Construction Rules of the Root Locus: Construction rules (or) properties of the root locus are derived from general control system transfer function i.e.,

$$M(s) = \frac{K\,G(s)}{1 + KG(s)H(s)}$$

$\rightarrow$ C.E = 1 + K G(s) H(s) = 0

Magnitude Condition

$|KG(s)\,H(s)|_{S = S1} = 1$

Angle Condition

$$\angle kG(s)H(s)\big|_{S = S_1} = \pm(2q + 1)\,180^{o}$$

q = 0, 1, 2,…..; where S_1 is a point on the root locus.

Note: For any point $S = S_1$ to be on root locus diagram for certain value of gain both magnitude and angle condition must be satisfied.

1. No. of branches of root locus = n

$\forall$ n > m where 'n' is no. of open loop poles, 'm' is no. of open loop zeroes.

2. Root Locus is Symmetrical about Real Axis. Reason: All physically reliable systems cannot have complex coefficient in their Characteristic equation.

3. Starting and Ending points of Root Locus: The root locus starts at finite and infinite open loop poles and it ends at finite and infinite open loop zeroes.

4. No. of branches of Root Locus Terminating at Infinity: No. of branches of root locus approaching / terminating at infinity = no. of asymptotes.

No. of asymptotes = (n – m); where n = no. of finite open loop poles; m = no of finite open loop zeroes.

The meeting point of asymptotes is called as centroid.

$$\text{Centroid } (\sigma) = \left(\frac{\Sigma \text{finite poles} - \Sigma \text{finite zeroes}}{\text{no. of finite poles} - \text{no. of finite zeroes}} \right)$$

The angle with which asymptotes approaches to infinity are called as angle of asymptotes (θ)

Angle of asymptotes (θ) = $\frac{(2q+1)180}{(\text{no. of finite poles} - \text{no. of finite zeros})}$

q = 0, 1, 2, ….. (n –m – 1)

Number of	Angle of asymptotes
1	-180^o
2	$+90^o, -90^o$
3	$+60^o, -60^o, 180^o$
4	$+45^o, -45^o, +135^o, -135^o$

5. **Real Axis Segments of Root Locus:** A real point to exist on root locus if the sum of finite open loop poles and finite open loop zeros to the right side of that point must be odd in number.

6. **Break-in and Breakaway points:** These are the points on the root locus at which the gain (k) is either maximum (or) minimum.

   $|k\ G(s)\ H(s)| = 1 \Rightarrow k = \left|\frac{1}{G(s)H(s)}\right|$ at break in (or) break away points 'k' is either maximum (or) minimum $\Rightarrow \left.\frac{dk}{ds}\right|_{S=S_1} = 0$ where S = S_1 either break in (or) break away point.

   Note: For a break point to be valid that must lie on root locus.

7. **jω – axis Crossings of Root Locus:** As and when gain 'k' increases from 0 to ∞ for some value of gain k some of the roots of the system tends to move towards right side (for order ≥ 3).