SOLUTION & ANSWER FOR CUSAT-2009
VERSION – A

[MATHEMATICS]

1. If $\frac{dy}{dx} = \infty$ at a point on the curve y = f(x), then

Ans: the normal at P to the curve y = f(x) is parallel to x – axis. and the tangent at P to the curve y = f(x) is parallel to y – axis.

Sol. : $\frac{dx}{dy} = 0$

$\therefore$ normal is parallel to x – axis and tangent is parallel to y – axis.

2. The slope of the tangent to the curve …..

Ans: 0

Sol. : $y = \tan^{-1}x + \tan^{-1}\frac{1}{x}$

$= \tan^{-1}x + \cot^{-1}x$ or $\tan^{-1}x + \cot^{-1}x - \pi$

$= \frac{\pi}{2}$. or $\frac{-\pi}{2}$

$\therefore \frac{dy}{dx} = 0$

$\therefore$ slope of tangent = 0.

3. The curves $y^2 = x$ and $x^2 = 4y$ intersect…..

Ans: two points

Sol.: $y^2 = x$, $x^2 = 4y$

$\Rightarrow y^4 = 4y$

which has 2 real roots

Hence intersect at 2 points.

4. The function (1 – cosx) is increasing …..

Ans: $(0, \pi)$

Sol: $\frac{d}{dx}(1 - \cos x) = \sin x$, is positive in $(0, \pi)$

5. The curve $(y - 5)^2 = 12(x - 3)$ is symmetrical ….

Ans: y – 5 = 0

Sol: The equation is

$Y^2 = 12X$ if Y = y – 5

$\therefore$ symmetric about Y = 0

6. The equation of the ellipse, whose foci are at…

Ans: $\frac{x^2}{144} + \frac{y^2}{128} = 1$

Sol: Equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

ae = 4, e = $\frac{1}{3}$ $\therefore$ a = 12

$\therefore b^2 = a^2 - a^2e^2$

= 144 – 16 = 128

7. If x + iy = $\sqrt{\frac{a+ib}{c+id}}$, then $(x^2 + y^2)$ is

Ans: $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Sol: $x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$

$\Rightarrow x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$.

8. When a complex number is multiplied by …

Ans: No answer

Sol: When a complex number is multiplied by –1 = cosπ + isinπ, the argument gets increased by 180°.

9. The least value of n, for whoch $[(1+i)/(1-i)]^n$ …

Ans: 4

Sol: $\left(\frac{1+i}{1-i}\right)^n = (i)^n = 1$

$\Rightarrow$ n is a multiple of 4

The least **positive** value is 4.

(∗ **"positive" is left out.)**

10. If α and β are the roots of $x^2 - 2x + 4 = 0$…

Ans: $2^{n+1}\cos\frac{n\pi}{3}$

Sol: Roots are $1 \pm i\sqrt{3} = 2\left(\text{Cis} \pm \frac{\pi}{3}\right)$

$\therefore \alpha^n + \beta^n = 2^n\left(2\cos\frac{n\pi}{3}\right)$.

11. Given $\vec{A}, \vec{B}, \vec{C}$ are three non zero, non-coplanar vectors…..

Ans: $m = n = p = 0$

Sol: Conceptual

12. Given $\vec{a} = \vec{i} + \vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$ and …..

Ans: $\vec{k}$

Sol: $\vec{a} + \vec{b} = 3\hat{j}$ and $\vec{b} + \vec{c} = -2\hat{i} + 4\hat{j}$
So required vector from options is $\vec{k}$.

13. Any vector can be expressed in terms of

Ans: any three non coplanar vectors.

Sol: Conceptual

14. The angle between the planes $4x - 6y + 2z = 3$…

Ans: $\frac{\pi}{2}$

Sol: $\cos\theta = \pm\frac{24 - 30 + 6}{\sqrt{4^2 + 6^2 + 2^2}\sqrt{6^2 + 5^2 + 3^2}} = 0$

$\theta = \frac{\pi}{2}$

15. The area between the curve $y = x^2$ and the …

Ans: $\frac{16}{3}$

Sol: Area = $\int_0^4 x\,dy = \int_0^4 \sqrt{y}\,dy = \frac{16}{3}$

16. $y = ae^x - be^{-x}$ is a solution of the differential …

Ans: $y'' - y = 0$

Sol: Differentiating twice, $y'' = y$ or
$y'' - y = 0$

17. The differential equation of all circles, which …

Ans: $y^2 - x^2 - 2xyy' = 0$

Sol: Let the circle be $x^2 + y^2 = \lambda x$
Differentiating, $2x + 2yy' = \lambda$
substituting and eliminating λ,
$x^2 - y^2 + 2xyy' = 0$

18. The differential equation formed by eliminating ..

Ans: $y_2 + 2y_1 + y = 0$

Sol: Differentiating given function twice and eliminating a and b, $y_2 + 2y_1 + y = 0$

19. With respect to multiplication, the set {0, 1, –1}..

Ans: existence of inverse

Sol: conceptual.
Inverse of 0 does not exist.

20. Which of the following is true ?

Ans: division is a binary operation in R – {0}

Sol: conceptual

21. In the group {(1, –1, i, –i), x} the order of ….

Ans: 4

Sol. : we find $(-i)^4 = 1$, which is the identity element of the group and $(-i)^n \neq 1$ for n = 1, 2, 3

22. If $\omega = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$ and G = ……

Ans: an abelian group

Sol. : n^{th} roots of unity form an abelian group under multiplication.

23. Given $f_1(x) = x$, $f_2(x) = -x$ ……

Ans: $f_3(x)$.

Sol.: $(f_4 \circ f_2)(x) = f_4(-x) = \frac{1}{x} = f_3(x)$.

24. Which statement is true, given H is a …..

Ans: inverse of a ∈ H is the same as the inverse of a ∈ G.

Sol: conceptual

25. $G = \{8^n \mid n \in Z\}$ is cyclic…..

Ans: 8 and $\frac{1}{8}$

Sol: conceptual, from options generators are 8 and $\frac{1}{8}$

26. $\Delta = \begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix}$ has the following ….

Ans: $m(x - a)$ and $(x - b)$

Sol: $\Delta = m\begin{vmatrix} a & a & x \\ 1 & 1 & 1 \\ b & x & b \end{vmatrix} = m\begin{vmatrix} a & 0 & x-a \\ 1 & 0 & 0 \\ b & x-b & b-x \end{vmatrix}$

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$

$= m(x - a)(x - b)$.

27. The number of permutations of the letter..

Ans: $\frac{9!}{2(4!)}$

Sol: No. of permutations = $\frac{9!}{2!4!} = \frac{9!}{2(4!)}$
since four A's and 2L's are there.

28. The rank of the matrix $\begin{pmatrix} 1 & -1 & i \\ -1 & i & 1 \\ i & 1 & -1 \end{pmatrix}$ is

Ans: 3

Sol: Determinant of the given matrix is non-zero. Hence there is a non-zero minor of order 3
Rank = 3.

29. The value of $\begin{vmatrix} \cos 10° & \sin 10° \\ -\cos 50° & \sin 50° \end{vmatrix}$ is

Ans: $\frac{\sqrt{3}}{2}$

Sol: $\begin{vmatrix} \cos 10° & \sin 10° \\ -\cos 50° & \sin 50° \end{vmatrix} = \sin(50° + 10°)$

$= \sin 60° = \frac{\sqrt{3}}{2}$.

30. If A and B are square matrices of the same order,

Ans: $|AB| = |A||B|$

Sol: conceptual

31. The least positive integer n such that n! is …

Ans: 10

Sol: $75 = 3 \times 5^2$
so 10! contains both 5^2 and 3
$\therefore$10 is the required answer.

32. The roots of the equation $x^2 - cx + c = 0$ $(c \neq 0)$ ………

Ans: 1

Sol: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = c^2 - 2c$
Back substituting from options c = 1 gives minimum value.

33. The number of terms in the expansion ……

Ans: 5

Sol: $[(1 + x)^2 (1 - x)^2 = [1 - x^2]^4$
so, 5 terms

34. A rectangular field is half as wide as it is long and …..

Ans: $\frac{p^2}{18}$

Sol: Length = 2l, breadth = l, Area = $2l^2$

$6l = p \Rightarrow l = \frac{p}{6}$

Area = $2l^2 = \frac{p^2}{18}$

35. Given that $\int_0^{\sqrt{3}} \frac{dx}{1+x^2} = 2\int_a^{\sqrt{3}} \frac{dx}{1+x^2}$ …..

Ans: $\frac{1}{\sqrt{3}}$

Sol: $[\tan^{-1} x]_0^{\sqrt{3}} = 2[\tan^{-1} x]_a^{\sqrt{3}}$
simplifying;
$\tan^{-1} a = \frac{\pi}{6} \Rightarrow a = \frac{1}{\sqrt{3}}$

36. For any complex number z the minimum value….
Ans: 1.

Sol: The minimum value happens when
$z \in [0, 1]$

37. In which polygon, the number of sides is equal to the ……….

Ans: pentagon

Sol: ${}^nC_2 - n = n$, ${}^nC_2 = 2n$.
$\Rightarrow n = 5 \Rightarrow$ pentagon

38. The number of solutions of the equation ….

Ans: 4

Sol: $(|x| - 1)(|x| - 2) = 0$ is the equation

$\Rightarrow |x| = 1$ or $|x| = 2$

$\Rightarrow x = \pm 1$ or $x = \pm 2$

39. If m, n are two positive integers ……

Ans: always even

Sol: m – n – 1, m – n are consecutive. Also m – n – 1, m + n + 1 differ by an even number. Also m + n + 1 and m + n + 2 are consecutive So, product in both terms are even.

40. A regular hexagon is inscribed in a circle

Ans: 3d

Sol: side of regular hexagon inscribed in a circle of diameter d = $\frac{d}{2}$

$\therefore$ perimeter= $6 \times \frac{d}{2}$ = 3d

41. The derivative w.r.t. x of the product ……

Ans: 1

Sol. : The derivative is $(1 + x^2)(1 + x^4)$ …..
…$(1 + x^{256}) + x \times$ a polynomial
At x = 0,. derivative = 1

42. $n^3 - n$ is divisible by

Ans: 6

Sol. : $n^3 - n = (n - 1)\, n(n + 1)$, divisible by 3! = 6.

43. The smallest positive integer n for …..

Ans: 17.

Sol.: $n^2 + n + 17 = n(n + 1) + 17$
verifying options, n = 17 is the answer

44. 220 cannot be the sum of first n cubes for

Ans: a perfect square

Sol: sum of cubes of n number

$= \left[\frac{n(n+1)}{2}\right]^2$, a perfect square.

But 220 is not a perfect square

45. The value of $\int_{-a}^{a} |x| dx$ is equal to

Ans: a^2

Sol: $\int_{-a}^{a} |x| dx = 2\int_{0}^{a} x dx = 2\left[\frac{x^2}{2}\right]_0^a = a^2$

46. The value of the product $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$….

Ans: $\frac{1}{n}$.

Sol: Product = $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times .... \frac{n-1}{n} = \frac{1}{n}$

47. If the ratio $\frac{z-i}{z+i}$ is purely imaginary, the …..

Ans: a circle

Sol: $\frac{z-i}{z+i} = \lambda i$ where λ is a constant

$\frac{2z}{-2i} = \frac{1+\lambda i}{\lambda i - 1}$

$z = \frac{-i(1+\lambda i)}{\lambda i - 1}$

$|z| = 1 \Rightarrow$ z be on unit circle

48. Which one of the following is true for ….

Ans:

Sol: No option satisfies
Note : Option (c) would be correct if it is corrected as $(n + 1)^n \ge 2^n\, n!$
The underlying principle used
s A.M $\ge$ G.M

49. Which one of the following equations cannot…

Ans: 6x + 4y = 91

Sol: G.C.D (6, 4) = 2 and 2 does not divide 91.
Hence 6x + 4y = 91 does not have any integral solution.

50. The probability distribution of X is …

Ans: $\frac{3}{4}$

Sol: solving $\sum P(x) = 1$, we get a = $\frac{1}{28}$

P(X < 6) = 21a = $\frac{21}{28} = \frac{3}{4}$

51. Arithmetic mean $\overline{X}$ of a random……..

Ans: E(X)

Sol: Standard result

52. A balanced coin is tossed 3 times. A man ….

Ans: –1.

Sol: probability of success p = $\frac{2}{8} = \frac{1}{4}$

probability of failure q = 1 – p = $\frac{3}{4}$

$\therefore$ expected gain = $5 \times \frac{1}{4} - 3 \times \frac{3}{4} = -1$.

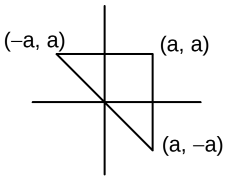
53. The random variable X has variance 4 ….

Ans: ± 2.

Sol: $E(X^2) - (E(X))^2 = 4$.
$8 - (E(X))^2 = 4$
$4 = (E(X))^2$
$E(X) = \pm 2$.

54. If the points (–a, a), (a, –a) and (a, a) enclose …

Ans: (1, 1) and (–1, –1)



Sol: Given $\Delta = 18$
ie $\frac{1}{2} \times 2a \times 2a = 18$
$a = \pm 3$
$\therefore$ vertices are (–3, 3), (3, 3), (3, –3),
(3, –3), (–3, –3) (–3, 3)
centroid is (1, 1) and (–1, –1).

55. The sum of the abscissa of …..

Ans: –4

Sol: $\pm 1 = \frac{4x + 3(4 - x) - 10}{5}$
Solving x = 3 or x = –7
$\therefore$ sum = –4

56. A section of a sphere by a plane, in ….

Ans: a circle.

Sol: conceptual

57. If $z = i^i$ (where $i^2 = -1$) …

Ans:

Sol: $z = i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{-\frac{\pi}{2}}$

$\therefore \bar{z} = e^{-\frac{\pi}{2}}$

$\frac{z}{\bar{z}} = 1$.

58. If f : R $\rightarrow$ R (where R is the set of all real numbers) ….

Ans: not one-one but continuous.

Sol: The function is clearly continuous. It is not one to one, since all negative numbers are mapped to zero..

59. Let f : R $\rightarrow$ R be such that f(x + y) = f(x) + f(y) ….

Ans: for all real x.

Sol:
$f(0) = 0$
$\underset{y \to 0}{\text{Lt}} \left[f(x+y) - f(x)\right]$
$= \underset{y \to 0}{\text{Lt}} f(y) = 0$
$\therefore f(x+y) \rightarrow f(x)$ as $y \rightarrow 0$
$\therefore$ f is continuous for all real values of x.

60. If f(a) = 4 and f'(a) = 1….

Ans: $4 - a$

Sol: Applying L' Hospital's Rule ,
$\underset{x \to a}{\text{Lt}} \frac{f(a) - af'(x)}{1}$
$\frac{4 - a \times 1}{1} = 4 - a$.

61. The value of *a* for which the function f(x) …..

Ans: –2.

Sol. : $f'(x) = -a\sin x - \frac{1}{3}(\sin 3x)3$
for Max/Min f'(x) = 0
$\therefore -a \times \sin\frac{\pi}{6} - \sin\frac{3\pi}{6} = 0$
$\therefore -\frac{a}{2} = 1 \therefore a = -2$.

62. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$ …..

Ans: in the 2nd quadrant

Sol. : $(\cos x + \sin x)^2 = \frac{1}{4}$
$1 + \sin 2x = \frac{1}{4}$
$\therefore \sin 2x = \frac{-3}{4}$
$\pi < 2x < 2\pi$
$\frac{\pi}{2} < x < \pi$
$\therefore$ in the2nd quadrant.

63. If $e^{\sin x} + e^{\cos x} = 2e^{\frac{-1}{\sqrt{2}}}$ then tanx is

Ans: 1.

Sol.: Back substitution
$e^{-1/\sqrt{2}} + e^{-1/\sqrt{2}} = 2e^{-1/\sqrt{2}}$

$= -\sin x = -\cos x = -\frac{1}{\sqrt{2}}$

$\therefore x = \frac{5\pi}{4}$

$\therefore \tan\frac{5\pi}{4} = 1.$

64. The domain of the function f(x) ……

Ans: $(\infty, -1] \cup [1, \infty)$.

Sol: standard result

65. The number of one-one, onto …..

Ans: n!

Sol: standard result

66. If $\vec{a} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ is a non zero ….

Ans: $\frac{1}{5\sqrt{2}}$

Sol: $m\vec{a} = m(3\hat{i} + 4\hat{j} + 5\hat{k})$

$1 = m^2 (9 + 16 + 25)$

$= 50m^2$

$m = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$.

67. One mapping is selected at random …..

Ans: $\frac{n!}{n^n}$

Sol: Total mapping in S = n^n

∴ total number of one to one mappings = n!

∴ probability of selection of one to one mappings = $\frac{n!}{n^n}$

68. If $\theta \in R$; then …..

Ans: [2, 4]

Sol: Expanding the determinant

$|D| = 1 + \cos^2\theta + 0 + 1 + \cos^2\theta$

$= 2 + 2\cos^2\theta$

$-1 \le \cos\theta \le 1 \Rightarrow 0 \le \cos^2\theta \le 1$

$0 + 2 \le 2 + 2\cos^2\theta < 2 + 2,$

So, [2, 4]

69. The system of equations $-4x + 3y + z = 1$; …

Ans: $\lambda = 1$

Sol: $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

$\begin{vmatrix} -4 & 3 & 1 \\ 2 & -6 & 1 \\ 2 & 3 & \lambda \end{vmatrix} = 0$

$24\lambda + 6 + 6 + 12 + 12 - 6\lambda = 0$

$\therefore \lambda = -2$

70. Let f(x) = $\begin{vmatrix} 1+\sin^2 x & \cos x & \sin x \\ 2-\sin^2 x & \cos^2 x & \tan x \\ \sin^2 x & \cos x & -\cot x \end{vmatrix}$ …

Ans: 0

Sol: f(–x) =

$\begin{vmatrix} 1+\sin^2 x & \cos x & -\sin x \\ 2-\sin^2 x & \cos^2 x & -\tan x \\ \sin^2 x & \cos x & \cot x \end{vmatrix} = -f(x)$

so this is an odd function

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$

71. If A = $\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}$ then ….

Ans: $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Sol: $A^2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2/2 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix}$

$\therefore A^n = \begin{bmatrix} 1 & 0 \\ n/2 & 1 \end{bmatrix}$ $\therefore A^{100} = \begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$.

72. The number of integers greater than 4000 …..

Ans: 192

Sol: Four digit numbers greater than 4000

□ □ □ □

3 4 3 2 $3 \times 4 \times 3 \times 2 = 72$

All five digit numbers are greater than 4000.

□ □ □ □ □

5 4 3 2 1

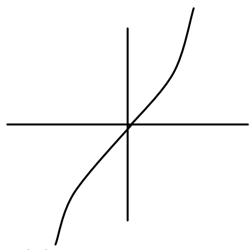
$5 \times 4 \times 3 \times 2 \times 1 = 120$

$\therefore$ Total = 120 + 72 = 192.

73. The set of all points where the function f(x) = x |x|, ……

Ans: $(-\infty, \infty)$

Sol: Standard result



f(x) = x $|x|$ is continuous and differentiable $\forall\, x \in R$

74. If x = $\sqrt{20+\sqrt{20+\sqrt{20+...\text{upto}\infty}}}$ ….

Ans: 5

Sol: $x = \sqrt{20+x}$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5, x = -4$

75. If the sum of 7 consecutive natural ….

Ans: 303

Sol: Let the numbers be a – 3d + a – 2d + a – d+ a + a + d + a + 2d + a + 3d = 2127

7a = 2121

Midde no, a = $\frac{2121}{7} = 303$

76. If the distance of the plane …..

Ans: 2**.**

Sol: Perpendicular distance = $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

$$\frac{4}{\sqrt{21}} = \left|\frac{+1}{\sqrt{1+\frac{1}{a^2}+\frac{1}{16}}}\right| \text{ squaring}$$

$$\frac{16}{21} = \frac{1}{1+\frac{1}{a^2}+\frac{1}{16}},\ 4a^2 = 16$$

$a = \pm 2.$

77. The equation of the curve whose subnormal …

Ans: a hyperbola

Sol: Give subnormal = 2x

ie $y\frac{dy}{dx} = 2x$

ydy = 2xdx. Integrating

$\frac{y^2}{2} = x^2 + C$ ie, $x^2 - \frac{y^2}{2} = k$

78. If $\omega(\neq 1)$ is a cube root of unity, ….

Ans: 81

Sol: $(1-\omega)(1-\omega^2)(1-\omega^3\omega)(1-\omega^3\omega^2)$
$(1-\omega^6\omega)(1-\omega^6\omega^2)(1-\omega^9\omega)$
$(1-\omega^9\omega^2)$

$[(1-\omega)(1-\omega^2)]^4$

$(1-\omega-\omega^2+1)^4$

$[2-(-1)]^4 = 3^4 = 81.$

79. If h(x) = min (x, x^2) …..

Ans: h(x) is neither increasing nor decreasing

Sol: h(x) = min (x, x^2) $x \in R$

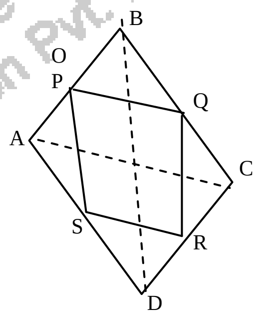
h(x) = x when $x \in I$

$= x^2$ when $x \in$ fraction.

80. If ABCD is a quadrilateral and …..

Ans: parallelogram

Sol: Standard result



81. The equation of the normal at the point ….

Ans: 25x – 8y = 60

Sol. : Equation of Tangent is $xx_1 + yy_1 = k$

$2 \times 4 \times x + 5 \times 5y = 20$

8x + 25y = 20

$\therefore$ equation of normal 25x – 8y + k = 0

$25 \times 4 - 8 \times 5 + k = 0$

$\therefore$ k = 60

required equation is 25x – 8y = 60.

82. If a and b are negative…

Ans: a + b

Sol. : Given a & b are negative

$\therefore\ |-a+-b| = |-(a+b)| = a+b.$

83. The greatest integer $\leq$ x for any ….

Ans: -1.

Sol.: $[-x] = -[x] - 1$
$\therefore [x] + [-x] = -1$.

84. If $f(x) = ax^3 + bx^2 + cx + d$ is a …..

Ans: No option satisfies**.**

Sol: $f(x) = a(x - \alpha)^3$ where α is a repeated root
$f'(x) = 3a(x - \alpha)^2$
$\therefore$ f'(x) has exactly two equal roots.

No option.

85. If x satisfies $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ ..

Ans: 4 or 6

Sol:
$$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$
$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0$$
$$\left(x + \frac{1}{x}\right)^2 - 2 - 10\left(x + \frac{1}{x}\right) + 26 = 0$$
$$\left(x + \frac{1}{x}\right)^2 - 10\left(x + \frac{1}{x}\right) + 24 = 0$$
$$x + \frac{1}{x} = 6 \text{ or } 4$$

86. A, B are two square matrices ……

Ans: Both (I – A) and (I – B) are invertible.

Sol: $A + B - AB = 0$
$I - A - B + AB = I \Rightarrow (I - A) - B(I - A) = I$
$(I - A)(I - B) = I$

87. The solutions of the equation $\frac{x}{y} + \frac{y}{x} - \frac{1}{xy} = 2$

Ans: (a, a+1), (a, a – 1)

Sol: $\frac{x}{y} + \frac{y}{x} - \frac{1}{xy} = 2 \rightarrow x^2 + y^2 - 2xy = 1$
$(x - y)^2 = 1 \Rightarrow x - y = \pm 1$
$x - y = 1$ or $x - y = -1$
$x = y + 1$ or $x = y - 1$
(c) satisfies the above relation.

88. The locus of a complex number ….

Ans: x - axis

Sol: $\left|\frac{z + 3i}{z - 3i}\right| = 1 \rightarrow \left|\frac{x + i(y + 3)}{x + i(y - 3)}\right|^2 = 1$
$\Rightarrow x^2 + (y + 3)^2 = x^2 + (y - 3)^2$
$\Rightarrow y = 0$ ie x axis

89. The roots of the equation $x^2 - px + q = 0$ ….

Ans: 1

Sol: $p = \cot 30 + \cot 15 = \sqrt{3} + 2 + \sqrt{3}$
$= 2 + 2\sqrt{3}$
$q = \cot 30.\cot 15 = \sqrt{3} + \left(2 + \sqrt{3}\right)$
$= 2\sqrt{3} + 3$
$q - p = 1$.

90. suppose $\begin{vmatrix} 1 & 2\lambda & 3 \\ 2 & 0 & 1 \\ 1 & \lambda & -1 \end{vmatrix}$ ……

Ans: -40.

Sol: Expanding we get,
$11\lambda - 10\lambda + 40 = 0$
$\lambda = -40$.

91. If $\omega \neq 1$ is a cube root of …..

Ans: 0

Sol: $D = -i\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ 1 & i & -i \end{vmatrix}$
$$= i\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 1 & i & -i \end{vmatrix} = -i(\omega - \omega) = 0$$

92. The graph of the function …..

Ans: a straight line passing through $(0, -\sin^2 1)$ parallel to x axis.

Sol: $y = \frac{\cos 2 - \cos 2(x - 1)}{2} - \frac{(1 - \cos 2(x - 1))}{2}$
$y = \frac{1}{2}(\cos 2 - 1)$
$y = k$
$\therefore$ straight line parallel to x axis, and passes through $(0, -\sin^2 1)$

93. If $\sin\alpha$, $\cos\alpha$ are the …..

Ans: $(a + c)^2 = b^2 + c^2$.

Sol: $\sin\alpha + \cos\alpha = \frac{-b}{a}$, $\sin\alpha \cos\alpha = \frac{c}{a}$
$(\sin\alpha + \cos\alpha)^2 = \frac{b^2}{a^2}$

$$1 + 2\frac{c}{a} = \frac{b^2}{a^2}$$

$$\frac{a+2c}{a} = \frac{b^2}{a^2}, a^2 + 2ac = b^2$$

$$a^2 + 2ac + c^2 = b^2 + c^2$$

$$(a + c)^2 = b^2 + c^2.$$

94. The function L(x) …..

Ans: $L(x - y) = L(x)/L(y)$

Sol: $L(x) = \int_0^x e^t dt + 1 = e^x$

$$\therefore L(x - y) = e^{x-y}$$

$$= \frac{e^x}{e^y}$$

$$= \frac{L(x)}{L(y)}.$$

95. The value of $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$ ….

Ans: $-\frac{1}{8}\cos 4x + C$

Sol: $\int \frac{2\cos^2 2x}{\cos 2x} dx$

$$= \int \cos 2x \sin 2x dx$$

$$= \frac{1}{2}\int \sin 4x dx = \frac{-\cos 4x}{8} + C$$

96. The number of distinct ….

Ans: $(n - 1)!$.

Sol: $(n - 1)!$ (Standard result)

97. The numbers e and π ……

Ans: Both irrationals

Sol: standard result

98. $\underset{x \to 0}{\text{Lt}} \left\{ \frac{\int_0^x \cos t^2 dt}{\sin x} \right\}$ ……

Ans: 1

Sol: $\underset{x \to 0}{\text{Lt}} \left\{ \frac{\int_0^x \cos t^2 dt}{\sin x} \right\} = \underset{x \to 0}{\text{Lt}} \frac{\cos x^2}{\cos x} = 1$

99. Let G be the set of all whole numbers,…

Ans: not a binary operation

Sol: G X G = {(0, 1), (0, 1), (1, 0) …}
(0, 0), (0, 1), (1, 0) do not satisfy the relation
$\Rightarrow$ not a binary operation.

100. Which of the following is divisible by 13?

Ans: No option satisfies.

Sol: None of the option is divisible by 13.

101. The mapping $f : R_0 \to R_0$ where R_0 is …

Ans: one-one onto

Sol. : $\frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$ and

$$f^{-1}(y) = \frac{1}{y}$$

102. The function $f : R \to R$ is defined …..

Ans: $f^{-1}(x) = \frac{x}{1+2x}, x \neq \frac{-1}{2}$

Sol. : $y = \frac{x}{1-2x} \Rightarrow y - 2xy = x$

$$\Rightarrow x = \frac{y}{1+2y}$$

$$f^{-1}(x) = \frac{x}{1+2x}, x \neq \frac{-1}{2}$$

103. If $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ then….

Ans: 0.

Sol.: $\underset{x \to 0}{\text{Lt}}\, x \sin\left(\frac{1}{x}\right) = 0$

$$\left(\because -1 \leq \sin\frac{1}{x} \leq 1\right)$$

104. If g[f(x)] = | sinx | and f [g(x)] = …..

Ans: $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$.

Sol: $f(g(x)) = f(\sqrt{x}) = (\sin\sqrt{x})^2$

$g(f(x)) = g(\sin^2 x) = \sqrt{\sin^2 x} = |\sin x|$

105. Let $f(x) = \frac{ax}{1+x}$, …….

Ans: -1

Sol: $\frac{\alpha\left(\frac{\alpha x}{1+x}\right)}{1+\frac{\alpha x}{1+x}} = x$

$\frac{\alpha^2}{1+(1+\alpha)x} = 1 \Rightarrow \alpha^2 - 1 = (1+\alpha)x$

106. When $|\sin x| + |\cos x| \ge 1$….

Ans: all real values

Sol: By verifying the options with n = 1 etc.

107. For $k \in N$, …..

Ans: k

Sol: $\underset{n\to\infty}{Lt} \frac{1}{\log_n(n-1)\log_{n+1} n \ldots \log_{n^k} n^k - 1}$

$= \underset{n\to\infty}{Lt} \frac{1}{\log_{n^k} n-1} = \underset{n\to\infty}{Lt}\ k\log_{(n-1)} n$

$= \underset{n\to\infty}{Lt} \frac{k}{\log_n(n-1)}$

$= \underset{n\to\infty}{Lt} \frac{k}{1+\log_n 1-\frac{1}{n}} = k$

108. If $f(x) = \frac{2-\sqrt{x+4}}{\sin(2x)}$ ..

Ans: $\frac{-1}{8}$

Sol: $\underset{x\to 0}{Lt} \frac{2-\sqrt{x+4}}{\sin(2x)} = \underset{x\to 0}{Lt} \frac{-x}{\sin 2x(2+\sqrt{x}+4)}$

$= \underset{x\to 0}{Lt} - \frac{1}{2\left(\frac{\sin 2x}{2x}\right)(2+\sqrt{x+4})}$

$= -\frac{1}{8} = f(0)$

109. The value of the derivative of ….

Ans: 2

Sol: $|x-1| = x-1$ at x = 2

$|x+3| = x+3$ at x = 2

$\therefore$ Derivative of $|x-1| + |x+3|$ at x = 2 is 2.

110. Let f and g be differentiable ….

Ans: $\frac{1}{2}$

Sol: f(g(x)) = x
f '(g(x).g'(x) = 1
f '(g(a)).g'(a) = 1
f '(b) = $\frac{1}{2}$

111. The locus of point z satisfying ….

Ans: circle

Sol: $\arg\left(\frac{x-1+iy}{x+1+iy}\right) = \frac{\pi}{3}$

ie, $\tan^{-1}\frac{y}{x-1} - \tan^{-1}\frac{y}{x+1} = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1+\frac{y^2}{x^2-1}}\right) = \frac{\pi}{3}$

$\frac{(x+1)y - y(x-1)}{x^2-1+y^2} = \sqrt{3}$

$\frac{2y}{x^2+y^2-1} = \sqrt{3}$

$\therefore$ Locus of z is a circle.

112. The inequality |z – 4| …

Ans: Re(z) > 3

Sol: By verifying options.

113. The value of the sum …..

Ans: i - 1

Sol: $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right) = i + i^2 = i - 1.$

114. If 2 + i$\sqrt{3}$ is a …..

Ans: (–4, 7).

Sol: $\left(2+i\sqrt{3}+2-i\sqrt{3}\right)=4$
$= -p$
or p = –4
$\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)=q=7$

115. If the roots of the equation …..

Ans: $\frac{1}{\alpha},\frac{1}{\beta}$

Sol: $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{-b}{c}$ and $\left(\frac{1}{\alpha}\right)\left(\frac{i}{\beta}\right)=\frac{a}{c}$
$\left(\because \alpha+\beta=\frac{-b}{a} \text{ and } \alpha\beta=\frac{c}{a}\right)$.

116. If f(x) = $\cos^2x + \sec^2x$, …

Ans: f(x) > 1.

Sol: conceptual

117. If tanθ + sinθ = m and ….

Ans: $m^2 - n^2 = 4\sqrt{mn}$

Sol: mn = $(\tan^2\theta - \sin^2\theta)$
$= \sin^2\theta \tan^2\theta = \left(\frac{m+n}{2}\right)^2\left(\frac{m-n}{2}\right)^2$
$m^2 - n^2 = 4\sqrt{mn}$

118. If $\tan\alpha = \frac{m}{m+1}$ …..

Ans: $\frac{\pi}{4}$

Sol: $\tan(\alpha+\beta) = \frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$
$= \alpha + \beta = \frac{\pi}{4}$.

119. Which of the following number(s)…

Ans: sin15° cos15°

Sol: sin15° cos15° = $\frac{1}{2}\sin 30 = \frac{1}{4}$

120. $\left(\frac{(1-\sin\theta)^2}{\cos^2\theta}\right)^{1/2}$ ….

Ans: secθ – tanθ

Sol: $\left(\frac{(1-\sin\theta)^2}{\cos^2\theta}\right)^{1/2} = \sec\theta - \tan\theta$

121. The number of diagonals in a …

Ans: $\frac{n}{2}(n-3)$

Sol. : $^nC_2 - n = \frac{n(n-3)}{2}$

122. The fourth, seventh and tenth …

Ans: q^2 = pr.

Sol. : $aR^3 = p$, $aR^6 = q$ and $aR^9 = r$
$\Rightarrow q^2 = pr$.

123. Sum of the n terms of the series

Ans: $4(2^n - 1) + 8n$

Sol.: By back substitution, with n = 1.

124. If H is the harmonic ….

Ans: 2.

Sol: $\frac{1}{P},\frac{1}{H},\frac{1}{Q}$ are in AP
$\Rightarrow \frac{2}{H}=\frac{1}{P}+\frac{1}{Q}$
$\Rightarrow \frac{H}{P}+\frac{H}{Q}=2$.

125. If $a^{1/x} = b^{1/y}$ …..

Ans: A. P

Sol: $a^{1/x} = b^{1/y} = c^{1/z} = k$
$\Rightarrow a = k^x, b = k^y, c = k^z$
a, b, c are in G.P
$\Rightarrow b^2 = ac \Rightarrow k^{2y} = k^{x+z}$
$\Rightarrow 2y = x + z \Rightarrow$ x, y, z are in A.P.