Ans: $\frac{3}{4} \leq A \leq 1$

Sol: $A = \sin^2 x + \cos^4 x$

$$= \cos^4 x - \cos^2 x + 1$$

$$= \left(\cos^2 x - \frac{1}{2} \right)^2 + \frac{3}{4}$$

$\therefore \frac{3}{4} \leq A \leq 1$

33. Ans: -144

Sol: $(1 - x - x^2 + x^3)^6 = (1 - x)^6 (1 - x^2)^6$

$$= (1 - 6x + \dots - 20x^3 \dots - 6x^5) x$$

$$(1 - 6x^2 + 75x^4 - 20x^6 \dots)$$

$$= 120 - 300 + 36$$

$$= 156 - 300 = -144$$

34. Ans: Does not exist

Sol: $\lim_{x \rightarrow 2} \sqrt{2} \left| \frac{\sin(x-2)}{(x-2)} \right|$

Limit does not exist

35. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1.

Sol: $x_1 + x_2 + x_3 + x_4 = 6$

$$x_1 \geq 0$$

no. of ways = 9C_3

S_2 is true
 S_1 is true
 S_1 follows from S_2

36. Ans: $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Sol: $\frac{d^2x}{dy^2} = \frac{d}{dy} \left(\frac{dx}{dy} \right)$

$$= \frac{d}{dy} \left[\frac{1}{\frac{dy}{dx}} \right]$$

$$= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{d}{dy} \left(\frac{dy}{dx} \right)$$

$$= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \frac{d^2y}{dx^2} \cdot \left(\frac{dx}{dy} \right)$$

$$= - \left(\frac{d^2y}{dx^2} \right) \left(\frac{dy}{dx} \right)^{-3}$$

37. Ans: 7

Sol: $\frac{dy}{dx} = y + 3$

$$\frac{dy}{y+3} = dx$$

$$\log(y+3) = x + c$$

$$\therefore y + 3 = c e^x$$

$$x = 0 \quad y = 2 \Rightarrow c = 5$$

$$\therefore y = 5 e^x - 3$$

$$\therefore y(\log 2) = 5 e^{\log 2} - 3$$

$$= 5 \times 2 - 3 = 7$$

38. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

Sol: A = (x, y) $y - x \in Z$
 B = (x, y) $x = \alpha y$ for rational α
 A : $x - x = 0 \in Z \Rightarrow (x, x) \in A$ reflexive
 $y - x \in Z \Rightarrow x - y \in Z$
 $\Rightarrow (y, x) \in A$ symmetric
 $y - x \in Z$ and $z - y \in Z \Rightarrow z - x \in Z$
 $\therefore (x, z) \in A$ transitive
 A is equivalence relation
 Statement - 1 is true
 B: $x = 1, x \Rightarrow (x, x) \in B$ reflexive
 $x = \alpha y \Rightarrow y = \frac{1}{\alpha} x \quad \therefore (y, x) \in B$
 $x = \alpha y$ and $y = \alpha z \Rightarrow x = \alpha^2 z$
 $\therefore (x, z) \in B$ transitive
 B is equivalence relation
 Statement - 2 is true but 1 does not follow from 2.

39. Ans: $\pi \log 2$

Sol: $I = 8 \int_0^1 \frac{\log(1+x)}{1+x^2} dx$

$$= 8 \int_0^{\pi/4} \log(1 + \tan \theta) d\theta$$

$$= \pi \log 2$$

40. Ans: $\beta \in (1, \infty)$

Sol: If $1 + ai$ is root (a, real)
 Then $(1 + ia)^2 + \alpha(1 + ia) + \beta = 0$
 $2a + \alpha a = 0 \Rightarrow \alpha = -2 \quad a \neq 0$
 $1 - a^2 + \alpha + \beta = 0$
 $1 - a^2 + \beta = 0$
 $\beta = a^2 + 1 > 1 \therefore \beta \in (1, \infty)$

41. Ans: $\left[0, \frac{1}{2} \right]$

Sol: $1 - P^5 \geq \frac{31}{32}$

$$P^5 \leq 1 - \frac{31}{32}$$

$$\leq \frac{1}{32}$$

$$P \leq \frac{1}{2} = \left[0, \frac{1}{2} \right]$$

Choice (3)

42. Ans: 21 months

Sol: Total savings = 11040
 Savings in the first 2 months = 400
 Hence, savings in the next n months = 10640

We have

$$\frac{n}{2} [400 + (n-1)40] = 10640$$

$$[200 + (n-1)20] n = 10640$$

$$200n + 20n^2 - 20n = 10640$$

$$20n^2 + 180n - 10640 = 0$$

$$n^2 + 9n - 532 = 0$$

$$n = \frac{9 \pm \sqrt{81 + 2128}}{2}$$

$$= \frac{-9 \pm \sqrt{2209}}{2} = \frac{-9 \pm 47}{2}$$

$$= 19$$

Therefore, answer is 21 months

43. Ans: $(-\infty, 0)$

Sol: $|x| - x > 0$

$$\Rightarrow |x| > x$$

$$\Rightarrow x \in (-\infty, 0)$$

44. Ans: $\frac{2}{3}$

Sol: The angle is $\sin^{-1} \frac{3}{\sqrt{14}}$

$$\therefore \frac{1+4+3\lambda}{\sqrt{(1+4+\lambda^2)(1+4+9)}} = \frac{3}{\sqrt{14}}$$

$$14(3\lambda+5)^2 = 9 \times 14(5+\lambda^2)$$

$$9\lambda^2 + 30\lambda + 25 = 9\lambda^2 + 45$$

$$\Rightarrow 30\lambda = 20 \Rightarrow \lambda = \frac{2}{3}$$

45. Ans: -5

Sol: $|a| = |b| = 1$ $a \cdot b = 0$
 $(2a - b) \cdot ((a \times b) \times (a + 2b))$
 $= (2a - b) \times$
 $[(a \cdot a) b - (a \cdot b) a + (2b \cdot a) b - (2b \cdot b)]$
 $(2a - b) \cdot (b - 2a) = -5$

46. Ans: $3x^2 + 5y^2 - 32 = 0$

Sol: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 $\frac{9}{a^2} + \frac{1}{b^2} = 1$
 $\frac{1}{b^2} = 1 - \frac{9}{a^2}$
 $\frac{1}{a^2(1-\frac{9}{a^2})} = \frac{a^2-9}{a^2}$
 $a^2 - 9 = \frac{3}{5}$
 $a^2 = 9 + \frac{3}{5} = \frac{32}{5}$
 $b^2 = a^2 \times \frac{3}{5} = \frac{32}{5} \times \frac{3}{5} = \frac{32}{5}$
 Equation of the ellipse is
 $\frac{x^2}{\frac{32}{5}} + \frac{y^2}{\frac{32}{5}} = 1$
 $3x^2 + 5y^2 - 32 = 0$

47. Ans: $I - \frac{kT^2}{2}$

Sol: $\frac{dv(t)}{dt} = -k(T-t)$
 $V(t) = \int -k(T-t) dt$
 $\frac{k(T-t)^2}{2} + C$
 $t = 0, V(t) = I$
 $\Rightarrow I = \frac{kT^2}{2} + C$
 $C = I - \frac{kT^2}{2}$
 Therefore,
 $V(t) = \frac{k(T-t)^2}{2} + I - \frac{kT^2}{2}$

$$\Rightarrow V(T) = 0 + I - \frac{kT^2}{2}$$

$$= I - \frac{kT^2}{2}$$

48. Ans: $\vec{c} - \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \vec{b}$

Sol: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$
 $\vec{a} \cdot \vec{d} = 0$
 $\vec{b} \times (\vec{c} - \vec{d}) = 0$
 \vec{b} and $(\vec{c} - \vec{d})$ are collinear
 $\vec{b} = k(\vec{c} - \vec{d})$
 $\vec{a} \cdot \vec{b} = k(\vec{a} \cdot \vec{c}) - \vec{a} \cdot \vec{d}$
 $k \left[\frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{c}} \right]$
 $k = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{c}}$
 $\vec{b} \cdot \vec{c} - \vec{d} = \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \vec{b}$
 $\vec{d} = \vec{c} - \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \vec{b}$

49. Ans: $|a| = c$

Sol: Two circle should touch each other
 Centres are $\left(\frac{a}{2}, 0\right)$ and $(0, 0)$
 \therefore also second circle passes through $(0, 0)$
 $\therefore c = a \Rightarrow |a| = c$

50. Ans: $P(C|D) \geq P(C)$

Sol: $P(C|D) = \frac{P(CD)}{P(D)}$
 $= \frac{P(C)}{P(D)}$
 $\geq P(C)$

51. Ans: 2

Sol: $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$
 $4(4-2) - k(k-2) + 2(2k-8) = 0$
 $= 8 - k^2 + 2k + 4k - 16 = 0$
 $\Rightarrow -k^2 + 6k - 8 = 0$
 $k^2 - 6k + 8 = 0$
 $\Rightarrow (k-4)(k-2) = 0$
 $\Rightarrow k = 2, 4$
 $\therefore k = 2$

52. Ans: $\sim(Q \leftrightarrow (P \wedge \sim R))$

Sol: The given statement is
 $(P \wedge \sim R) \leftrightarrow Q \equiv Q \leftrightarrow (P \wedge \sim R)$

∴ The required negative is
 $\sim [Q \leftrightarrow (P \wedge \sim R)]$

53. Ans: $\frac{3\sqrt{2}}{8}$

Sol: Slope of the line perpendicular to $y - x = 1$ is (-1)

Hence $t = 1$

Point on the parabola corresponding to $t = 1$ is

$\Rightarrow \left(\frac{1}{4}, \frac{1}{2}\right)$

∴ shortest distance = $\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}$

54. Ans: 4

Sol: Median = $\frac{25a + 26b}{2}$
 $= \frac{51a}{2}$

Numerical value of the sum of the derivation

$= \left| 2a \left\{ \frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \dots + \frac{49}{2} \right\} \right|$
 $= \left| \frac{2a \times 25^2}{2} \right| = |25^2 a|$

Mean derivation about median = $\left| \frac{25^2 a}{50} \right| = \frac{1}{2}$

$\left| \frac{25^2 a}{50} \right| = 50$

$|a| = \frac{50 \times 50}{25 \times 25} = 4$

55. Ans: Statement-1 is true, Statement-2 is true;
 Statement -2 is **not** a correct explanation for Statement-1.

Sol: A (1, 0, 7) B, (1, 6, 3)

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{5}$

P ($\lambda, 2\lambda + 1, 3\lambda + 2$)

drs ($\lambda - 1, 2\lambda + 1, 3\lambda - 5$)

∴ $\lambda - 1 + 2(2\lambda + 1) + 3(3\lambda - 5) = 0$

$14\lambda - 14 = 0 \Rightarrow \lambda = 1$

P (1, 3, 5) is mid point of A and B

Statement-1 is true

Statement-2 is also true but

statement-1 does not follow from 2

56. Ans: Statement-1 is true, Statement-2 is true;
 Statement -2 is **not** a correct explanation for Statement-1.

Sol: if $AB = BA$
 $(AB)^T = A^T B^T$

$\Rightarrow AB$ is symmetric

Statement-2 is true

$(ABA)^T = A^T B^T A^T$

Take $A = I$ and $B =$ some non - symmetric

∴ ABA always

∴ $A(BA)$ and $(AB)A$ are symmetric

Statement-1 is true but does not depend on Statement-2

57. Ans: (1, 1)

Sol: $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^{14} = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = A + B\omega$

∴ $A = 1$ $B = 1$

∴ (1, 1)

58. Ans: $p = -\frac{3}{2}, q = \frac{1}{2}$

Sol: $f(x) = \frac{\sin(p+1)x + \sin x}{x}, x < 0$

$= q, x = 0$

$\frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, x > 0$

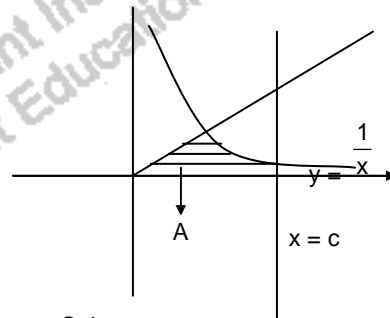
is continuous.

$\Rightarrow p + 1 + 1 = q = \lim_{x \rightarrow 0} \frac{x}{x^{3/2}(\sqrt{x+x^2} + \sqrt{x})}$

$= \frac{1}{2}$

∴ $p = -\frac{3}{2}, q = \frac{1}{2}$

59. Ans: $\frac{3}{2}$ square units



Sol: $y = x$
 $y = \frac{1}{x} \Rightarrow x^2 = 1$
 $\Rightarrow x = 1 (x > 0)$

$y = \frac{1}{x}, x = e \Rightarrow x = e$

∴ area $A = \int_1^e \left(x - \frac{1}{x}\right) dx$

$= \frac{e^2 - 1}{2} - \log e$

$= \frac{e^2 - 3}{2}$

$$\text{Required area} = \frac{1}{2} \cdot e^2 - \frac{e^2 - 3}{2} = \frac{3}{2}$$

60. Ans: local maximum at π and local minimum at 2π

Sol: $f'(x) = \sqrt{x} \sin x$

$$f''(x) = \frac{2x \cos x + \sin x}{2\sqrt{x}}$$

$$f'(x) = 0 \Rightarrow x = n\pi, n \in \mathbb{Z}$$

$$\text{i.e., } x = \pi, 2\pi \text{ in } \left(0, \frac{5\pi}{2}\right)$$

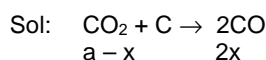
$$f''(\pi) < 0 \text{ and } f''(2\pi) > 0$$

$\therefore f(x)$ has maximum at $x = \pi$

And minimum at $x = 2\pi$

Sol: 'a' is a measure of attraction between the molecules and 'b' the size of the molecules.

69. Ans: 1.8 atm



$$a = 0.5 \text{ atm}$$

$$a + x = 0.8 \text{ atm}$$

$$x = 0.3 \text{ atm}$$

$$K_p = \frac{p_{\text{CO}}^2}{p_{\text{CO}_2}} = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$$

PART C – CHEMISTRY

61. Ans: AlCl_3

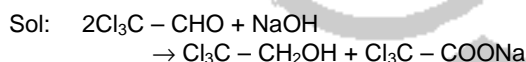
Sol: Fajan's rules.

Al^{3+} is the smallest cation and it has high charge.

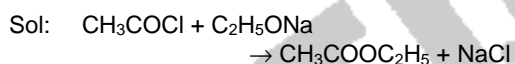
62. Ans: 2nd

Sol: RNA contains β -D-ribose while DNA contains β -D-2-deoxyribose.

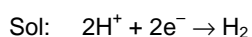
63. Ans: 2, 2, 2-Trichloroethanol



64. Ans: Ethyl ethanoate



65. Ans: $p(\text{H}_2) = 2 \text{ atm}$ and $[\text{H}^+] = 1.0 \text{ M}$



$$E_{\text{Cl}} = \frac{0.0591}{2} \log \frac{[\text{H}^+]^2}{[\text{H}_2]}$$

$$[\text{H}_2] > [\text{H}^+]^2$$

66. Ans: $\text{CH}_3\text{CH}_2\text{CH}(\text{Cl})\text{CO}_2\text{H}$

Sol: Presence of Cl having -I effect on the α -carbon makes 2-chlorobutanoic acid the strongest acid among the given compounds.

67. Ans: $\alpha = \frac{i - 1}{(x + y - 1)}$

Sol: $i = 1 - \alpha + n\alpha; n = x + y$

$$\alpha = \frac{i - 1}{x + y - 1}$$

68. Ans: a for $\text{Cl}_2 > a$ for C_2H_6 but b for $\text{Cl}_2 < b$ for C_2H_6

70. Ans: BF_6^{3-}

Sol: Boron cannot form BF_6^{3-} since boron has no available d-orbitals.

71. Ans: The complex is an outer orbital complex

Sol: $[\text{Cr}(\text{NH}_3)_6]\text{Cl}_3$ is not an outer orbital complex.

72. Ans: 804.32 g

Sol: $\Delta T_f = K_f \times \frac{W_2}{M_2} \times \frac{1}{W_1}$

$$6 = 1.86 \times \frac{W_2}{62} \times \frac{1}{4}$$

$$W_2 = 800 \text{ g}$$

Wt. of glycol required is more than 800 g

73. Ans: $\text{Al}_2\text{O}_3 < \text{MgO} < \text{Na}_2\text{O} < \text{K}_2\text{O}$

Sol: K_2O is more basic than Na_2O . Al_2O_3 is amphoteric.

74. Ans: 32 times

Sol: 2 times increase for 10°C

$$2^5 = 32 \text{ times increase for } 50^\circ\text{C}$$

75. Ans: 2.82 BM

Sol: There are two unpaired electrons in $[\text{NiCl}_4]^{2-}$ hence the paramagnetic moment is 2.82 BM.

76. Ans: sp^2, sp, sp^3

Sol: $\text{NO}_3^- - sp^2, \text{NO}_2^+ - sp$ and $\text{NH}_4^+ - sp^3$

77. Ans: Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series

Sol: All the lanthanoids does not exhibit +4 oxidation state.

78. Ans: 0.086

Sol: Mole fraction of methanol
$$= \frac{\text{moles of methanol}}{\text{total moles}} = \frac{5.2}{5.2 + \frac{1000}{18}}$$
$$= 0.086$$

79. Ans: The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.

Sol: Stability of hydrides decreases from NH_3 to BiH_3 .

80. Ans: $4f^7 5d^1 6s^2$

Sol: The outer electronic configuration of ${}_{64}\text{Gd}$ is $4f^7 5d^1 6s^2$

81. Ans: The oxidation state of sulphur is never less than +4 in its compounds

Sol: Sulphur exhibits oxidation state lower than +4 in its compounds.

82. Ans: pentagonal bipyramid

Sol: IF_7 is pentagonal bipyramidal.

83. Ans: a vinyl group

Sol: Formation of HCHO in ozonolysis shows the presence of $\text{CH}_2 = \text{CH}$ - group.

84. Ans: 743 nm

Sol:
$$\frac{1}{355} = \frac{1}{680} - \frac{1}{\lambda}$$
$$\lambda = 743 \text{ nm}$$

85. Ans: Acetaldehyde

Sol: Acetaldehyde reduces tollens' reagent to metallic silver on warming.

86. Ans: Neutral FeCl_3

Sol: Neutral FeCl_3 solution gives violet colour with phenol.

87. Ans: 2, 4, 6-Tribromophenol

Sol: Phenol forms 2, 4, 6-tribromophenol when treated with a mixture of KBr , KBrO_3 and HCl .

88. Ans: A_2B_5

Sol: $\text{A}_1\text{B}_{5/2} = \text{A}_2\text{B}_5$

89. Ans: $38.3 \text{ J mol}^{-1} \text{ K}^{-1}$

Sol:
$$\Delta S = 2.303 nR \log \frac{V_2}{V_1}$$
$$= 2.303 \times 2 \times 8.314 \times \log 10$$
$$= 38.3 \text{ J K}^{-1}$$

90. Ans: 2-Pentanone

