T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE 2011

Paper I – Code-0

1	2	3	4	5	6	7
A	**D**	**A**	**B**	**D**	**C**	**C**

8	9	10	11
A, B, D	**A, B, C, D**	**A, C, D**	**B, C**

12	13	14	15	16
D	**B**	**B**	**A**	**C**

17	18	19	20	21	22	23
9	**5**	**5**	**4**	**6**	**5**	**7**

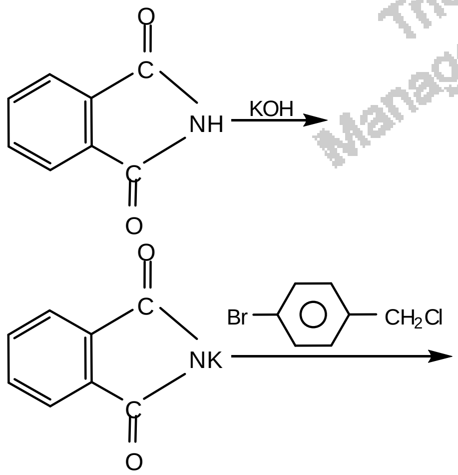
Section I

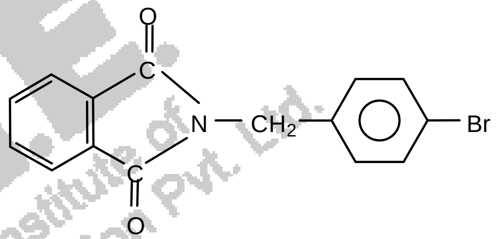
1. $^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{15}P + ^{1}_{0}n(Y)$

   $\downarrow$ $\qquad\qquad\qquad\qquad \downarrow$

   $^{30}_{14}Si + ^{1}_{1}H(X) \quad ^{30}_{14}Si + ^{0}_{1}e(Z)$

2. Since the mobility of K^+ is nearly same as that of Ag^+ which it replaces, the conductance will remain as more or less constant and will increase only after the end point.

3. 



4. $[NiCl_4]^{2-}$ is tetrahedral, $[Ni(CN)_4]^{2-}$ is square planar & $[Ni(H_2O)_6]^{2+}$ is octahedral.

5. $Ba(N_3)_2 \rightarrow Ba + 3N_2$
   Very pure N_2 is produced by the thermal decomposition of Barium or sodium azide.

6. $M = \dfrac{2 \times 1.15 \times 1000}{1120}$
   $= 2.05$ M

7. o-hydroxy benzoic acid: pKa = 2.99
   p–hydroxy benzoic acid: 4.58
   p–toluic acid = 4.34
   p–nitrophenol = 7.15

Section II

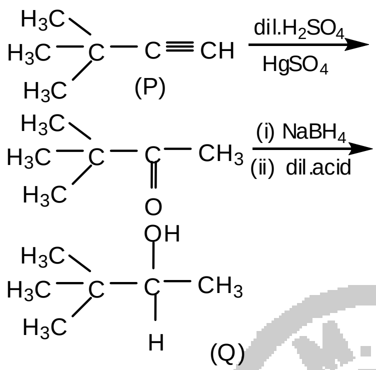
8. Adsorption is always exothermic.
   Chemisorption is more exothermic than physisoprtion and it requires activation energy.

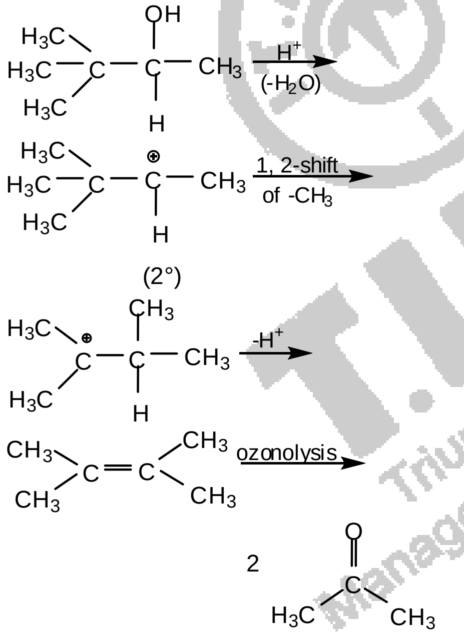
9. All the options A, B, C and D are the postulates of kinetic theory of gases and its extension.

10. Cassiterite contain 0.5 – 10% of metal as SnO_2 the rest being impurities of pyrites of Fe, Cu & wulframite.

11. In B & C all atoms are in the same plane.

Section III

12. 

13. 

14. $2AgNO_3 + Cu \rightarrow Cu(NO_3)_2 + \frac{1}{2}Ag$

The metal dipped is Cu.
Blue colour due to formation of Cu^{2+}

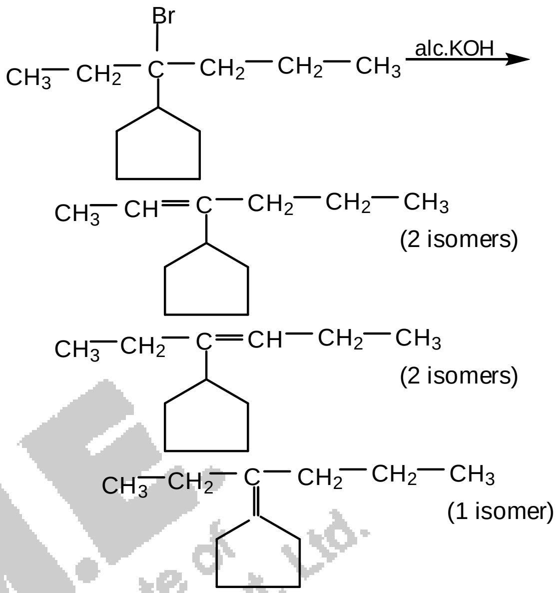
15. The solution in which the metal is dipped in $AgNO_3$.

16. The deep blue colour due to formation of $[Cu(NH_3)_4]^{2+}$ and the white precipitate of AgCl dissolve due to formation of $[Ag(NH_3)_2]^+$

Section IV

17. Maximum no. of electron with n = 3 is 18, of which are having $-\frac{1}{2}$ spin

18. $3Na_2CO_3 + 3Br_2 \rightarrow 5NaBr + NaBrO_3 + 3CO_2$

19. 

20. $h\upsilon = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9} \times 1.6 \times 10^{-19}}$

$= 4.14$ eV

$E = h\upsilon - h\upsilon_0$
For Li, Na, K and Mg $h\upsilon_0$ values are less than 4.14 eV

21. Let the number of glycine units be x
Total mass of the hydrolysed products
$= 796 + 9 \times 18 = 958$
% by mass of glycine $= \frac{75x \times 100}{958} = 47$
$\therefore$ x = 6

22. $S_4O_6^{2-}$ has sulphur atoms with oxidation states '0' & +5.

23. Vol. of 0.1 mole at 0.32 atm $= \frac{2.24}{0.32} = 7$ L

PART II

24	25	26	27	28	29	30
C	**D**	**D**	**A**	**A**	**A**	**B**

31	32	33	34
B, D	**A, B, C, D**	**A, B, C, D**	**A, D**

35	36	37	38	39
C	**B**	**D**	**C**	**B**

40	41	42	43	44	45	46
3	**5**	**6**	**3**	**9**	**4**	**1**

Section I

24. x component of area vector is a^2 (The given surface is a rectangle and not a square). Hence flux = E_0a^2.

25. $T\sin\theta = m\omega^2 L\sin\theta$

$\omega = \sqrt{\dfrac{324}{0.5\times 0.5}}$

$= 36$

26. Position (1): Let charge on 2 μF be Q. Then energy = $\dfrac{Q^2}{4}$ μJ

Position (2): C_{eq} = 10 μF

Total charge = Q. ∴ Energy = $\dfrac{Q^2}{20}$ μJ

Loss % = 80%

27. $n = \dfrac{1}{4}$ (∵ 22.4 ℓ = 1 mole)

$W = nC_V\Delta T = \dfrac{1}{4}\times\dfrac{3}{2}R.\Delta T$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow$

$T_2 = 4T_1$ ∴ $\Delta T = 3T_1$

28. $f' = \left(\dfrac{f}{1-\dfrac{v_s}{c}}\right)\times\left(1+\dfrac{v_0}{c}\right)$

$= \dfrac{8\times(320+10)}{320-10}$

29. $\dfrac{1}{\lambda_1} = R\left(\dfrac{1}{2^2}-\dfrac{1}{3^2}\right)$

$\dfrac{1}{\lambda_2} = Z^2R\left(\dfrac{1}{2^2}-\dfrac{1}{4^2}\right)$

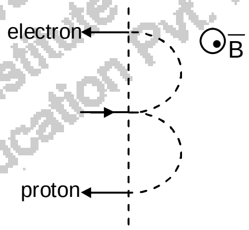
$\lambda_2 = \lambda_1 \times \dfrac{\frac{5}{36}}{\frac{3}{16}} \times 2^2$

$= 1215\ \overset{0}{A}$

30. $\dfrac{X}{10} = \dfrac{52+1}{48+2}$

Section II

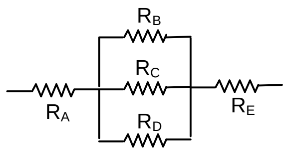
31.



$T = \dfrac{2\pi m}{qB}$, different for them.

32. $\dfrac{dQ}{dt}$ is same for A and E and both are maximum.

Thermal resistances $\left(\dfrac{\ell}{KA}\right)$ are as below.



$R_A = \dfrac{1}{8}$, $R_B = \dfrac{4}{3}$, $R_C = \dfrac{1}{2}$, $R_D = \dfrac{4}{5}$

$R_E = \dfrac{1}{24}$. So C is also correct.

(∵ Eq.R (R_C, R_B, R_D) is $\dfrac{1}{4}$.

D also correct $\because$ R_B parallel
R_D is equal R_C

Note: It is assumed that there is no radiation loss.

33. On interconnecting V same

$\Rightarrow \frac{Q_A}{R_A} = \frac{Q_B}{R_B}$ $\therefore$ (B) correct.

$V = \frac{\sigma R}{\varepsilon_0}$ is standard formula

$\therefore$ (C) correct

V same $\Rightarrow E_A = \frac{V}{R_A}$, $E_B = \frac{V}{R_B}$

$\therefore$ (D) correct.

34. In case (A)

$$mg\left(\frac{\ell}{2}\right)\sin\theta + Mg\ell\sin\theta = \left[\frac{m\ell^2}{3} + \frac{MR}{2} + M\ell^2\right]\alpha_A$$

In case (B)

$$mg\left(\frac{\ell}{2}\right)\sin\theta + Mg\ell\sin\theta = \left[\frac{m\ell^2}{3} + M\ell^2\right]\alpha_B$$

Hence A, D

Section III

35. $[N] = L^{-3}\left[\frac{e^2}{\varepsilon_0}\right]$ = Force $\times$ area

36. Find ω by the given formula

$$\lambda = \frac{c}{\frac{\omega}{2\pi}}$$

37. As ball goes up, x is positive and increasing, v is positive and decreasing. Symmetrical for ball coming down.

38. At x = 0, E = K.E = $\frac{p^2}{2m}$

$\therefore$ E varies as p^2

$\therefore E_1 = 4E_2$

39. Starting from positive positions (i.e. in air), amplitude in air will be more than amplitude in water. Momentum is negative for downward journey. Also water produces damping.

$\Rightarrow$ graph is spiralling in.

Section IV

40. $U = \frac{q^2}{4\pi\varepsilon_0 a}\left[4+\sqrt{2}\right] + 2a^2r$

For equilibrium, $\frac{dU}{da} = 0$

$\Rightarrow a = \left(\frac{q^2}{r}\right)^{1/3}\left(\frac{4+\sqrt{2}}{16\pi\varepsilon_0}\right)$

$\Rightarrow$ N = 3

41. $F_d = mg\sin\theta - \mu mg\cos\theta$

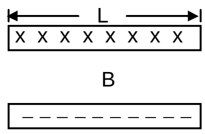
$F_u = mg\sin\theta + \mu mg\cos\theta$

$F_u = 3F_d$

$\Rightarrow \mu = \frac{1}{2}$

$\therefore N = 10\,\mu = 10 \times \frac{1}{2} = 5$

42.



$B = \frac{\mu_0 I}{L}$

$\phi_{coil} = B\pi r^2 = \frac{\mu_0 \pi r^2 g I}{L}$

$-\frac{d\phi}{dt} = E$, $i = \frac{E}{R}$; $M = i\pi r^2$

$\Rightarrow$ N = 6

43. $\Delta\ell = \frac{MgL}{AY}$

$L' = L + \Delta\ell$

$\Rightarrow \frac{L'}{L} = \left(1 + \frac{Mg}{AY}\right)$

$$\alpha = \frac{L' - L}{(L \times 40 - L' \times 30)}$$

$$= \frac{Mg}{AY\left[40 - \left(1 + \frac{Mg}{AY}\right)30\right]}$$

Solving, we get M $\cong$ 3 kg

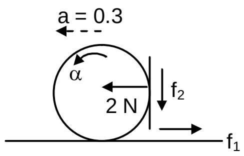
44. $N \times 10^{-4} = 2\left[\frac{2}{5}mr^2 + md^2\right] + 2\left[\frac{2}{5}mr^2\right]$;

Here $d = \frac{4\sqrt{2}}{2} \times 10^{-2} = 2\sqrt{2} \times 10^{-2}$ m

$r = \frac{\sqrt{5}}{2} \times 10^{-2}$ m; $m = \frac{1}{2}$ kg

$\Rightarrow$ N = 9

45.



$\alpha = \dfrac{f_1R - f_2R}{mR^2}$, $\alpha = \dfrac{a}{R}$

$\Rightarrow \dfrac{f_1 - f_2}{m} = a = 0.3 = \dfrac{2 - f_1}{m}$, m = 2

$\Rightarrow f_1 = 1.4$, $f_2 = 0.8 \Rightarrow \mu.2 = 0.8 \Rightarrow \mu = 0.4$

P = 4

46. $A_0 = |-\lambda N_0| = 10^{10}\ s^{-1}$

$\therefore N_0 = \dfrac{10^{10}}{\lambda}$

$= 10^{10} \times 10^{9} = 10^{19}$

$\therefore M = mN_0 = 10^{-25} \times 10^{19}$ kg

$= 10^{-25} \times 10^{19} \times 10^{6}$ mg

= 1 mg

PART III

47	48	49	50	51	52	53
C	**B**	**B**	**A**	**C**	**D**	**C**

54	55	56	57
QUESTION INCORRECT	**B,D**	**A,D**	**B,C/ B, C, D**

58	59	60	61	62
B	**D**	**D**	**A**	**B**

63	64	65	66	67	68	69
7	**9/ 3/ 3 & 9**	**8**	**QUESTION INCORRECT**	**1**	**2**	**5**

Section I

47. Let $\vec{v} = A\bar{i} + B\bar{j} + C\bar{k}$

$\bar{a} \times \bar{b} . \bar{v} = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ A & B & C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 0 & 2 \\ 1 & -1 & 1 \\ A & B & C \end{vmatrix} = 0$$

$\Rightarrow 2\,(-C - B) + 2\,(B + A) = 0$

$\Rightarrow A = C$

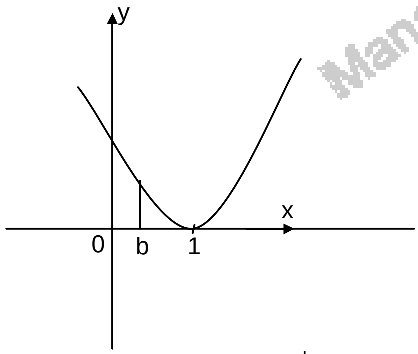
For the vector in (C),

A = C

Projection of $\left(3\bar{i} - \bar{j} + 3\bar{k}\right)$ on $\bar{C}$

$= \dfrac{3+1-3}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

48.



$$R_1 = \int_0^b (x-1)^2 dx = \left[\frac{(x-1)^3}{3}\right]_0^b$$

$$= \frac{(b-1)^3}{3} + \frac{1}{3}$$

$$R_2 = \int_b^1 (x-1)^2 dx = \left[\frac{(x-1)^3}{3}\right]_b^1$$

$R_1 - R_2 = \dfrac{1}{4}$ gives $= 0 - \left\{\dfrac{(b-1)^3}{3}\right\}$

$$\frac{2(b-1)^3}{3} + \frac{1}{3} = \frac{1}{4}$$

$\Rightarrow b = \dfrac{1}{2}$ satisfies the above equation.

49. $y + 2 = m\,(x - 3)$

and $m = \sqrt{3}$

$y + 2 = \sqrt{3}\,(x - 3)$

$y - \sqrt{3}\,x + 2 + 3\sqrt{3} = 0$

50. Let $I = \displaystyle\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)}\,dx$

Set $x = \sqrt{t}$

$dx = \dfrac{1}{2\sqrt{t}}\,dt$

$x = \sqrt{\ln 2}$, $t = \ln 2$

$x = \sqrt{\ln 3}$, $t = \ln 3$

$$\Rightarrow I = \int_{\ln 2}^{\ln 3} \frac{\sqrt{t}\,\sin t}{\sin t + \sin(\ln 6 - t)} \times \frac{1}{2\sqrt{t}}\,dt$$

$$= \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t\,dt}{\sin t + \sin(\ln 6 - t)}$$

$$= \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t}\,dt$$

$$2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} dt$$

$$= \frac{1}{2}\log\frac{3}{2}$$

$$I = \frac{1}{4}\log 3/2$$

51. $\frac{\log x}{\log y} = \frac{\log 2}{\log 3}$

$\Rightarrow \log x = k \log 2 \Rightarrow x = 2^k$

$\log y = k \log 3 \Rightarrow y = 3^k$

$\left(2^{k+1}\right)\log 2 = \left(3^{k+1}\right)\log 3$

$2^{\log 2^{k+1}} = 3^{\log 3^{k+1}}$

$\Rightarrow (k+1)(\log 2)^2 = (k+1)(\log 3)^2$

$\Rightarrow k = -1$

$\Rightarrow x_0 = 2^{-1} = 1/2$

52. $P = \left\{\theta : \sin\left(\theta - \frac{\pi}{4}\right) = \cos\theta\right\}$

$\Rightarrow P = \left\{\theta : \cos\theta = \cos\left(\frac{3\pi}{4} - \theta\right)\right\}$

$\therefore P = \left\{\theta : \theta = 2n\pi \pm \left(\frac{3\pi}{4} - \theta\right)\right\}$

$= \left\{\theta : \theta = n\pi + \frac{3\pi}{8}\right\}$

$Q = \left\{\theta : \theta - \frac{\pi}{4} = 2n\pi \pm \left(\frac{\pi}{2} - \theta\right)\right\}$

$= \left\{\theta : \theta = n\pi + \frac{3\pi}{8}\right\}$

$\therefore P = Q$

53. $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2\left(\alpha^8 - \beta^8\right)}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$

$= \frac{\alpha^8 ,6\alpha - \beta^8\, 6\beta}{2\left(\alpha^9 - \beta^9\right)}$

$= \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)} = 3$

Section II

54. Skew symmetric matrix of order 3 is <u>singular.</u> Its inverse does not exist. Therefore there is **NO SOLUTION TO THIS QUESTION.**

But if

The question is reframed in the following manner;

" Let M and N be two nonsingular skew-symmetric matrices………..."

<u>The solution is</u>

$M^2 N^2 (M^{-1} N)^{-1} (MN^{-1})^T$

$= M^2 N^2 (-MN)^{-1} (N^{-1})^T M^T$

$= M^2 N^2 (-N^{-1} M^{-1})^{-1} (N^T)^{-1} (-M)$

$= M^2 N^2 N^{-1} M^{-1} (-N)^{-1} M$

$= - M^2 N^2 N^{-1} M^{-1} N^{-1} M = - M^2 N M^{-1} N^{-1} M$

$= - M^2 N (N M)^{-1} M$

$= - M^2 N(M N)^{-1} M$

$= - M^2 N N^{-1} M^{-1} M$

$= - M^2$

Choice (c)

55. For $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$e = \frac{\sqrt{3}}{2}$ and focus = $(\pm\sqrt{3}, 0)$

$\therefore$ for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, e = \frac{2}{\sqrt{3}}$

$\Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$

Substitution $\left(\sqrt{3}, 0\right)$, $a^2 = 3$, $b^2 = 1$

Focus (2, 0)

$\therefore$ required hyperbola is $\frac{x^2}{3} - y^2 = 1$

56. $j - k = i + j + k — (i + j + 2k)$

$\&\ k - j = i + j + 2k — (i + 2j + k)$

$\therefore$ $j - k$, $k - j$ coplanar with the given vectors

and $\pm\begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \pm\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 3 \\ 0 & 1 & -1 \end{vmatrix} \neq 0$

(A) and (D) true.

57. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(x) = kx$

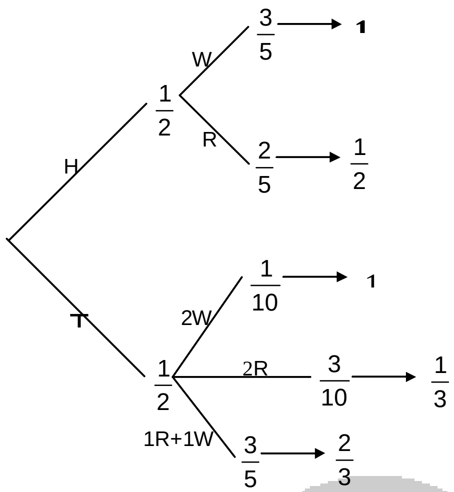
$f(x)$ is continuous $\forall\, x \in R$

and $f'(x) = k$.

* IIT-JEE has recommended option (D) also. The function is NOT differentiable at 'zero' points, which can be expressed as 'finitely many' points.

Section III

For problems (58) and (59)



58. Probability

$= \frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}+\frac{3}{10}\times 1+\frac{1}{10}\times\frac{1}{3}+\frac{3}{5}\times\frac{2}{3}\right]$

$= \frac{1}{2}\left[\frac{3}{5}+\frac{1}{5}+\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right]$

$= \frac{1}{2}\times\frac{46}{30}=\frac{23}{30}$

59. Probability $= \dfrac{\frac{1}{2}\left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{23}{30}}$

$= \frac{12}{23}$

60. 7(a + b+ c) = 0 , $\Rightarrow$ a + b + c = 0

also 2a + b + c = 1

$\Rightarrow$ a = 1, b + c = –1

$\therefore$ 7a + b + c = 7 – 1 = 6

61. $x^3 - 1 = 0$ and Im(ω) > 0 $\Rightarrow \omega = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$

a = 2 $\Rightarrow$ 2 + 8b + 7c = 0

14 + 7b + 7c = 0

(taking first and third columns)

Solving we get b = 12 and c = – 14

$\therefore$ The equation in $\frac{3}{\omega^2}+\frac{1}{\omega^{12}}+\frac{3}{\omega^{-14}}$

$= 3\omega + 1 + 3\omega^2$

$= 3(\omega + \omega^2) + 1 = -2$

Correct choice (a)

62. b = 6. Taking 1st and 3rd columns we get

a + 48 + 7c = 0

a + 6 + c = 0

Solving a = 1 and c = - 7.

$\therefore$ The equation is $x^2 + 6x - 7 = 0$

$\therefore \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{6}{7}<1$

$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ (is an infinite geometric progression)

$= \frac{a}{1-r}=\frac{1}{1-\frac{6}{7}}=7$

Section IV

63. $\theta = \frac{\pi}{n}$

$\frac{1}{\sin\theta}=\frac{1}{\sin 2\theta}+\frac{1}{\sin 3\theta}$

$\frac{1}{\sin\theta}-\frac{1}{\sin 3\theta}=\frac{1}{\sin 2\theta}$

$\frac{\sin 3\theta-\sin\theta}{\sin\theta\sin 3\theta}=\frac{1}{\sin 2\theta}$

$\frac{2\cos 2\theta}{\sin 3\theta}=\frac{1}{\sin 2\theta}$

$\Rightarrow 2\sin 2\theta\cos 2\theta = \sin 3\theta$

$\sin 4\theta - \sin 3\theta = 0$

$\Rightarrow 2\cos\frac{7\theta}{2}\sin\frac{\theta}{2}=0$

$\Rightarrow 2\cos\frac{7\pi}{2n}\sin\frac{\pi}{2n}=0$

$\frac{7\pi}{2n}=\left((2k+1)\frac{\pi}{2}\right)$

$\Rightarrow n = \frac{7}{2k+1}$

For positive integral values of n

k = 0 or 3

put k = 0, n = 7

64. m = 5n; $\frac{s_m}{s_n}=\dfrac{\frac{5n}{2}\left[6+(5n-1)(a_2-3)\right]}{\frac{n}{2}\left[6+(n-1)(a_2-3)\right]}$

$= 5\left[\frac{9-a_2+5n(a_2-3)}{9-a_2+n(a_2-3)}\right]$

$\therefore \quad \frac{9-a_2+5(a_2-3)}{9-a_2+a_2-3}=\frac{(9-a_2)+10(a_2-3)}{9-a_2+2(a_2-3)}$, is independent of n

$\frac{4a_2-6}{6}=\frac{9a_2-21}{a_2+3}$

$(2a_2 - 3)(a_2 + 3) = (3a_2 - 7)$

$2a_2^2 - 24a_2 + 54 = 0$

$a_2^2 - 12a_2 + 27 = 0$

$a_2 \neq 3$, $a_2 = 9$.
But if we take d = 0, then $a_2 = 3$.
So IIT-JEE has recommended the key to be either 3 or 9 or 3 and 9.

65. $\dfrac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+1+a^{8}+a^{10}}{8}$

$$\geq \left(a^{-5}a^{-4}\left(a^{-3}\right)^3 1\, a^8a^{10}\right)^{\frac{1}{8}} = 1$$

$\Rightarrow$ minimum value of sum = 8.

66. $6\int_1^x f(t)dt = 3x\,f(x) - x^3$

At x = 1, 0 = 3 f(1) – 1 = 5, initial condition given in the problem is NOT satisfied.
Without considering the initial condition, we may proceed as follows:
$6f(x) = 3xf'(x) + 3f(x) - 3x^2$
$f(x) = xf'(x) - x^2$
$y = x\dfrac{dy}{dx} - x^2$
$x\dfrac{dy}{dx} - y = x^2 \Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = x$
$I.F = \int_e -\dfrac{1}{x}dx$
$\therefore \quad y.\dfrac{1}{x} = \int x\dfrac{1}{x}dx = x + c$
$y = x(x + c) = f(x)$
$f(1) = 2 \Rightarrow 2 = 1 + c \Rightarrow c = 1$
$\therefore f(x) = x(x + 1) \Rightarrow f(2) = 6$

67. $f(\theta) = \sin\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)$

$$= \frac{\tan\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)}{\sqrt{1+\tan^2(\tan^{-1}\frac{\sin\theta}{\sqrt{\cos 2\theta}}}}$$

$$= \frac{\frac{\sin\theta}{\sqrt{\cos 2\theta}}}{\sqrt{\frac{\cos 2\theta + \sin^2\theta}{\cos 2\theta}}} = \frac{\sin\theta}{\sqrt{\cos^2\theta - \sin^2\theta + \sin^2\theta}}$$

$$= \tan\theta$$

$\Rightarrow \dfrac{d(f(\theta))}{d(\tan\theta)} = 1$

68. Extremum point of latus rectum are (2,4) and (2,–4)
$\therefore$ Area of triangle so formed with $\left(\dfrac{1}{2}, 2\right)$

$\Delta_1 = \dfrac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$

$= \dfrac{1}{8.2}(4+4)(-4-2)(2-4) = 6$

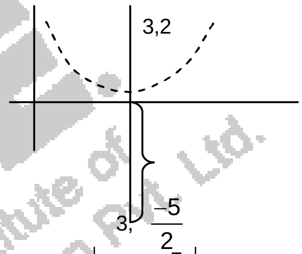
Eqn. of tangent at (2,4) is y = x +2 ________(1)
Eqn. of tangent at (2, –4) is –y = x +2 ______(2)
Eqn. of tangent at $\left(\dfrac{1}{2}, 2\right)$ is y = 2x +1 ______(3)

Tangent at the extremities of latus rectum intersect directrix at (–2, 0).
Point of intersection of (1) and (2) is (1,3) and of (2) and (3) is (–1, –1)

$$\Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & -2 & 0 \\ 1 & 1 & 3 \\ 1 & -1 & -1 \end{vmatrix} = |-3| = 3$$

$\therefore \dfrac{\Delta_1}{\Delta_2} = \dfrac{6}{3} = 2$

69.



minimum of $2\left|z - 3 + \dfrac{5}{2}i\right|$

$= 2\left|\left(\dfrac{5}{2} + 2\right) - 2\right| = 5$