Probability Distributions

Advanced Probability

Random Variable: A random variable is a real valued function defined over the sample space. (discrete or continuous).

A **discrete random variable** takes the values that are finite or countable. For example when we consider the experiment of tossing of 3 coins, the number of heads can be appreciated as a discrete random variable (X). X would take 0, 1, 2 and 3 as possible values.

A continuous random variable takes values in the form of intervals. Also, in the case of a **Continuous Random Variable** P(X = c) = 0, where c is a specified point.

Heights and weights of people, area of land held by individuals, etc., are examples of continuous random variables.

Probability Mass Function (p.m.f): If X is a discrete random variable, which can take the values x_1, x_2,……. and f(x) denote the probability that X takes the value x_i, then p(x) is called the **Probability Mass Function** (p.m.f) of X. $p(x_i) = P(x = x_i)$. The values that X can take and the corresponding probabilities determine the probability distribution of X. We also have

(i) $p(x) \geq 0$; (ii) $\sum p(x) = 1$.

Probability density function (P.d.f): If X is a continuous random variable then a function f(x), $x \in I$ (interval) is called a probability density function. The probability statements are made as $P(x \in I) = \int_I f(x)\,dx$

We also have,

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

The probability $P(X \leq x)$ is called the cumulative distribution function (c.d.f) of X and is denoted by F(X). It is a point function. It is defined for discrete and continuous random variables.

The following are the properties of probability distribution function F(x),

(i) $F(x) \geq 0$.

(ii) F(x) is non-decreasing
i.e., for $x > y$, $F(x) \geq F(y)$.

(iii) F(x) is right continuous.

(iv) $F(-\infty) = 0$ and $F(+\infty) = 1$.
Also,

(v) $P(a < x \leq b) = F(b) - F(a)$.
For a continuous random variable

(vi) $Pr\{x < X \leq x + dx\} = F(x + dx) - F(x) = f(x)\,dx$; where dx is very small

(vii) $f(x) = \frac{d}{dx}\left[F(x)\right]$ where;

(a) $f(x) \geq 0 \ \forall \ x \in R$.

(b) $\int_R f(x)\,dx = 1$.

Mathematical Expectation [E(X)]

Mathematical Expectation is the weighted mean of values of a variable.

If X is a random variable which can assume any one of the values x_1, x_2, … x_n with the respective probabilities p_1, p_2, ... p_n, then the mathematical expectation of X is given by $E(X) = p_1x_1 + p_2x_2 + \ldots + p_n x_n$

For a continuous random variable,

$E(X) = \int_{-\infty}^{+\infty} x\, f(x)\, dx$ where f(x) is the p.d.f. of X.