SOLUTIONS & ANSWERS FOR AIEEE-2011
VERSION – R

Part – A – Mathematics

1. Ans: $\left[0, \frac{1}{2}\right]$

Sol: $1 - P^5 \geq \frac{31}{32}$

$P^5 \leq 1 - \frac{31}{32}$

$\leq \frac{1}{32}$

$P \leq \frac{1}{2} = \left[0, \frac{1}{2}\right]$

2. Ans: – 144

Sol: $(1 - x - x^2 + x^3)^6 = (1 - x)^6 (1 - x^2)^6$
$= (1 - 6x + ..- 20x^3 \ldots - 6x^5)\ x$
$(1 - 6x^2 + 75x^4 - 20x^6 \ldots.)$
$= 120 - 300 + 36$
$= 156 - 300 = -144$

3. Ans: Does not exist

Sol: $\lim_{x \to 2} \sqrt{2}\left|\frac{\sin(x-2)}{(x-2)}\right|$
Limit does not exist

4. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

Sol: A = (x, y) $y - x \in z$
B = (x , y) x = αy for rational α
A : $x - x = 0 \in z \Rightarrow (x, x) \in$ A reflexive
$y - x \in z \Rightarrow x - y \in z$
$\Rightarrow (y, x) \in$ A symmetric
$y - x \in z$ and $z - y \in z \Rightarrow z - x \in z$
$\therefore (x, z) \in$ A transitive
A is equivalence relation
Statement – 1 is true
B: x = 1, x $\Rightarrow (x, x) \in$ B reflexive
$x = \alpha y \Rightarrow y = \frac{1}{\alpha} x$ $\quad \therefore (y, x) \in$ B symmetric
$x = \alpha y$ and $y = \alpha z \Rightarrow x = \alpha^2 z$
$\therefore (x, z) \in$ B transitive
B is equivalence relation
Statement – 2 is true but I does not follow from 2.

5. Ans: $\beta \in (1, \infty)$

Sol: If 1 + ai is root (a, real)
Then $(1 + i\,a)^2 + \alpha (1 + I\,a) + \beta = 0$
$2a + a\alpha = 0 \Rightarrow \alpha = -2\,a \neq 0$
$1 - a^2 + \alpha + \beta = 0$
$1 - a^2 + \beta = 0$
$\beta = a^2 + 1 > 1 \therefore \beta \in (1, \infty)$

6. Ans: $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Sol: $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right)$

$= \frac{d}{dy}\left[\frac{1}{dy/dx}\right]$

$= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{d}{dy}\left(\frac{dy}{dx}\right)$

$= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \frac{d^2y}{dx^2} \cdot \left(\frac{dx}{dy}\right)$

$= -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

7. Ans: 2

Sol: $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$4(4 - 2) - k\,(k - 2) + 2\,(2k - 8) = 0$
$= 8 - k^2 + 2k + 4k - 16 = 0$
$\Rightarrow -k^2 + 6k - 8 = 0$
$k^2 - 6k + 8 = 0$
$\Rightarrow (k - 4)\,(k - 2) = 0$
$\Rightarrow k = 2, 4$
$\therefore k = 2$

8. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

Sol: A (1, 0, 7) B,(1, 6, 3)
$\frac{x}{1} = \frac{y-1}{2} = \frac{2-2}{5}$
P $(\lambda, 2\lambda + 1, 3\lambda + 2)$
drs $(\lambda - 1, 2\lambda + 1, 3\lambda - 5)$
$\therefore \lambda - 1 + 2\,(2\lambda + 1) + 3(3\lambda - 5) = 0$
$14\lambda - 14 = 0 \Rightarrow \lambda = 1$
P (1, 3, 5) is mid point of A and B
Statement-1 is true
Statement-2 is also true but
statement-1does not follow from 2

9. Ans: $\sim (Q \leftrightarrow (P \wedge {\sim}R))$

Sol: The given statement is

(P ∧~R) ↔ Q ≡ Q ↔(P ∧~R)
∴ The required negative is
~ [Q ↔ (P ∧~R)]

10. Ans: Statement-1 is true, Statement-2 is false.

Sol: P is (-2, -2) and Q (-1, 2) since R bisect ∠POQ, PR ¨RQ = OP : OQ
$= \sqrt{4+4} : \sqrt{1+4} = \sqrt{8} : \sqrt{5}$
∴ Statement 1 is true
But statement 2 is false.

11. Ans: 21 months

Sol: Total savings = 11040
Savings in the first 2 months = 400
Hence, savings in the next n months = 10640
We have
$\frac{n}{2}[400+(n-1)40] = 10640$
[200 + (n–1) 20] n = 10640
$200n + 20n^2 - 20n = 10640$
$20n^2 + 180n - 10640 = 0$
$n^2 + 9n - 532 = 0$
$n = \frac{9 \pm \sqrt{81+2128}}{2}$
$= \frac{-9 \pm \sqrt{2209}}{2} = \frac{-9 \pm 47}{2}$
= 19
Therefore, answer is 21 months

12. Ans: $3x^2 + 5y^2 - 32 = 0$

Sol: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$\frac{9}{a^2} + \frac{1}{b^2} = 1$
$\frac{1}{b^2} = 1 - \frac{9}{a}$
$\frac{1}{a^1(1-2/5)} = \frac{a^2-9}{a^2}$
$a^2 - 9 = \frac{3}{5}$
$a^2 = 9 + \frac{3}{5} = \frac{32}{3}$
$b^2 = a^2 \times \frac{3}{5} = \frac{32}{3} \times \frac{3}{5} = \frac{32}{5}$
Equation of the ellipse is
$\frac{x}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$
$3x^2 + 5y^2 - 32 = 0$

13. Ans: $\frac{3}{4} \le A \le 1$

Sol: $A = \sin^2 x + \cos^4 x$
$= \cos^4 x - \cos^2 x + 1$
$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$\therefore \frac{3}{4} \le A \le 1$

14. Ans: π log 2

Sol: $I = 8\int_0^1 \frac{\log(1+x)}{1+x^2}$
$= 8\int_0^{\pi/4} \text{Log}(1+\tan\theta)\, d\theta$
= π log 2

15. Ans: $\frac{2}{3}$

Sol: The angle is $\sin^{-1} \frac{3}{\sqrt{14}}$
$\therefore \frac{1+4+3\lambda}{\sqrt{(1+4+\lambda^2)(1+4+9)}} = \frac{3}{\sqrt{14}}$
$14(3\lambda+5)^2 = 9 \times 14(5+\lambda^2)$
$9\lambda^2 + 30\lambda + 25 = 9\lambda^2 + 45$
$\Rightarrow 30\lambda = 20 \Rightarrow \lambda = \frac{2}{3}$

16. Ans: local maximum at π and local minimum at 2π

Sol: $f'(x) = \sqrt{x}\sin x$
$f''(x) = \frac{2x\cos x + \sin x}{2\sqrt{x}}$
$f'(x) = 0 \Rightarrow x = n\pi, n \in Z$
ie., $x = \pi, 2\pi$ in $\left(0\ 5\pi/2\right)$
$f''(\pi) < 0$ and $f''(2\pi) > 0$
∴ f(x) has maximum at x = π
And minimum at x = 2π

17. Ans: (-∞, 0)

Sol: $|x| - x > 0$
$\Rightarrow |x| > x$
$\Rightarrow x \in (-\infty, 0)$

18. Ans: 4

Sol: Median $= \frac{25a+26b}{2}$
$= \frac{51a}{2}$
Numerical value of the sum of the derivation
$= \left|2a\left\{\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+....+\frac{49}{2}\right\}\right|$

$$= \left| \frac{2a \times 25^2}{2} \right| = \left| 25^2 a \right|$$

Mean derivation about median = $\left| \frac{25^2 a}{50} \right|$

$$\left| \frac{25^2 a}{50} \right| = 50$$

$$|a| = \frac{50 \times 50}{25 \times 25} = 4$$

19. Ans: -5

Sol: $|a| = |b| = 1$ a. b = 0

$(2a - b).\ ((a \times b) \times (a + 2b))$

$= (2a - b) \times$

$[(a.\ a)\ b - (a.\ b)\ a + (2b.\ a)\ b - (2b.\ b)]$

$(2a - b).\ (b - 2a) = -5$

20. Ans: $p = -\frac{3}{2}, q = \frac{1}{2}$

Sol: $f(x) = \frac{\sin(p+1)x + \sin x}{x}$, $x < 0$

$= q$, $x = 0$

$\frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$, $x > 0$

is continuous.

$$\Rightarrow p + 1 + 1 = q = \lim_{x \to 0} \frac{x}{x^{3/2}\left(\sqrt{x + x^2} + \sqrt{x}\right)}$$

$$= \frac{1}{2}$$

$\therefore p = -\frac{3}{2}, q = \frac{1}{2}$

21. Ans: $|a| = c$

Sol: Two circle should touch each other

Centres are $\left(\frac{a}{2}, 0\right)$ and (0,0)

$\therefore$ also second circle passes through (0, 0)

$\therefore c = a \Rightarrow |a| = c$

22. Ans: $I - \frac{kT^2}{2}$

Sol: $\frac{dv(t)}{dt} = -k(T - t)$

$V(t) = \int -k(T - t)\,dt$

$\frac{k(T - t)^2}{2} + C$

$t = 0, V(t) = I$

$\Rightarrow I = \frac{kT^2}{2} + C$

$C = I - \frac{kT^2}{2}$

Therefore,

$V(t) = \frac{k(T - t)^2}{2} + I - \frac{kT^2}{2}$

$\Rightarrow V(T) = 0 + I - \frac{kT^2}{2}$

$= I - \frac{kT^2}{2}$

23. Ans: $P(C|D) \geq P(C)$

Sol: $P(C \mid D) = \frac{P(CD)}{P(D)}$

$= \frac{P(C)}{P(D)}$

$\geq P(C)$

24. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

Sol: if AB = BA

$(AB)^T = A^T B^T$

$\Rightarrow$ AB is symmetric

Statement-2 is true

$(ABA)^T = A^T B^T A^T$

Take A = I and B = some non – symmetric

$\therefore$ABA always

$\therefore$A(BA) and (AB)A are symmetric

Statement-1 is true nut does not depend on Statement-2

25. Ans: (1, 1)

Sol: $(1 + \omega)^7 = A + B\omega$

$(-\omega^2)^7 = A + B\omega$

$-\omega^{14} = A + B\omega$

$-\omega^2 = A + B\omega$

$1 + \omega = = A + B\omega$

$\therefore$ A = 1 B = 1

$\therefore$ (1, 1)

26. Ans: Statement-1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1.

Sol: $x_1 + x_2 + x_3 + x_4 = 6$

$x_i \geq 0$

no. of ways = ${}^9C_3$

S_2 is true

S_1 is true

S_1 follows from S_2

27. Ans: $\frac{3\sqrt{2}}{8}$

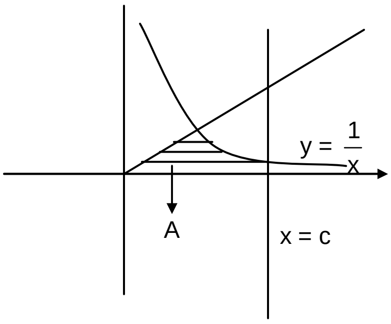
Sol: Slope of the line perpendicular to $y - x = 1$is (-1)

Hence t = 1

Point on the parabola corresponding to t = 1 is $\Rightarrow \left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ shortest distance = $\dfrac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}} = \dfrac{3\sqrt{2}}{8}$

28. Ans: $\frac{3}{2}$ square units



Sol: $y = x$

$y = \frac{1}{x} \Rightarrow x^2 = 1$

$\Rightarrow x = 1\ (x > 0)$

$y = \frac{1}{x}$, $x = e \Rightarrow x = e$

$\therefore$area A = $\int_1^e \left(x - \frac{1}{x}\right) dx$

$= \frac{e^2 - 1}{2} - \log e$

$= \frac{e^2 - 3}{2}$

Required area = $\frac{1}{2}.e^2 - \frac{e^2 - 3}{2} = \frac{3}{2}$

29. Ans: 7

Sol: $\frac{dy}{dx} = y + 3$

$\frac{dy}{y+3} = dx$

$\log (y + 3) = x + c$

$\therefore y + 3 = c\, e^x$

$x = 0 \;\; y = 2 \;\Rightarrow\; c = 5$

$\therefore y = 5\, e^x - 3$

$\therefore y (\log 2) = 5\, e^{\log 2} - 3$

$= 5 \times 2 - 3 = 7$

30. Ans: $\bar{c} - \frac{\bar{a} \cdot \bar{c}}{\bar{a}.\bar{b}}\bar{b}$

Sol: $\bar{b} \times \bar{c} = \bar{b} \times \bar{d}$

$\bar{a} \cdot \bar{d} = 0$

$\bar{b} \times \left(\bar{c} - \bar{d}\right) = 0$

$\bar{b}$ and $\left(\bar{c} - \bar{d}\right)$ are collinear

$\bar{b} = k\left(\bar{c} - \bar{d}\right)$

$\bar{a} \cdot \bar{b} = k\left(\bar{c} - \bar{c}\right) - \bar{a} \cdot \bar{d}]$

$k\ [\bar{c} - \bar{c}]$

$k = \frac{\bar{a} \cdot \bar{b}}{\bar{a} \cdot \bar{c}}$

$\bar{b}\ \bar{c} - \bar{d} = \frac{\bar{a}\bar{c}}{\bar{a} \cdot \bar{b}}\bar{b}$

$\bar{d} = \bar{c} - \frac{\bar{a} \cdot \bar{c}}{\bar{a}.\bar{b}}\bar{b}$

PART B – CHEMISTRY

31. Ans: Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series

Sol: All the lanthanoids does not exhibit +4 oxidation state.

32. Ans: A_2B_5

Sol: $A_1B_{5/2} = A_2B_5$

33. Ans: 2.82 BM

Sol: There are two unpaired electrons is $[NiCl_4]^{2-}$ hence the paramagnetic moment is 2.82 BM.

34. Ans: The complex is an outer orbital complex

Sol: $[Cr(NH_3)_6]Cl_3$ is not an outer orbital complex.

35. Ans: 32 times

Sol: 2 times increase for 10°C

2^5 = 32 times increase for 50°C

36. Ans: a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

Sol: 'a' is a measure of attraction between the molecules and 'b' the size of the molecules.

37. Ans: sp^2, sp, sp^3

Sol: $NO_3^- - sp^2$, $NO_2^+ -$ sp and $NH_4^+ - sp^3$

38. Ans: 804.32 g

Sol: $\Delta T_f = K_f \times \frac{W_2}{M_2} \times \frac{1}{W_1}$

$6 = 1.86 \times \frac{W_2}{62} \times \frac{1}{4}$

$W_2 = 800$ g

Wt. of glycol required is more than 800 g

39. Ans: $4f^7 5d^1 6s^2$

Sol: The outer electronic configuration of $_{64}Gd$ is $4f^7 5d^1 6s^2$

40. Ans: pentagonal bipyramid

Sol: IF_7 is pentagonal bipyramidal.

41. Ans: a vinyl group

Sol: Formation of HCHO in ozonolysis shows the presence of $CH_2 = CH -$ group.

42. Ans: $\alpha = \frac{i - 1}{(x + y - 1)}$

Sol: $i = 1 - \alpha + n\alpha$; $n = x + y$

$$\alpha = \frac{i - 1}{x + y - 1}$$

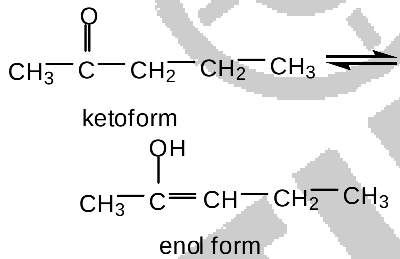
43. Ans: 743 nm

Sol: $\frac{1}{355} = \frac{1}{680} - \frac{1}{\lambda}$

$\lambda = 743$ nm

44. Ans: 2–Pentanone

Sol:



45. Ans: $38.3 \text{ J mol}^{-1} \text{ K}^{-1}$

Sol: $\Delta S = 2.303 \text{ nR} \log \frac{V_2}{V_1}$

$= 2.303 \times 2 \times 8.314 \times \log 10$

$= 38.3 \text{ J K}^{-1}$

46. Ans: Acetaldehyde

Sol: Acetaldehyde reduces tollen's reagent to metallic silver on warming.

47. Ans: 2, 2, 2–Trichloroethanol

Sol: $2Cl_3C - CHO + NaOH$
$\rightarrow Cl_3C - CH_2OH + Cl_3C - COONa$

48. Ans: $p(H_2) = 2$ atm and $[H^+] = 1.0$ M

Sol: $2H^+ + 2e^- \rightarrow H_2$

$$E_{Cl} = \frac{0.0591}{2} \log \frac{[H^+]^2}{[H_2]}$$

$[H_2] > [H^+]^2$

49. Ans: 2, 4, 6–Tribromophenol

Sol: Phenol forms 2, 4, 6–tribromophenol when treated with a mixture of KBr, $KBrO_3$ and HCl.

50. Ans: $AlCl_3$

Sol: Fajan's rules.
Al^{3+} is the smallest cation and it has high charge.

51. Ans: BF_6^{3-}

Sol: Boron cannot form BF_6^{3-} since boron has no available d–orbitals.

52. Ans: Ethyl ethanoate

Sol: $CH_3COCl + C_2H_5ONa$
$\rightarrow CH_3COOC_2H_5 + NaCl$

53. Ans: Neutral $FeCl_3$

Sol: Neutral $FeCl_3$ solution gives violet colour with phenol.

54. Ans: 1.8 atm

Sol: $CO_2 + C \rightarrow 2CO$
a – x   2x

a = 0.5 atm
a + x = 0.8 atm
x =0.3 atm

$$K_p = \frac{p^2_{CO}}{p_{CO_2}} = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$$

55. Ans: $CH_3CH_2CH(Cl)CO_2H$

Sol: Presence of Cl having –I effect on the α–carbon makes 2–chlorobutanoic acid the strongest acid among the given compounds.

56. Ans: $Al_2O_3 < MgO < Na_2O < K_2O$

Sol: K_2O is more basic than Na_2O. Al_2O_3 is amphoteric.

57. Ans: 0 .086

Sol: Mole fraction of methanol

$$= \frac{\text{moles of methanol}}{\text{total moles}} = \frac{5.2}{5.2 + \frac{1000}{18}}$$

= 0.086

58. Ans: 2^{nd}

Sol: RNA contains β–D–ribose while DNA contains β–D–2–deoxyribose.

59. Ans: The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.

Sol: Stability of hydrides decreases from NH_3 to BiH_3.

60. Ans: The oxidation state of sulphur is never less than +4 in its compounds

Sol: Sulphur exhibits oxidation state lower than +4 in its compounds.

PART – B – PHYSICS

61. Ans: 372 K and 310 K

Sol: $1-\frac{T_2}{T_1}=\frac{1}{6}$

$1-\frac{T_2-62}{T_1}=\frac{1}{3}$

$\frac{T_2}{T_1}=\frac{5}{6}$

$\frac{T_2-62}{T_1}=\frac{2}{3}$

$\frac{T_2}{T_2-62}=\frac{5}{4}$

$4T_2 = 5T_2 - 310$

$T_2 = 310$ K

$\Rightarrow T_1 = 372$ K

62. Ans: more than 3 but less than 6.

Sol: $\tau = Fr = 40t - 10t^2$

$\alpha = \frac{\tau}{I} = 4t - t^2$

$\frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = 2t^2 - \frac{t^3}{3}$

($\because$ At t = 0, ω = 0)

At t = 6 s. ω again become zero

$\omega = \frac{d\theta}{dt} = 2t^3 - \frac{t^3}{3} \Rightarrow \theta - \frac{2t^3}{3} - \frac{t^4}{12}$

$\therefore$ θ in 6 s = (144 – 108) = 36 rad

$\Rightarrow N = \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.72$rotation.

63. Ans: $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

Sol: $P_1V = n_1KT_1$

$P_2V = n_2KT_2$

$P_3V = n_3KT_3$

$\frac{1}{2}mv^2 = \frac{3}{2}KT_1 \times n_1 + \frac{3}{2}KT_2n_2 + \frac{3}{2}K - T_3n_3$

$= \frac{3}{2}K(n_1 + n_2 + n_3)T$

$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

64. Ans: 0.15 mV

Sol: $\mathcal{E} = B\lambda v$

$= 5 \times 10^{-5} \times 2 \times 1.50$

$= 0.15$ mV

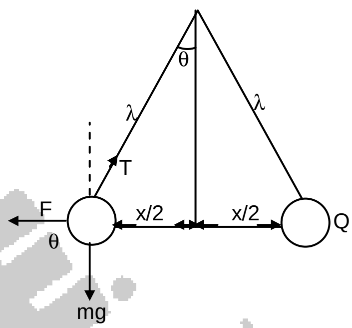
65. Ans: First increases and then decreases.

Sol: Angular momentum is conserved.

I decreases ω increases then I increases ω decreases.

66. Ans: $x^{-1/2} \propto v$

Sol:



$\tan\theta = \frac{x}{2\lambda}$ — (i)

$T\sin\theta = F = \frac{kQ^2}{x^2}$

$T\cos\theta = mg$

$\therefore \tan\theta = \frac{F}{mg} = \frac{kQ^2}{x^2mg}$ — (ii)

From (i) and (ii) $\frac{kQ^2}{x^2mg} = \frac{x}{2\lambda}$

$\Rightarrow Q^2 \propto x^3$ — (iii)

$\therefore 2Q\frac{dQ}{dt} \propto 3x^2\frac{dx}{dt}$

$\Rightarrow Q\frac{dQ}{dt} \propto x^2v \left(\because \frac{dx}{dt} = v\right)$

$\Rightarrow x^{3/2} \propto x^2v$

$\left(\because Q \propto x^{3/2} \text{ and } \frac{dQ}{dt} = \text{constant}\right)$

$\Rightarrow x^{-1/2} \propto v$

67. Ans: 8.4 kJ

Sol: $\Delta U = mC\Delta T$

$= 4184 \times 20 \times 0.1$

$= 8.4$ kJ

68. Ans: 20 min

Sol: $N = \frac{N_0}{2^{t/T_{1/2}}}$

$\frac{N_0}{3} = \frac{N_0}{2^{t_2/20}} \Rightarrow t_2 = 20\frac{\log 3}{\log 2}$

$N_0\frac{2}{3} = \frac{N_0}{2^{t_1/20}} \Rightarrow t_1 = \frac{20(\log 3 - \log 2)}{\log 2}$

$t_2 - t_1 = \frac{20}{\log 2}$ (log 3 – log 3 + log 2)

= 20 min

69. Ans: 108.8 eV

Sol: $\frac{13.6\,Z^2}{n^2} = 13.6\times 9\left[1-\frac{1}{9}\right]$

$= 13.6\times 9\times\frac{8}{9}$

= 108.8 eV

70. Ans: $-6\,\varepsilon_0 a$

Sol: $V = ar^2 + b$

$E = -\frac{dV}{dr} = -2ar$

$4\pi r^2.E = \frac{Q}{\varepsilon_0}$

$Q = -4\pi r^2.2ar.\varepsilon_0$

$\rho = \frac{-8\pi a r^3\varepsilon_0}{\frac{4}{3}\pi r^3}$

$= -6\,\varepsilon_0 a$

71. Ans: 0.4π mJ

Sol: $E = T.8\pi(r_2^2 - r_1^2)$

$= 8\pi T\left(\frac{25}{10^4} - \frac{9}{10^4}\right)$

$= 8\times 16\times\pi\times 0.03\times 10^{-4}$

$= 0.4\pi$ mJ

72. Ans: $2.7\times 10^6\ \Omega$

Sol: $V = V_0(1 - e^{-t/RC})$

$120 = 200(1 - e^{-t/RC})$

$e^{-t/RC} = \frac{2}{5}$

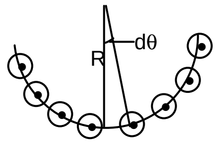
$e^{t/RC} = 2.5$

$\frac{t}{RC} = 0.4\times 2.5\times 2.303$

$\Rightarrow R = 2.7\times 10^6\ \Omega$

73. Ans: $\frac{\mu_0 I}{\pi^2 R}$

Sol: $B = \frac{I}{\pi R}Rd\theta\frac{\mu_0}{2\pi R}\sin\theta$



$= \frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi/2}\sin\theta d\theta$

$= \frac{\mu_0 I}{\pi^2 R}$

74. Ans: 2 s

Sol: $\frac{dv}{dt} = -2.5\sqrt{v}$

$\frac{dv}{\sqrt{v}} = -2.5\ dt$

$\Rightarrow -2.5t = \left|2\sqrt{v}\right|_{6.25}^{0}$

$t = \frac{2\sqrt{6.25}}{2.5}$

$= \frac{2\times 2.5}{2.5} = 2$

75. Ans: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

Sol: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

76. Ans: $\frac{-9\,Gm}{r}$

Sol: $\frac{Gm}{x^2} = \frac{G4m}{(r-x)^2}$

$\frac{(r-x)^2}{x^2} = 4$

$r - x = 2x$

$x = \frac{r}{3}$

$V = \frac{-Gm}{\frac{r}{3}} - \frac{G4m}{\frac{2r}{3}}$

$= -\frac{Gm}{r}(3+6)$

$= \frac{-9\,Gm}{r}$

77. Ans: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement – 1.

Sol: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement – 1.

78. Ans: $\frac{\pi}{4}\sqrt{LC}$

Sol: $q' = q_0 \cos\omega t$

$E = \frac{q_0^2}{2C}$

$\frac{E}{2} = \frac{1}{2}\frac{q_0^2}{2C}$

i.e. $q' = \frac{q_0}{\sqrt{2}}$

$\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$

$\Rightarrow \omega t = \frac{\pi}{4}$

$t = \frac{\pi}{4}\sqrt{LC}$

79. Ans: Statement – 1 is false, Statement-2 is true.

Sol: If $\upsilon \Rightarrow 2\upsilon$,
$V_0' > 2V_0$, well known result
$\Rightarrow$ Statement 1 is wrong.
Statement 2 is true.

80. Ans: 3.6×10^{-3} m

Sol: $P_0 + \frac{1}{2}\rho v_1^2 + \rho g h$

$= P_0 + \frac{1}{2}\rho v_2^2$

$\Rightarrow 2gh = (v_2^2 - v_1^2)$
$\Rightarrow 2gh + v_1^2 = v_2^2$;
$v_1 = 0.4$ m s^{-1}, $h_2 = 0.2$ m
$\Rightarrow v_2 = 2.0396$ m s^{-1}

$A_1v_1 = A_2v_2 \Rightarrow d_2^2 = \frac{d_1^2 v_1}{v_2}$

$\Rightarrow d_2 = d_1 . \sqrt{\frac{v_1}{v_2}}$

$= 8 \times 10^{-3} \times \sqrt{\frac{0.4}{2.0396}}$

$\cong 3.6 \times 10^{-3}$ m

81. Ans: $\left(\frac{M+m}{M}\right)^{1/2}$

Sol: $Mv_1 = (M + m)\, v_2$

$\frac{v_1}{v_2} = \frac{M+m}{M}$

$\frac{1}{2}(M+m)v_2^2 = \frac{1}{2}KA_2^2$

$\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$

$\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$

$\Rightarrow \frac{A_1^2}{A_2^2} = \frac{M}{M+m}\left(\frac{M+m}{M}\right)^2$

$= \frac{M+m}{M}$

$\therefore \frac{A_1}{A_2} = \left(\frac{M+m}{M}\right)^{1/2}$

82. Ans: $\frac{\pi}{3}$ rad

Sol: $x_1 = A \sin\omega t$
$x_2 = x_0 + A \sin(\omega t + \phi)$
$d = x_2 - x_1 = x_0 + A [\sin(\omega t + \phi) - \sin\omega t]$

$\therefore d = x_0 - 2A \cos\left(\omega t + \frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)$ —(i)

d is maximum when $\cos\left(\omega t + \frac{\phi}{2}\right) = -1$

Given $d_{(max)} = x_0 + A$

$\therefore$ (i) $\Rightarrow x_0 + A = x_0 - 2A \times -1. \sin\frac{\phi}{2}$

$= x_0 + 2A \sin\frac{\phi}{2}$

$\Rightarrow \frac{1}{2} = \sin\frac{\phi}{2}$ or $\frac{\phi}{2} = \frac{\pi}{6}$ rad

$\Rightarrow \phi = \frac{\pi}{3}$ rad

83. Ans: Increases by 0.2%

Sol: $R \propto \lambda^2$
$R' \propto \lambda'^2$
$\propto (1.001)^2\lambda^2$

$\frac{\Delta R}{R} = 0.002$

$\therefore 0.002 \times 100$
$= 0.2\%$

84. Ans: $\frac{\pi v^4}{g^2}$

Sol: $R_{max} = \frac{v^2}{g}$

Area $= \pi(R_{max})^2$

$= \frac{\pi v^4}{g^2}$

85. Ans: $\frac{1}{2}\frac{Mv^2(\gamma - 1)}{R}$

Sol: Volume is constant

$C_v = \frac{R}{(\gamma - 1)}$

$KE = \frac{1}{2}Mv^2$

$\Delta Q = nC_v\,\Delta\theta = 1 \times C_v\,\Delta\theta$

$\therefore \Delta\theta = \frac{KE}{C_v} = \frac{1}{2}\frac{Mv^2(\gamma - 1)}{R}$

86. Ans: 0.052 cm

Sol: $LC = \frac{1}{100} = 0.01$ mm

Reading = PSR × pitch + CSR × LC

= 0 + 52 × 0.01

= 0.52 mm

= 0.052 cm

87. Ans: $\frac{2}{3}g$

Sol: $mg - T = ma$

$TR = \frac{mR^2}{2}.\frac{a}{R}$

$\Rightarrow mg = \frac{3}{2}ma$

$\Rightarrow a = \frac{2}{3}g$

88. Ans: Wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

Sol: $y(x, t) = e^{-\left(\sqrt{a}x + \sqrt{b}t\right)^2}$

This is of the form y(x, t) = f(x + vt), where

$v = \frac{\sqrt{b}}{\sqrt{a}}$ travels in negative x direction.

89. Ans: $\frac{1}{15^2} \times 15 = \frac{1}{15}$ m s^{-1}

Sol: $\frac{1}{v} + \frac{1}{-2.8} = \frac{1}{0.2}$

$\Rightarrow \frac{1}{v} = \frac{15}{2.8}$

$v = \frac{2.8}{15}$

$\frac{v}{u} = \frac{1}{15}$

$\frac{v^2}{u^2} = \frac{1}{15^2}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{\frac{dv}{dt}}{\frac{du}{dt}} = -\frac{v^2}{u^2}$

$\left|\frac{dv}{dt}\right| = \frac{v^2}{u^2}.\frac{du}{dt}$

$= \frac{1}{15^2} \times 15 = \frac{1}{15}$ m s^{-1}

90. Ans: 45°

Sol: $\mu_1\left[\hat{N} \times K_1\right] = \mu_2\left[\hat{N} \times K_2\right]$. But plane of separation need to be XY.