T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE 2009

Paper II

PART I

1	2	3	4
D	**A**	**D**	**B**

5	6	7	8	9
A, B, D	**A, D**	**A**	**A, B, C**	**B, C**

10	11
A – p, s	**A – p, q**
B – q, s	**B – p, s, t**
C – r, t	**C – r, s**
D – q, t	**D – p**

12	13	14	15	16	17	18	19
9	**4**	**8**	**8**	**6**	**4**	**6**	**9**

Section I

1. $\log K = \log A - \frac{Ea}{2.303\,RT}$

   Given $\log K = 6 - \frac{2000}{T}$

   $\therefore A = 1 \times 10^6\ s^{-1}$

   $\frac{Ea}{2.303\,R} = 2000$

   $Ea = 2000 \times 2.303 \times 8.314 \times 10^{-3}$ kJ mol^{-1}

   $= 38.3$ kJ mol^{-1}

2. CO is a strong legand. Configuration of Cr in $Cr(CO)_6$ is $t_{2g}^6\ e_g^0$. All electrons are paired. So diamagnetic, $\mu = 0$ BM.

3. If H at C – 2 migrates, a positive charge will develop at C –2 containing –OH group which can be stabilized by resonance involving the lone pair on oxygen.

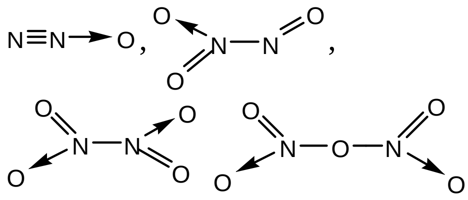
4. Between structures I & III, I is most stable as more electro negative nitrogen carries negative charge than less electro negative carbon. Between II & IV, II is more stable due to the same reason. In I & III all the atoms/ions have completely filled octet.

Section II

5. Metals with lesser reduction potential than the NO_3^- ions get oxidised ie. V, Fe and Hg are oxidised in combination with $NO_{3(aq)}^-$

6. Among the given options, the state functions are internal energy and molar enthalpy

7. Ammonia forms the ionic adduct $[BH_2(NH_3)_2]^+[BH_4^-]$
Amines form the simple adduct $RH_2N \rightarrow BH_3$

8. 

9. X is a non reducing sugar because the aldehyde and keto group of the two monosaccharide units are involved in glycosidic linkage. But Y is a reducing sugar because the two monosaccharide units are joined only by 1, 4 – glycosidic linkage.

Section III

10. (A) $3Cu + 8dil.HNO_3$
$\rightarrow 3Cu(NO_3)_2 + 4H_2O + 2NO$
(B) $Cu + 4HNO_3(conc)$
$\rightarrow Cu(NO_3)_2 + 2H_2O + 2NO_2$
(C) $4Zn + 10HNO_3(dil)$
$\rightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$
(D) $Zn + 4HNO_3(conc)$
$\rightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

11. The compound (A) is a bromo compound which undergoes nucleophilic substitution and dehydrohalogenation (elimination). It cannot undergo dehydrogenation.
(B) can undergo nucleophilic substitution, esterification and dehydrogenation.
(C) can undergo nucleophilic addition at carbonyl group and esterification at – OH group.
(D) can undergo nucleophilic substitution of bromine atom because of the presence of electron withdrawing $-NO_2$ group at the ortho position.

Section IV

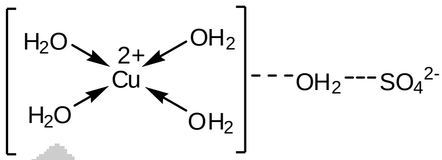
12. $Z = 2.5 \text{ kJ K}^{-1}$
$\theta = 0.45K$
$M = 28 g \text{ mol}^{-1}$
$m = 3.5$ g
$Qv = Z \times \theta \times \frac{M}{m}$
$= 9 \text{ kJ mol}^{-1}$

13. $\sqrt{\frac{3RT_x}{M_x}} = \sqrt{\frac{2RT_y}{M_y}}$
$\frac{3 \times 400}{40} = \frac{2 \times 60}{M_y}$
$M_y = 4$

14. $pH = \frac{1}{2}[pK_w + pK_a + \log C]$
$= \frac{1}{2}[14 + 4 - 2] = 8$

15. Total number of α - particles $= \frac{238 - 214}{4} = 6$
Total number of β–particles = $2 \times 6 - 10 = 2$
Answer = 6 + 2 = 8

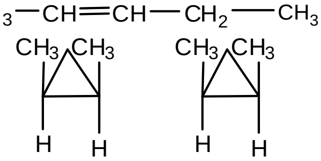
16. On fusing MnO_2 with KOH K_2MnO_4 is formed.

17. 
Ans = 4

18. In the crystalline $AlCl_3$, the C.N of Al is 6 and it changes to four coordinated dimer at its melting point.

19. Cyclic Structures – 5
Cyclo pentane, methyl cyclobutane, ethylcyclopropane, 1, 2 – dimethyl cyclopropane and 1,1- dimethylcyclopropane.
Stereoisomers
Cis and trans – (2 isomers)

$CH_3-CH=CH-CH_2-CH_3$



cis - isomer   trans - isomer

trans- 1, 2 – dimethyl cyclopropane is optically active and exists in (+) and (–) forms.

The question is ambiguous. If it is interpreted as "total number of cyclic structural as well as cyclic stereo isomers possible" there are only 7 isomers.

(1) Cyclopentane
(2) methyl cyclobutane
(3) ethyl cyclopropane

(4) 1, 1-dimethyl cyclopropane
(5) Cis-1,2-dimethylcyclopropane (meso)
(6) d-trans-1,2-dimethyl cyclopropane
(7) l-trans-1,2-dimethyl cyclopropane
If it is interpreted as "total number of stereo isomers and cyclic structures" there are 9 isomers i.e., in addition to the above 7 isomers there are (8) Cis-2-pentene and (9) trans-2-pentene.

PART II

20	21	22	23
C	**C**	**C**	**D**

24	25	26	27	28
A, B, C	**A, B**	**B, C, D**	**A, D**	**C, D**

29	30
A – q, s	**A – p**
B – p, r, st	**B – q, s**
C – t	**C – q, r, s, t**
D – r	**D – r**

31	32	33	34	35	36	37	38
7	**7**	**4**	**0**	**0**	**8**	**2**	**2**

Section I

20. Given that $\frac{n}{2}(2+(n-1)d) = cn^2$

$\Rightarrow \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n = cn^2$

$\Rightarrow$ d = 2c and a = $\frac{d}{2} = c$

$\therefore$ The AP is c, 3c, 5c, 7c

Sum of the squares of the terms of the A.P

$= c^2 + (3c)^2 + (5c)^2 + \ldots\ (2n-1)^2c^2$

$= c^2\left[1+9+25+\ldots+(2n-1)^2\right]$

$= c^2\sum_{r=1}^{n}(2r-1)^2$

$= c^2\sum_{r=1}^{n}(4r^2-4r+1)$

$= c^2\left[4\sum_{r=1}^{n}r^2 - 4\sum_{r=1}^{n}r + \sum_{r=1}^{n}1\right]$

$= c^2\left[4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n\right]$

$= \left\{4n\left[\frac{(n+1)(2n+1)-3(n+1)}{6}\right]+n\right\}c^2$

$= \left[\frac{2n(2n^2+3n-1-3n-3)}{3}+n\right]c^2$

$= \frac{2n(2n^2-2)+3n}{3}c^2 = \frac{n(4n^2-4+3)}{3}c^2$

$= \frac{n(4n^2-1)}{3}c^2$

21. Equating the line is

$\frac{x-2}{\ell} = \frac{y+1}{\ell} = \frac{3-2}{\ell} = k$

$\therefore$ The point θ on the line is

$(k\ell + 2,\ k\ell - 1,\ k\ell + 2)$

since θ lies on $2x + y + z = 9$,

we have $k\ell = 1$, $\Rightarrow$ Q is (3, 0, 3)

$\therefore$ PQ = $\sqrt{3}$

22. $\frac{x^2}{16}+\frac{y^2}{4}=1$

$(4\cos\theta, 2\sin\theta)$

$\frac{x}{8}+\frac{yy'}{2}=0$

$y' = -\frac{x}{8}\times\frac{2}{y}$

$= -\frac{1}{4}\ \frac{4\cos\theta}{2\sin\theta}$

$= -\frac{\cot\theta}{2}$

Slope of normal = $2\tan\theta$

$y - 2\sin\theta = 2\tan\theta\ (x - 4\tan\theta)$

$y = 2\sin\theta + 2\tan\theta x - 8\sin\theta$

$= 2\tan\theta x - 6\sin\theta$

$y = 0$

$x = \frac{6\sin\theta}{2\tan\theta} = 3\cos\theta$

$Q(3\cos\theta, 0)$

$P(4\cos\theta, 2\sin\theta)$

Midpoint of PQ $\left(\frac{7\cos\theta}{2}, \sin\theta\right)$

$h = \frac{7}{2}\cos\theta$; $k = \sin\theta$

$\left(\frac{2h}{7}\right)\cos\theta; k = \sin\theta$

$\frac{4h^2}{49}+k^2=1$

Locus of M $\frac{4x^2}{49}+y^2=1$

$a^2 = 16$, $b^2 = 4$

$b^2 = a^2(1 - e^2)$

$4 = 16(1 - e^2)$

$1 - e^2 = \frac{1}{4} \Rightarrow \quad e = \frac{\sqrt{3}}{2}$

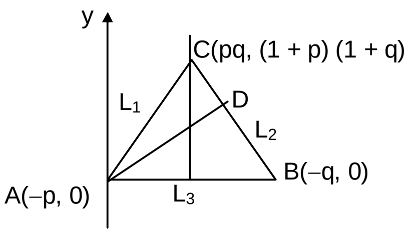
Equation of the latus rectum is $x = \pm 2\sqrt{3}$

$\frac{4}{49}\times 4\times 3+4^2=1$

$y^2 = 1 - \frac{48}{49}=\frac{1}{49}$

$y = \pm\frac{1}{7}$

23.



$x = -p$

$q(1 + p)x - pqy = -pq\ (1 + p)$

$p(1 + q)x - pqy = -pq(1 + q)$

$(q\ (1 + p) - p(1 + q))]x = -pq\ [1 + p - (1 + q)]$

$(q - p)x = -pq[p - q]$

$x = pq$

$py = (1 + p)\ pq + p(1 + p)$

$= p(1 + p)\ (q + 1)$

$y = (1 + p)\ (1 + q)$

$x = pq$

Equation of the AD

$y - 0 = \frac{-q}{1+q}(x+p)$

$y = -\frac{qx}{1+q}-\frac{-pq}{1+q}$

$= -\frac{p^2q^2-pq}{1+q}$

$\therefore$ $x = pq$, $y = -pq$

$\therefore$ $x + y = 0 \Rightarrow$ Straight line

Section II

24. $I_n = \int_{-\pi}^{\pi}\frac{\sin(nx)}{(1+\pi^x)\sin x}dx$

$= \int_{-\pi}^{0}\frac{\sin(nx)}{(1+\pi^x)\sin x}dx + \int_{0}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$

$= \int_{0}^{\pi}\frac{\pi^x\sin nx}{(1+\pi^x)\sin x}dx + \int_{0}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$

(by replacing x with −x in the first integral)

$= \int_{0}^{\pi}\frac{\sin nx}{\sin x}dx$

$\therefore I_{n+2} = \int_{0}^{\pi}\frac{\sin(n+2)x}{\sin x}dx$

$= \int_{0}^{\pi}\frac{\sin(n+1)x\cos x}{\sin x}dx + \int_{0}^{\pi}\frac{\cos(n+1)x\sin x}{\sin x}dx$

$= \int_{0}^{\pi}\frac{[\sin nx\cos x+\cos nx\sin x]}{\sin x}\cos x dx + 0$

$= \int_{0}^{\pi}\frac{\sin nx[1-\sin^2 x]}{\sin x}dx + \int_{0}^{\pi}\cos nx\cos x dx$

$=$

$\int_{0}^{\pi}\frac{\sin nx}{\sin x}dx - \int_{0}^{\pi}\sin nx\sin x\,dx + \int_{0}^{\pi}\cos nx\cos x dx$

$= I_n + \int_{0}^{\pi}\cos(n+1)x\,dx = I_n$

$$\sum_{m=1}^{10} I_{2m+1} = I_3 + I_5 + ...+ I_{21} = 10I_3$$

$(\because I_3 = I_5 = ...I_{21})$

$$= 10\int_0^{\pi} \frac{\sin 3x}{\sin x}dx$$

$$= 10\int_0^{\pi}\left[\frac{3\sin x - 4\sin^3 x}{\sin x}\right]dx$$

$$= 10\left[\int_0^{\pi} 3 - 4\int_0^{\pi}\sin^2 x\, dx\right]$$

$$= 30\pi - 40\int_0^{\pi}\frac{1-\cos 2x}{2}dx$$

$$= 30\pi - 20\int_0^{\pi}dx + 20\int_0^{\pi}\cos 2x\, dx$$

$$= 10\pi$$

$$\sum_{m=1}^{10} I_{2m} = I_2 + I_4 + ...+ I_{20}$$

$$= 10\, I_2$$

$$= 10\int_0^{\pi}\frac{\sin 2x}{\sin x}dx = 10\int_0^{\pi}\cos x\, dx$$

$$= 0$$

25. Eccentricity of the ellipse = $\frac{1}{\sqrt{2}}$

Consider the ellipse $x^2 + 2y^2 = 2$

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

$b^2 = a^2(1 - e^2)$

$1 = 2(1 - e^2) \Rightarrow 1 - e^2 = \frac{1}{2}$

$$e = \frac{1}{\sqrt{2}}$$

Solving the two equations,

$x^2 + 2y^2 = 2$

$x^2 - y^2 = \frac{1}{2}$

$\Rightarrow 3y^2 = \frac{3}{2} \Rightarrow y^2 = \frac{1}{2}$

$x^2 = \frac{1}{2} + y^2 = 1$

Product of the slopes of the curves

$x^2 - y^2 = \frac{1}{2}$ and $x^2 + 2y^2 = 2$

at a point of intersection (x, y)

$$= \left(\frac{x}{y}\right)\left(\frac{-x}{2y}\right) = \frac{-x^2}{2y^2}$$

$$= \frac{-1}{2\times\frac{1}{2}} = -1$$

Foci of the ellipse $x^2 + 2y^2 = 2$ are at $(\pm 1, 0)$

26. $f(x) = x\cos\frac{1}{x}, x \ge 1$

$$f'(x) = x \times \left(-\sin\frac{1}{x}\right)\times\left(\frac{-1}{x^2}\right) + \cos\frac{1}{x}$$

$$= \frac{x\sin\frac{1}{x}}{x^2} + \cos\frac{1}{x}$$

$$= \frac{\sin\frac{1}{x}}{x} + \cos\frac{1}{x}$$

$$= \frac{\sin\frac{1}{x}}{\frac{1}{x}} \times \left(\frac{1}{x^2}\right) + \cos\frac{1}{x}$$

$$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty}\left(\frac{\sin 1/x}{1/x}\right)\times\lim_{x\to\infty}\left(\frac{1}{x^2}\right) + \lim_{x\to\infty}\cos\frac{1}{x}$$

$$= 1 \times 0 + 1 = 1$$

Again $f(x) \approx x\left\{1 - \frac{\frac{1}{x^2}}{2}\right\}$, since $\frac{1}{x} < 1$

$$= x - \frac{1}{2x}$$

$$f(x + 2) = (x + 2) - \frac{1}{2x+4}$$

$$f(x + 2) - f(x) = 2 - \left\{\frac{1}{2(x+2)} - \frac{1}{2x}\right\}$$

$$= 2 - \frac{1}{2}\left\{\frac{-2}{x(x+2)}\right\}$$

$$= 2 + \frac{1}{x(x+2)}$$

$$> 2$$

$$f''(x) = \frac{x\left(\cos\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) - \sin\frac{1}{x}}{x^2} - \left(\sin\frac{1}{x}\right)\times\frac{-1}{x^2}$$

$$= \frac{-\cos\frac{1}{x}}{x^3} - \frac{\sin\frac{1}{x}}{x^2} + \frac{\sin\frac{1}{x}}{x^2}$$

$$= \frac{-\cos\frac{1}{x}}{x^3} < 0$$

$f'(x)$ is decreasing in $[1, \infty)$

27. Let P be $(at^2, 2at)$

Tangent at P is

$y = \frac{x}{t} + at$ — (1)

Normal at P is

$y + xt = 2at + at^3$ — (2)

Put y = 0 in (1) $\Rightarrow$ T is $(-at^2, 0)$

Put y = 0 in (2) $\Rightarrow$ N is $(2a + at^2, 0)$

Let (x, y) denote the centroid of Δ PTN

$x = \frac{at^2 - at^2 + 2a + at^2}{3} = \frac{2a + at^2}{3}$

$y = \frac{2at}{3}$

$t = \frac{3y}{2a}$

$3x = 2a + at^2$

$= 2a + \frac{ax9y^2}{4a^2} \quad = 2a + \frac{9y^2}{4a}$

$12ax = 8a^2 + 9y^2$

$9y^2 = 8a^2 - 12ax$

$= 4a(2a - 3x)$

$= -12a\left(x - \frac{2a}{3}\right)$

Parabola whose vertex is at $\left(\frac{2a}{3}, 0\right)$

Latus rectum = $\frac{4a}{3}$, Focus is at (a, 0)

28. $S = \sum_{m=1}^{6} \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right) . \sin\left(\theta + \frac{m\pi}{4}\right)}$

$= \sum_{m=1}^{6} \frac{1}{\sin A_{m-1} \sin A_m}$

$= \frac{1}{\sin\frac{\pi}{4}} \sum_{m=1}^{6} \frac{\sin\frac{\pi}{4}}{\sin A_{m-1} \sin A_m}$

But $A_m - A_{m-1} = \frac{\pi}{4}$.

$\therefore S = \sqrt{2} \sum_{m=1}^{6} \cot A_m - \cot A_{m-1}$

On further simplification,

$4 = \cot A_0 - \cot A_6$

$\therefore 4 = \frac{\sin(A_6 - A_0)}{\sin A_0 \sin A_6}$

We have $A_6 = \theta + \frac{3\pi}{2}$ and $A_0 = \theta$;

Simplifying we get, $\sin 2\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$ and the correct options are $\frac{\pi}{12}$ and $\frac{5\pi}{12}$.

Section III

29. (A) - Given equation is

$2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$

$\Rightarrow \sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$

$\Rightarrow 2\sin^4\theta - 3\sin^2\theta + 1 = 0$

$\Rightarrow (\sin^2\theta - 1)(2\sin^2\theta - 1) = 0$

$\Rightarrow \sin^2\theta = 1, \frac{1}{2}$

$\Rightarrow \sin\theta = \pm 1, \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{\pi}{4}$

$\therefore$ A $\rightarrow$ (p, s)

(B) – We note that [n] is discontinuous at all integer values of n.

$\therefore$ Given function is discontinuous wherever $\left[\frac{6x}{\pi}\right]$ or $\left[\frac{3x}{\pi}\right]$ is an integer.

Hence the values are $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and π.

(C) - $V = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$

(C) – t

(D) - $\bar{a} + \bar{b} = -\sqrt{3}\,c$

$(\bar{a} + \bar{b})^2 = 3\bar{c}.\bar{c}$

$1 + 1 + 2\bar{a}.\bar{b} = 3.1$

$\bar{a}.\bar{b} = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\therefore$ D $\rightarrow$ (r)

30. (A) – $f(x) = xe^{\sin x} - \cos x$

$f'(x) = xe^{\sin x} . \cos x - \sin x$

$f'(x) > 0$ in $\left(0, \frac{\pi}{2}\right)$

$\therefore$ f(x) is increasing in $\left(0, \frac{\pi}{2}\right)$;

$f(0) < 0$ and $f\left(\frac{\pi}{2}\right) > 0$

$\Rightarrow$ f(x) has exactly one root in $\left(0, \frac{\pi}{2}\right)$

(A) $\rightarrow$ (p)

(B) - The planes intersect in the line

$\Rightarrow$ corresponding homogeneous system has non-trivial solutions

$$\Rightarrow \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$k(k - 4) - 4(4 - 4) + 1(8 - 2k) = 0$

$k^7 - 4k + 8 - 2k = 0$

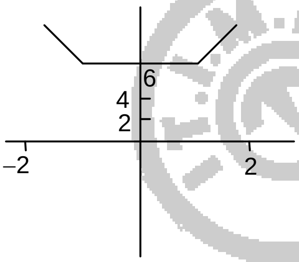
$k^7 - 6k + 8 = 0$

$(k - 2)(k - 4) = 0$

$k = 2, 4$

(B) – (q, s)

(C) -



$$f(x) = \begin{cases} -4x & x < -2 \\ 4 - 2x & -2 < x < -1 \\ 6 & -1 < x < 1 \\ 2x + 4 & 1 < x < 2 \\ 4x & x > 2 \end{cases}$$

$\therefore 4k \geq 6$

$k \geq \frac{3}{2}$

$\therefore$ (C) $\rightarrow$ q, r, s, t

(D) - $\frac{dy}{dx} = y + 1$

$\frac{dy}{y+1} = dx \Rightarrow \log(y + 1) = x + C$

$y(0) = 1$

$\Rightarrow \log 2 = C$

$\therefore \log\left(\frac{y+1}{2}\right) = x$

$\frac{y+1}{2} = e^x$

$y + 1 = 2e^x$

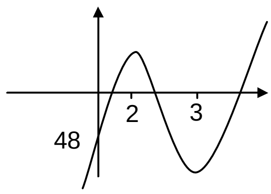
$\therefore y = 2e^x - 1$

$\therefore y(\log 2) = 4 - 1 = 3$

(D) $\rightarrow$ (r)

Section IV

31.



$x^2 + 20 - 9x \leq 0$

$D = 81 - 80 = 1 \Rightarrow$ Roots are 5 and 4

$\therefore x^2 + 20 - 9x \leq 0$ when $x \in [4, 5]$

Consider $f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6x^2 - 30x + 36$

Let $f'(x) = 0 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

so the graph of f(x) will be as given above

In the interval [4, 5]

f(x) is increasing and the maximum value is at x = 5

$f(5) = 2(5)^3 - 15(5)^2 + 36(5) - 48$

$= 250 - 375 + 180 - 48$

$= 7$

32. $\begin{vmatrix} 3 & -1 & -1 \\ -3 & 0 & 1 \\ -3 & 2 & 1 \end{vmatrix} = 0$

$\therefore$ System has infinite solutions

Let z = k

$\therefore x = \frac{k}{3}, y = 0, z = k$

But x, y, z are integer values

$\therefore x = k, y = 0; z = 3k$

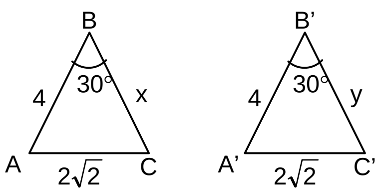
But $x^2 + y^2 + z^2 \leq 100$

$10k^2 \leq 100$

$k^2 \leq 10 \Rightarrow -3 \leq k \leq 3$

7 integer values are there.

33.



The 2 triangles are ΔABC and ΔA'B'C' with $\angle$ABC = 30° = $\angle$A'B'C'

AB = A'B' = 4, AC = A'C' = $2\sqrt{2}$

(Note that it means $\angle A + \angle C = 150° = \angle A' + \angle C'$)

Using sine law, for the two Δ's we have

$$\frac{\sin A}{x} = \frac{\sin 30^\circ}{2\sqrt{2}} = \frac{\sin C}{4}$$

And

$\frac{\sin A'}{y} = \frac{\sin 30^\circ}{2\sqrt{2}} = \frac{\sin C'}{4}$

$\Rightarrow \left(\sin A = \frac{x}{4\sqrt{2}}, \sin C = \frac{1}{\sqrt{2}}\right)$

and

$\left(\sin A' = \frac{y}{4\sqrt{2}}, \sin C' = \frac{1}{\sqrt{2}}\right)$

As the triangles are non congruent,

sin C = sinC' = $\frac{1}{\sqrt{2}}$

$\Rightarrow$ C = $\frac{\pi}{4}$, $C' = \frac{3\pi}{4}$ or C = $\frac{3\pi}{4}$, $C' = \frac{\pi}{4}$

We consider C = $\frac{\pi}{4}$ $\Rightarrow \angle BAC = 105°$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times 4 \times 2\sqrt{2} \sin 105^\circ$

$= 4\sqrt{2}\cos 15^\circ = 4\sqrt{2}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$= 2\left(\sqrt{3}+1\right)$

C' = $3\frac{\pi}{4}$ $\Rightarrow$ A = 15°

$\therefore$ Area of $\Delta A'B'C' = \frac{1}{2} \times 4 \times 2\sqrt{2} \sin 15^\circ$

$= 4\sqrt{2}\sin 15^\circ$

$= 4\sqrt{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = 2\left(\sqrt{3}-1\right)$

$\left|\Delta ABC - \Delta A'B'C'\right| = \left|2\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}-1\right)\right|$

= 4

34. Note that

$\lim_{x \to 0} \frac{p(x)}{x^2} = 1$

$\Rightarrow$ p(0) = 0
p'(0) = 0
and p''(0) = 2

If $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

$p'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$p''(x) = 12ax^2 + 6bx + 2c$

p(0) = 0 gives e = 0
p'(0) = 0 gives d = 0
p''(0) = 2 gives e = 1

$p(x) = ax^4 + bx^3 + x^2$
$p'(x) = 4ax^3 + 3bx^2 + 2x$
p'(1) = 0, p'(2) = 0
4a + 3b + 2 = 0 — (1)
32a + 12b + 4 = 0 — (2)

Solving, a = $\frac{1}{4}$

3b = – 2 – 4a = – 3 $\Rightarrow$ b = – 1

p(2) = $\frac{1}{4} \times 2^4 - 2^3 + 2^2 = 4 - 8 + 4 = 0$

35. f(x) = $\int_0^x f(t)dt$

Differentiating w.r.t. x

$f'(x) = f(x)$

$\frac{dy}{dx} = y$

$\frac{dy}{y} = dx$

$y = Ce^x$

Since y must be zero when x = 0,

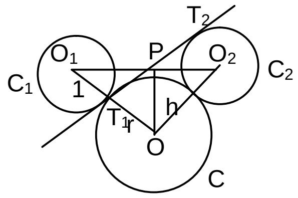
(because f(0) = $\int_0^0 f(t)dt = 0$)

C must be zero

$\Rightarrow$ f(x) = 0

f(ℓn 5) = 0

36.



Let O_1, O_2 and O be the centres of the circles C_1, C_2 and C respectively. Then $O_1P = PO_2$ = 3 units

Let T_1T_2 be the common tangent

$O_1T_1 = O_2T_2$ = 1 unit;

Let OT_1 = r units. Then

$T_1P = \sqrt{3^2 - 1^2} = 2\sqrt{2}$

Now O_1O_2O being an isosceles triangle and P is the mid point of O_1O_2, therefore OP is the altitude Let OP = h units. Then,

from ΔO_1PO, $(1 + r)^2 = 3^2 + h^2$ — (1)

from ΔOT_1P_1, $h^2 = \left(2\sqrt{2}\right)^2 + r^2$ — (2)

From (1) and (2) we get

$(1 + r)^2 - 3^2 = \left(2\sqrt{2}\right)^2 + r^2$ …

$\Rightarrow$ 2r = 16 $\Rightarrow$ r = 8 units

37. $x^2 - 8kx + 16(k^2 - k + 1) = 0$

Roots are real and distinct $\Rightarrow$ D > 0

$64k^2 - 4 \times 16(k^2 - k + 1) > 0$

$64k^2 - 64k^2 - 64k + 64 > 0$

k > 1 — (1)

$f(4) \geq 0$

$16 - 32k + 16(k^2 - k + 1) \geq 0$

$32 - 32k + 16k^2 - 16k \geq 0$

$16k^2 - 48k + 32 \geq 0$

$k^2 - 3k + 2 \geq 0$

k must lie beyond 1 and 2 — (2)

Using (1) and (2),

$k \geq 2$ — (3)

Again, the minimum point x = 4k must be > 4

k > 1

Smallest value of k = 2

38. $g(x) = f^{-1}(x) \Rightarrow f(g(x)) = x$
Differentiating w.r.t x
$f'(g(x))g'(x) = 1$
$\therefore g'(1) = \frac{1}{f'(g(1))}$ — (1)
To find g(1)
As $g \equiv f^{-1}$, g(1) is the value of x for which
f(x) = 1
i.e., $x^3 + e^{x/2} = 1$
$\phi(x) = x^3 + e^{x/2} - 1$ is an increasing function
$(\because \phi'(x) = 3x^2 + \frac{1}{2}e^{x/2} > 0)$
$\Rightarrow \phi(x)$ has atmost one zero
We observe that x = 0 makes $\phi(x) = 0$
$\Rightarrow g(x) = 0$
$\therefore g'(1) = \frac{1}{f'(0)}$
But, $f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$ so that $f'(0) = \frac{1}{2}$
$\therefore g'(1) = 2$

PART III

39	40	41	42
D	**B**	**A**	**C**

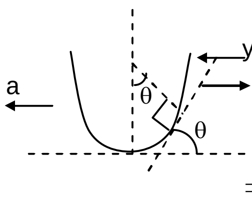
43	44	45	46	47
B, D	**D**	**B, D**	**A**	**B, C**

48	49
A –P, Q. T	**A – P, S**
B – Q	**B – Q**
C – S	**C – T**
D – S	**D – R, S, T**

50	51	52	53	54	55	56	57
9	**6**	**6**	**4**	**7**	**8**	**2**	**5**

Section I

39. $k_{eff} = \frac{k_1k_2}{k_1 + k_2}$ Let Amplitude A
$\therefore F = \frac{K_1K_2}{(K_1 + K_2)} A = K_1A' \Rightarrow A' = \frac{K_2}{K_1 + K_2} A$

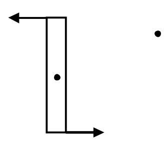
40.


$y = Kx^2$
Slope = $2kx = \frac{a}{g}$
$\Rightarrow x = \frac{a}{2kg}$

41.

	λ ev:	550 2.25	450 2.75	350 3.65
ϕ_p	2	✓	✓	✓
ϕ_q	2.5		✓	✓
ϕ_r	3.0			✓

$\therefore$ Saturation currents will be the ratio 3:2:1 for p : q : r

42.

Angular SHM : Torque = $-2 \times Kr\theta.r$
$= I\alpha = \frac{m(2r)^2}{12}\alpha$
$\Rightarrow \omega = \sqrt{\frac{6k}{m}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$

Section II

43. $\alpha = \frac{d\phi}{dt}$ same $i \propto \frac{\varepsilon}{\rho}$

$F \propto i\alpha \frac{1}{\rho}$ $a = \frac{F}{m} \propto \frac{1}{\rho m}$

$h_A > h_B \Rightarrow \frac{a_A}{a_B} > 1 \Rightarrow \frac{\rho_B m_B}{\rho_A m_A} > 1$

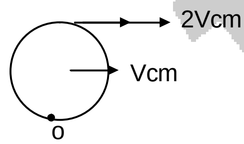
44. Amplitude of vibration much less than 1 cm Intensity $\propto$ (frequency)^2 $\Rightarrow$ same prongs in vertical plane. Closed pipe with end correction $\Rightarrow$ Air column < $\frac{1}{4}\lambda$

Obviously (B) and (C) cannot be a match (knowledge based). Now option (A) is also ruled out because; Consider the condition for forced resonance, at steady state energy input to the air column by the tuning fork is equal to energy output at resonance whatever be the harmonics. Intensity received by the student is the same.

45. AB is not shaped like a rectangular hyperbola
$\Rightarrow$ not isothermal
BCD : W : negative, T decreases $\Rightarrow \Delta u$: negative
$W_{ABC} \neq 0$ (Area)
$W_{cycle} > 0$, (Area)

46. Similar to Kepler's law – Torque is zero

47.



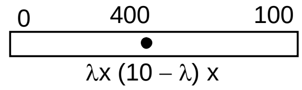
Section III

48. Knowledge based

49. Introducing film will shift central fringe up working out for each A, B, C, D based on $I = I_0 \cos^2 \frac{\theta}{2}$, where θ is phase difference I_0 : Maximum in tensity

Section IV

50.



$$\frac{\left(\frac{400-0}{\lambda x}\right)}{80} = \frac{\left(\frac{400-100}{(10-\lambda)}\right)}{540} \Rightarrow \lambda = 9$$

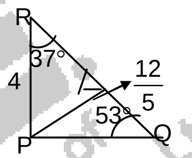
51. $p_0 \times (500 - H) = (p_0 - 200) . 300$
$\Rightarrow H = 206 \Rightarrow H - 200 = 6$ mm

52. $\Delta p = \frac{4T}{r}$, $p = p_0 + \Delta p$
$v \alpha r^3$, pv = constant
$\Rightarrow \frac{n_B}{n_A} = 6$

53. $v_{2m} = \frac{2m - m}{2m + m}.9 = 3 \Rightarrow p_{2m} = 6m\ s^{-1}$

Final velocity $= \frac{6 \times 2m}{3m} = 4\,m\,s^{-1}$

54.



$B_p = \frac{\mu 0\, i[\sin(37°)}{4\pi} \times \frac{i}{\left(\frac{12}{5}\right)} + (\sin 37° + \sin 53°)$

$\frac{\mu_0 I.k}{48\pi x} \Rightarrow k = 7$

55. $a = \frac{2m - m}{3m} g$, $(m = 0.36)$

$= \frac{g}{3} \Rightarrow s = \frac{g}{6}$ m

$T = \frac{2m(2m)}{3m}\ g = \frac{4}{3} m.g;\ w = \frac{4}{3} mg.\frac{g}{6} = 8$ J

56. $E \times 4\pi r^2 = \int_0^r kr^a.\,4\pi r^2 dr$

$E\ \alpha\ r^{a+1}$ E at $\frac{R}{2} = \frac{1}{8}E$ at R

$\Rightarrow a = 2.$

57. $v = \sqrt{\frac{T}{\mu}}, \lambda = \frac{v}{\nu}, \frac{\lambda}{2} = 5$cm