# SOLUTION & ANSWER FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2009 VERSION – B1

## [MATHEMATICS]

**1.** Ans. : $(4, \infty)$

Sol. : $\log_3(\log_4 x) > 0$
$\Rightarrow \log_4 x > 1$
$\Rightarrow x > 4$
$\Rightarrow (4, \infty)$ is the domain.

**2.** Ans. : 0, 6

Sol. : $(x-3)^2 + 1 = 10$
$\Rightarrow x - 3 = \pm 3$
$\Rightarrow x = 0$ or 6.

**3.** Ans. : 7

Sol. : $2^m - 2^n = 112$
$2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$
$\Rightarrow m = 7$.

**4.** Ans. : $\frac{x^2+2x-1}{3}$ \

Sol. : $2f(1-x) + f(x) = (1-x)^2$
$2[x^2 - 2f(x)] + f(x) = (1-x)^2]$
solving; $f(x) = \frac{x^2+2x-1}{3}$.

**5.** Ans. : $\left[\frac{1}{3}, 3\right]$

Sol. : $y = \frac{x^2-x+1}{x^2+x+1}$
$\Rightarrow (y-1)x^2 + (y+1)x + y - 1 = 0$
$\therefore (y+1)^2 - 4(y-1)^2 \geq 0$
$\Rightarrow y \in \left[\frac{1}{3}, 3\right]$.

**6.** Ans. : 10

Sol. : $\frac{1}{2}|z|^2 = 50$
$\Rightarrow |z| = 10$.

**7.** Ans. : line not passing through the origin.

Sol. : $\arg[(1-2i)(x+iy) - 2 + 5i] = \frac{\pi}{4}$
$\Rightarrow \arg[(x+2y-2) + i(y-2x+5)] = \frac{\pi}{4}$
$\Rightarrow x + 2y - 2 = y - 2x + 5$
$\Rightarrow 3x + y - 7 = 0$
a straight line not passing through the origin.

**8.** Ans. : $\frac{\pi}{3}$

Sol. : $\arg(z^2 e^{z-i}) = 2\arg z + 0$,
since $z - i = \sqrt{3}$
$= 2 \times \frac{\pi}{6} = \frac{\pi}{3}$.

**9.** Ans. : $2\omega^2$.

Sol. : Expression
$= \frac{\omega^2(c + b\omega + a\omega^2)}{a\omega^2 + b\omega + c} + \frac{\omega^2(a + b\omega + c\omega^2)}{a + b\omega + c\omega^2}$
$= 2\omega^2$.

**10.** Ans. : $1 + i(1 \pm \sqrt{3})$.

Sol. : Vertices are $i + (2 + i - i)\text{cis}\left(\frac{\pm\pi}{3}\right)$
ie, vertices are $1 + i(1 \pm \sqrt{3})$.

**11.** Ans. : 2r

Sol. : Coefficient of x = 0
$\Rightarrow p + q = 2r$.

**12.** Ans. : $\pm 3, \pm\sqrt{7}$.

Sol. : $\left(3 + 2\sqrt{2}\right)^{x^2-8} + \left(3 - 2\sqrt{2}\right)^{x^2-8} = 6$
$\Rightarrow x^2 - 8 = \pm 1$
$\Rightarrow x = \pm 3, \pm\sqrt{7}$.

**13.** Ans. : 45

Sol. : 2 + i is also a root
$\Rightarrow a = -3, b = -15$
$\therefore ab = 45$.

**14.** Ans. : 85

Sol. : Roots are equal since $m^2 = 4nl$
$\therefore$ roots are $\frac{9}{2}$ and $\frac{9}{2}$
$l = 4, n = 81$
$\Rightarrow l + n = 85$.

**15.** Ans. : $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Sol. : $16a^2 < 8$

$a^2 < \frac{1}{2}$

$a \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

**16.** Ans. : $\frac{2}{b}$

Sol. : Put a = b = c = 1

$\Rightarrow x = y = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = 2$

From options, only (A) satisfies

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{2}{b}$.

**17.** Ans. : 240

Sol. : $\left(a - \frac{27}{2}\right)10 = -30$

$a = \frac{21}{2}$

$S_{10} = 10\left(\frac{21}{2} + \frac{27}{2}\right) = 240$.

**18.** Ans. : 120

Sol. : $\frac{a_5}{a_1} = \frac{a_5}{a_4} \times \frac{a_4}{a_3} \times \frac{a_3}{a_2} \times \frac{a_2}{a_1}$

$= 5 \times 4 \times 3 \times 2$

$= 120$.

**19.** Ans. : $\tan a_n - \tan a_1$

Sol. : $\sin d \sec a_1 . \sec a_2 = \frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2}$

$= \tan a_2 - \tan a_1$ etc

$\therefore$ Expression = $\tan a_n - \tan a_1$

**20.** Ans. : $\frac{1000! - 1}{1000!}$

Sol. : $t_n = \frac{1}{n!} - \frac{1}{(n+1)!}$

sum = $S_{999} = 1 - \frac{1}{1000!}$

$= \frac{1000! - 1}{1000!}$

**21.** Ans. :

Sol. : log(1 + 3x) – log(1 – 2x)

$3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - ....$

$+ \left[2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + ....\right]$

$= 5x - \frac{5x^2}{2} + \frac{35x^3}{3} - ......$

**22.** Ans. : $\frac{1}{2}\log\left(\frac{5}{3}\right)$

Sol. : Expression = $\frac{1}{2}\log\left(1 + \frac{1}{3}\right) + \frac{1}{2}\log\left(1 + \frac{1}{4}\right)$

$= \frac{1}{2}\log\left[\frac{4}{3} \times \frac{5}{4}\right] = \frac{1}{2}\log\left(\frac{5}{3}\right)$

**23.** Ans. : $\frac{(e^2 - 1)^2}{2e^2}$

Sol. : Expression = $\frac{1}{2}\left[\frac{e^2 + e^{-2}}{2}\right] - 1$

$= \frac{(e^2 - 1)^2}{2e^2}$.

**24.** Ans. : 3

Sol. : Given expression = $\frac{(1-x)^3}{(1-x^3)^3}$

$= (1-x)^3(1-x^3)^{-3}$

Coefficient of $x^6$

$= \frac{1 \times (-3)(-4)}{2!} + (-1)(3)$

$= 6 - 3 = 3$.

**25.** Ans. : $\frac{20!}{5!15!} - \frac{15!}{5!10!}$

Sol. : Expression =

coefficient of $x^5$ in $(1 + x)^{15}(1+x)^5$

– Coefficient of $x^5$ in $(1 + x)^{15}$

$= {}^{20}C_5 - {}^{15}C_5$

$= \frac{20!}{5!15!} - \frac{15!}{5!10!}$.

**26.** Ans. : 4

Sol. : Back substituting for n from the options n = 4 satisfies the equation.

**27.** Ans. : 152

Sol. : $x = (\sqrt{3} + 1)^5 = I + f$ where $0 \le f < 1$ and $I \in Z$

$f' = (\sqrt{3} - 1)^5; 0 \le f' < 1$

$I + f - f' = 2\left[5(\sqrt{3})^4 + 10(\sqrt{3})^2 + 1\right]$

which is an integer.

$\therefore I = 152$.

**28.** Ans. : 576

Sol. : Put n = 1 only option (E) satisfies

**29.** Ans. : 8

Sol. : $^{n+1}C_3 - ^{n}C_3 = 28$

$\Rightarrow \frac{n(n-1)(3)}{6} = 28$

$\Rightarrow$ n = 8 from options.

**30.** Ans. : 0

Sol. : Given determinant

$= e^{\alpha+\beta+\gamma}\begin{vmatrix} 1 & e^{\alpha} & e^{2\alpha} \\ 1 & e^{\beta} & e^{2\beta} \\ 1 & e^{\gamma} & e^{2\gamma} \end{vmatrix} - \begin{vmatrix} e^{\alpha} & e^{2\alpha} & 1 \\ e^{\beta} & e^{2\beta} & 1 \\ e^{\gamma} & e^{2\gamma} & 1 \end{vmatrix}$

= 0 since $\alpha + \beta + \gamma = 0$.

**31.** Ans. : det(A)

Sol. : Obviously det $(B^{-1}AB)$ = det(A)

**32.** Ans. : $1 + \omega$.

Sol. : $1 + \omega + a = 0$

$\Rightarrow a = -\omega^2$ and b = 1

$a^2 + b^2 = \omega^4 + 1 = \omega + 1$.

**33.** Ans. : 2ac.

Sol. : $\begin{vmatrix} \frac{1}{a} & 4 & 1 \\ \frac{1}{b} & 3 & 1 \\ \frac{1}{c} & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \Rightarrow ab + bc = 2ac$.

**34.** Ans. : I

Sol. : $A^3 - 3A^2 + 3A = (A - I)^3 + I$

$= \begin{bmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ x & x & 0 \end{bmatrix}^3 + I$

= I.

**35.** Ans. : –6

Sol. : $A^2 - 5A = A + I$

$\therefore A^{2009} - 5A^{2008} = (A + I)A^{2007}$

$|A^{2009} - 5A^{2008}| = |A + I| \cdot |A|^{2007}$

$= (-1)^{2007} \cdot 6 = -6$.

**36.** Ans. : $(-\infty, -3)$

Sol. : $2x - 7 < 11 \Rightarrow x < 9$

$3x + 4 < -5 \Rightarrow x < -3$

$\therefore x < -3$

Answer is $(-\infty, -3)$

**37.** Ans. : $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$

Sol. : $(3 - |x|)(4 - |x|) \geq 0$ and $|x| \neq 4$

$\Rightarrow |x| \leq 3$ or $|x| > 4$

$\Rightarrow x \in [-3, 3] \cup (-\infty, -4) \cup (4, \infty)$

**38.** Ans. : $\sim(p \vee q) \cong (\sim p) \vee (\sim q)$

Sol. : $\sim(P \vee q) \cong (\sim p) \wedge (\sim q)$

so, option (E) is false.

**39.** Ans. : $[x_1 . (x_2 + x_3')] + x_2$.

Sol. : The expression is obviously $[x_1 . (x_2 + x_3')] + x_2$.

**40.** Ans. : x + x' = 1 and x. x' = 0

Sol. : Obviously x + x' = 1 and x. x' = 0

**41.** Ans: 1

Sol: $\cos A = \frac{5}{13} \Rightarrow \sin A = \frac{12}{13}$

$\sin B = \frac{12}{13} \Rightarrow \cos B = \frac{5}{13}$

$\cos(A - B) = \frac{5}{13} \cdot \frac{5}{13} + \frac{12}{13} \cdot \frac{12}{13} = 1$

$\Rightarrow x = 1$

**42.** Ans: $\sqrt{\frac{1+x^2}{2+x^2}}$

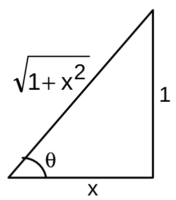
Sol: $\cos \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

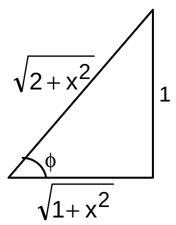
$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

$= \sqrt{\frac{1+x^2}{2+x^2}}$

$(\cot^{-1} x = \theta$

$\cot\theta = x)$




**43.** Ans: $\sqrt{a^2 - b^2}$

Sol: $f(\theta) = a\sec\theta - b\tan\theta$

$f'(\theta) = \sec\theta\,(a\tan\theta - b\sec\theta) = 0$

$\Rightarrow \theta = \frac{\pi}{2}$ or $\sin\theta = \frac{b}{a}$

$\theta \neq \frac{\pi}{2} \Rightarrow \sin\theta = \frac{b}{a}$

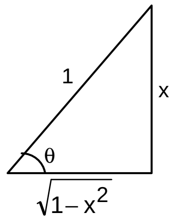
Min. value of $f(\theta) = \frac{a^2}{\sqrt{a^2 - b^2}} - \frac{b^2}{\sqrt{a^2 - b^2}}$

$= \sqrt{a^2 - b^2}$

**44.** Ans: $\frac{x}{\sqrt{1-x^2}}$

Sol: $\tan(\sin^{-1}x)$

$= \frac{x}{\sqrt{1-x^2}}$




**45.** Ans: 2

Sol: $A + B = 45^{o} \Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$

$(\cot A - 1)(\cot B - 1)$

$= \cot A \cot B - (\cot A + \cot B) + 1 = 2$

**46.** Ans: $\frac{\pi}{4}$

Sol: By back substitution

At $x = \frac{\pi}{4}$, $(\sin x + \cos x)^{1 + \sin 2x}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)^{1+1} = \left(\sqrt{2}\right)^2 = 2$

**47.** Ans.:

Sol.: If $x = \cos A + \sqrt{6}\sin A$

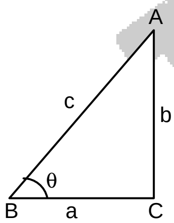
$x^2 + 7\cos^2 A = 1 + 6$

$x^2 = 7\sin^2 A$

$x = \sqrt{7}\sin A$.

**48.** Ans: 2

Sol:




$\tan A + \tan B = \frac{c^2}{ab}$ ----- (1)

$\tan A \tan B = 1 \Rightarrow A + B = \frac{\pi}{2} \rightarrow C = \frac{\pi}{2}$

$\tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

$= \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 A + \cos^2 A + \sin^2\frac{\pi}{2} = 2$

**49.** Ans: $\frac{\sqrt{5}}{2}$

Sol: If the sides containing the right angle are along the x and y axis, incentre is (1, 1) since $\Delta = s = 6$

circumcentre is $\left(\frac{3}{2}, 2\right)$

so required distance $= \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$.

**50.** Ans: $\frac{2c}{a+b+c}$

Sol: $\frac{\cot\frac{A}{2}\cot\frac{B}{2} - 1}{\cot\frac{A}{2}\cot\frac{B}{2}} = 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$= 1 - \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

$= 1 - \frac{s-c}{s} = \frac{c}{s}$

$= \frac{2c}{a+b+c}$

**51.** Ans: $c^2 - 4c - 9 = 0$

Sol: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

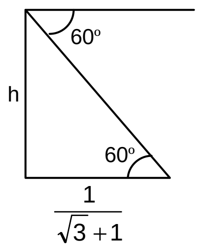
$\Rightarrow \frac{1}{2} = \frac{16 + c^2 - 25}{8c}$

$\Rightarrow 4c = 16 + c^2 - 25$

$c^2 - 4c - 9 = 0$

**52.** Ans: $\frac{3-\sqrt{3}}{2}$ metres

Sol:



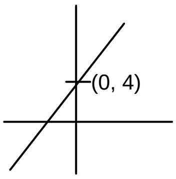

$\tan 60 = \frac{h}{\frac{1}{\sqrt{3}+1}}$

$h = \frac{1}{\sqrt{3}+1}\sqrt{3} = \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{2}$

$= \frac{3-\sqrt{3}}{2}$

**53.** Ans: 2

Sol:




y = mx + 4 i.e. mx – y + 4 = 0

perpendicular distance = $\frac{4}{\sqrt{m^2+1}}$

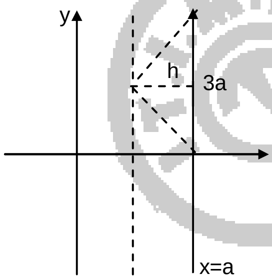
**54.** Ans: $\frac{4}{\sqrt{m^2+1}}$

Sol: conceptual line is y = mx + 4

Distance = $\frac{4}{\sqrt{m^2+1}}$.

**55.** Ans: $x = \frac{a}{3}$

Sol:




Let h be the altitude of the triangle.

Area = $\frac{1}{2} 3a \times h = a^2$

$h \Rightarrow \frac{2a}{3}$

$\therefore$ The 3rd vertex lies on $x = \frac{a}{3}$

**56.** Ans: $\sqrt{40}$

Sol: A line parallel to 3x – y = 7 is 3x – y + K = 0

If it passes through (1, 2), K = –1

$\therefore$ 3x – y – 1 = 0 and x + y + 5 = 0

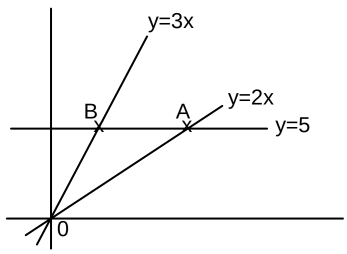
$\Rightarrow$ x = –1, y = –4

Distance of (1, 2) from (–1, –4) is

$\sqrt{4+36} = \sqrt{40}$

**57.** Ans: $\frac{25}{12}$

Sol:



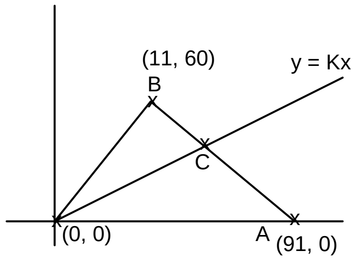

O(0, 0), $A\left(\frac{5}{2}, 5\right)$, $B\left(\frac{5}{3}, 5\right)$ are the vertices

Area = $\frac{1}{2}(x_1y_2 - x_2y_1)$

$= \frac{1}{2}\left(\frac{25}{2} - \frac{25}{3}\right) = \frac{1}{2} \times 25 . \frac{1}{6} = \frac{25}{12}$

**58.** Ans: $\frac{30}{51}$

Sol:




y = Kx cuts the triangle into two triangles of equal area when the line passes through the mid point C(51, 30) of AB

$\Rightarrow$ 30 = 51 K i.e. K = $\frac{30}{51}$

**59.** Ans: $\frac{1}{7}\left(1 \pm 5\sqrt{2}\right)$

Sol: $\frac{3-m}{1+3m} = \pm\left(\frac{\frac{1}{2}-m}{1+\frac{1}{2}m}\right)$

$\Rightarrow m^2 + 1 = 0$ or $7m^2 - 2m - 7 = 0$

$\Rightarrow m = \frac{1}{7}\left(1 \pm 5\sqrt{2}\right)$

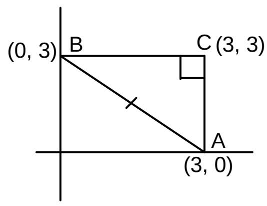
**Note:**

But $\frac{1-5\sqrt{2}}{7} < 0$

So, one cannot say that option (C) is the correct solution, since it is partially correct.

**60.** Ans: $\left(\frac{3}{2}, \frac{3}{2}\right)$

Sol:

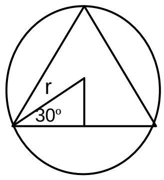



Circumcentre is the mid point of AB

i.e. $\left(\frac{3}{2}, \frac{3}{2}\right)$

**61.** Ans: $\frac{165}{8}\sqrt{3}$ square units

Sol:




Radius of the circle

$= \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{9^2}{2}\right) - 5} = \sqrt{\frac{110}{4}}$

Side of the triangle = $2 \times r\frac{\sqrt{3}}{2} = \sqrt{3}\,r$

Area=

$\frac{\sqrt{3}}{4}\left(\sqrt{3}\,r\right)^2 = \frac{\sqrt{3}}{4} \times 3 \times \frac{110}{4} = \frac{165}{8}\sqrt{3}$

**62.** Ans: $x + 3y = 0$

Sol: Centre of the circle is (3, –1) which lies on $x + 3y = 0$
$\therefore$ Diameter is $x + 3y = 0$

**63.** Ans: $\frac{15}{2}$

Sol: $a^2 - 1 = 3(a + 1)$
$a^2 - 3a - 4 = 0 \Rightarrow a = 4, -1$
$r = \frac{a^2 - 1}{2} = \frac{15}{2}$

**64.** Ans: $y^2 - 4y - 8x + 12 = 0$

Sol: Parabola is $(y - 2)^2 = 4a\,(x - 1) = 8\,(x - 1)$
i.e. $y^2 - 4y + 4 - 8x + 8 = 0$
$y^2 - 4y - 8x + 12 = 0$

**65.** Ans: 6

Sol: Centre is (2, –3)
Required distance is 2ae.
Here a = 5, e = $\frac{3}{5}$
so, sum of distance required =
$2 \times 5 \times \frac{3}{5} = 6.$

**66.** Ans: $x - y + 1 = 0$

Sol: Equation to the tangent is of the form $y = mx + K$ ----(1) (1) will be a tangent to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ if $K^2 = 3 - 2 = 1 \Rightarrow K = \pm 1$
$\Rightarrow$ Tangent is $x - y + 1 = 0$

**67.** Ans: $\frac{25}{12}$

Sol: $e_1{}^2 = \frac{a^2 - b^2}{a^2} = \frac{16 - 7}{16} = \frac{9}{16} \Rightarrow e_1 = \frac{3}{4}$

$e_2{}^2 = \frac{a^2 + b^2}{a^2} = \frac{9 + 7}{9} = \frac{16}{9} \Rightarrow e_2 = \frac{4}{3}$

$e_1 + e_2 = \frac{25}{12}$

**68.** Ans: q = 1, p = r

Sol: $\bar{p} \times \bar{q} = \bar{r};\ \bar{q} \times \bar{r} = \bar{p}$
$\bar{q} \times (\bar{p} \times \bar{q}) = p \Rightarrow (\bar{q}.\bar{q})\bar{p} - (\bar{q}.\bar{p})\bar{p} = \bar{p}$
$\Rightarrow \bar{q}.\bar{q} = 1 \Rightarrow q = 1$
p = r

**69.** Ans: 192

Sol: $\left[\left(\bar{a} + 3\bar{b}\right) \times \left(3\bar{a} + \bar{b}\right)\right]^2 = \left[8\left(\bar{b} \times \bar{a}\right)\right]^2$
$= 64\left[a^2b^2 - \left(\bar{b}.\bar{a}\right)^2\right]$
$\bar{b}.\bar{a} = |\bar{b}||\bar{a}|\cos 120 = 2.1.\left(-\frac{1}{2}\right) = -1$
$= 64[4 - 1] = 64 \times 3 = 192$

**70.** Ans: $\frac{\pi}{3}$

Sol: $\frac{\overline{a.b}}{|\bar{b}|} = |\bar{a} \times \bar{b}|$ -----(1)

$3\bar{b} = \hat{i} + \hat{j} + \hat{k} \Rightarrow |\bar{b}| = \frac{1}{\sqrt{3}}$

(1) $\Rightarrow \sqrt{3}\left(\bar{a}.\bar{b}\right) = |\bar{a} \times \bar{b}| \Rightarrow 3\left(\bar{a}.\bar{b}\right)^2$
$= \overline{a^2}\,\overline{b^2} - \left(\bar{a}.\bar{b}\right)^2$
$\left(\bar{a}.\bar{b}\right)^2 = a^2b^2 \Rightarrow \frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|} = \frac{1}{2} - \cos\theta$

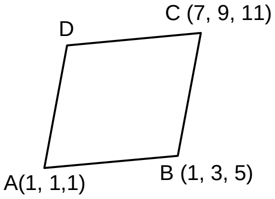
$\theta = \frac{\pi}{3}$

**71.** Ans: $6\sqrt{3}$

Sol: $\bar{x}$ and $\bar{y}$ are diagonals of parallelogram whose sides are $\bar{a}$ and $\bar{b}$.
$|\bar{x} \times \bar{y}| = 2|\bar{a} \times \bar{b}|$
$= 2 \times 2 \times 3 \times \frac{\sqrt{3}}{2}$
$= 6\sqrt{3}$.

**72.** Ans: (7, 7, 7)

Sol:




D is A + C –B i.e. (7, 7, 7)

**73.** Ans: Hyperbola

Sol: $\left(3x\,\hat{i} + y\hat{j} - 3\hat{k}\right).\left(x\,\hat{i} - 4y\hat{j} + 4\hat{k}\right) = 0$

$\Rightarrow 3x^2 - 4y^2 - 12 = 0$

$\frac{x}{4} - \frac{y^2}{3} = 1 \Rightarrow$ Hyperbola

**74.** Ans: $\frac{1}{12}$

Sol: $\left[\bar{a}\bar{b}\bar{c}\right] \neq 0$

$\left(\bar{a} + \lambda\bar{b}\right).\left[\left(\bar{b} + 3\hat{i}\right) \times \left(\bar{c} - 4\bar{a}\right)\right] = 0$

$\left[\bar{a}\bar{b}\bar{c}\right] - 12\lambda\left[\bar{b}\bar{c}\bar{a}\right] = 0$

i.e. $\left[\bar{a}\bar{b}\bar{c}\right]\left[1 - 12\lambda\right] = 0$

$1 - 12\lambda = 0$

$\lambda = \frac{1}{12}$.

**75.** Ans: $\frac{\pi}{2}$

Sol: $\frac{x - \frac{1}{3}}{1} = \frac{y+3}{-1} = \frac{z - \frac{5}{2}}{-2}$ ----- (1)

$3x - 3y - 6z = 10$

D. R's of line are 1, –1, –2

D. R's of normal to the plane are 1, –1, –2

$\therefore$ Line is parallel to the normal

$\therefore$ angle = $\frac{\pi}{2}$

**76.** Ans: $\cos^{-1}\left(\frac{34}{63}\right)$

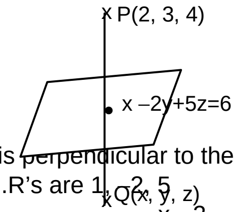
Sol: $\bar{r} = \left(2\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(-3\hat{i} + 2\hat{j} + 6\hat{k}\right)$

$\bar{r} = \left(\hat{i} + 2\hat{j} - \hat{k}\right) + s\left(4\hat{i} - \hat{j} + 8\hat{k}\right)$

$\cos\theta = \frac{-12 - 2 + 48}{\sqrt{49}\sqrt{81}} = \frac{34}{7 \times 9} = \frac{34}{63}$

**77.** Ans: $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$

Sol:




PQ is perpendicular to the plane

$\therefore$ D.R's are 1, –2, 5

Equation to PQ is $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$

**78.** Ans: 13

Sol: Any point on the line is

$(3\lambda + 2,\ 4\lambda - 1,\ 12\lambda + 2)$

If it lies on plane,

$3\lambda + 2, -4\lambda + 1 + 12\lambda + 2 = 5$

$11\lambda = 0 \quad \lambda = 0$

Point is (2, –1, 2)

Distance = 13

**79.** Ans: $c = \pm\sqrt{3}$

Sol: $\frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow \frac{3}{c^2} = 1$

$c^2 = 3$

$c = \pm\sqrt{3}$

**80.** Ans: $\bar{r}.\left[\bar{r} - \left(2\hat{i} - 3\hat{j} - 4\hat{k}\right)\right] = \frac{5}{2}$

Sol: $x^2 + y^2 + z^2 - (2x - 3y - 4z) = \frac{5}{2}$

$\bar{r}.\left[\bar{r} - \left(2\hat{i} - 3\hat{j} - 4\hat{k}\right)\right] = \frac{5}{2}$

**81.** Ans. : $\frac{-10}{7}$

Sol. : $-3 \times 3\alpha + 2\alpha \times 1 + 2 \times -5 = 0$

$7\alpha = -10 \Rightarrow \alpha = \frac{-10}{7}$.

**82.** Ans. : 0

Sol. : $\frac{\left(\bar{a}_2 - \bar{a}_1\right) \times \bar{b}}{\left|\bar{b}\right|}$

$= \frac{\left(-3\hat{i} + 7\hat{j} - 4\hat{k}\right) \times \left(3\hat{i} - 7\hat{j} + 4\hat{k}\right)}{\sqrt{9 + 49 + 16}} = 0.$

**83.** Ans. : $\frac{3\sqrt{33}}{2}$

Sol. : Standard deviation of 12, 3, … =

$\sqrt{\frac{99}{12}} = \frac{\sqrt{33}}{2}$

standard deviation of 3, 6, 9, ….

$= \frac{3\sqrt{33}}{2}$.

**84.** Ans. : $\frac{n}{2}$

Sol. : $\frac{0.^nC_0 + 1.^nC_1 + 2.^nC_2 + ..... + n.^nC_n}{2^n}$

$= \frac{1}{2^n}\left(n.2^{n-1}\right) = \frac{n}{2}$.

**85.** Ans. : $\frac{7}{12}$

Sol. : P(Light will not be green) = $\frac{30+5}{60}$

$= \frac{7}{12}$.

**86.** Ans. : $\frac{1}{55}$

Sol. : k(1 + 2 + 3+ …+ 10) = 1

$k\left(\frac{10\times 11}{2}\right) = 1$

$k = \frac{1}{55}$.

**87.** Ans. : $a^2\frac{(\alpha-\beta)^2}{2}$

Sol. : $\underset{x\to\alpha}{\text{Lt}} \frac{2\sin^2 \frac{a(x-\alpha)(x-\beta)}{2}}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{2^2}} \times \frac{a^2(x-\beta)^2}{4}$

$= 2.1.\frac{a^2(\alpha-\beta)^2}{4} = a^2\frac{(\alpha-\beta)^2}{2}$

**88.** Ans. : 103.

Sol. : There are 103 integers in $\left(\frac{-7}{2}, 100\right)$

Required no. of discontinuities = 103.

**89.**

Ans. : $\frac{3\pi}{10}$

Sol. : $\underset{x\to 0}{\text{Lt}} \frac{3\sin\pi x}{5x} = 2k$

$\underset{x\to 0}{\text{Lt}} \frac{3}{5}.\frac{\sin\pi x}{\pi x}\times\pi = 2k$

$\frac{3\pi}{5} = 2k$

$\therefore k = \frac{3\pi}{10}$.

**90.** Ans. : $\frac{3}{2}$

Sol. : f(f(x)) = x + 1

f(f(0)) = 0 + 1 = 1

$f\left(\frac{1}{2}\right) = 1$

$f(1) = f\left(f\left(\frac{1}{2}\right)\right)$

$= \frac{1}{2} + 1 = \frac{3}{2}$.

**91.** Ans. : $\frac{\log_2 e}{x\log_e x}$

Sol. : $y = \log_2(\log_2 x)$

$= \log_2\left(\frac{\log x}{\log 2}\right)$

$= \log_2 \log x - \log_2 \log 2$

$= \frac{\log(\log x)}{\log_e 2}$ – constant

$\therefore \frac{dy}{dx} = \frac{1}{\log 2}\times\frac{1}{\log x}\times\frac{1}{x}$

$= \frac{\log_2 e}{x\log_e x}$.

**92.** Ans. : $\frac{3x^2}{1+x^6}$.

Sol. : $\frac{d}{dx}f(x) = \frac{1}{1+x^2}$

$\frac{d}{dx}f(x^3) = f'(x^3)3x^2$

$= \frac{3x^2}{1+x^6}$.

**93.** Ans. : 1

Sol. : $y = \sin\left\{\cos^{-1}\left[\sin\left(\sin^{-1}\sqrt{1-x^2}\right)\right]\right\}$

$= \sin\left\{\cos^{-1}\sqrt{1-x^2}\right\}$

$= \sin\ \sin^{-1}x = x$

$\therefore \frac{dy}{dx} = 1$.

**94.** Ans. : $\frac{1}{yx^3}$

Sol. : $\left(t - \frac{1}{t}\right)^2 = t^2 + \frac{1}{t^2} - 2$

$\Rightarrow (x^2+y^2)^2 = x^4 + y^4 - 2$

$\Rightarrow x^2y^2 = -1 \Rightarrow y^2 = \frac{-1}{x^2}$

$\Rightarrow 2yy' = \dfrac{2}{x^3} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{yx^3}$ .

**95.** Ans. : –4.

Sol. : $y = 2\cos^{-1}(\sin x) + 2\sin^{-1}(\cos x)$

$= 2\cos^{-1}\cos\left(\dfrac{\pi}{2} - x\right)$

$+ 2\sin^{-1}\sin\left(\dfrac{\pi}{2} - x\right)$

$= \pi - 2x + \pi - 2x$

$y = 2\pi - 4x$

$y' = -4$.

**96.** Ans. : 4e.

Sol. : $y = e^x.e^{x^2}.e^{x^3}.....e^{x^n}....$

$= e^{x+x^2+x^3+...+x^n...}$

$y = e^{x\left(\frac{1}{1-x}\right)}$

$\log y = \dfrac{x}{1-x}$

$\dfrac{1}{y}y' = \dfrac{1-x+x}{(1-x)^2}$

$y' = y \times 4$

$(y')_{x=\frac{1}{2}} = 4e$ since $y = e$ at $x = \dfrac{1}{2}$ .

**97.** Ans. : $\dfrac{2\sqrt{1-x^2}}{1+x^2}$

Sol. : $\dfrac{d\left(\tan^{-1}\dfrac{2x}{1-x^2}\right)}{d\left(\cos^{-1}\sqrt{1-x^2}\right)}$

$= \dfrac{d\left(2\tan^{-1}x\right)}{d\left(\sin^{-1}x\right)} = \dfrac{\left(\dfrac{2}{1+x^2}\right)}{\left(\dfrac{1}{\sqrt{1-x^2}}\right)}$

$= \dfrac{2\sqrt{1-x^2}}{1+x^2}$ .

**98.** Ans. : 4

Sol. : $y_1 = \dfrac{\dfrac{-2x}{a^2}}{\dfrac{2y}{12}}$

$m_1 = \dfrac{-12x}{a^{2y}}, m_2 = \dfrac{8}{3y^2}$

$3y^2y_1 = 8$

$\therefore y_1 = \dfrac{8}{3y^2}$,

$\dfrac{-12x}{a^2y} \times \dfrac{8}{3y^2} = -1$

$\dfrac{32}{8a^2} = 1$

$a^2 = 4$

**99.** Ans. : $\dfrac{1}{6}$

Sol. : $f'(x) = 3x^2 - 24ax + 36a^2$

$0 = 3(x^2 - 8ax + 12a^2)$

ie $(x - 6a)(x - 2a) = 0$

$x = 6a, 2a$

$f''(x) = 6x - 24a$

$f'(6a) > 0$, $f''(2a) < 0$

$3(2a) = 36a^2$

$6a = 36a^2$

$\therefore a = \dfrac{1}{6}$ .

**100.** Ans. : x + y = 4.

Sol. : Curve crosses y axis

$\therefore$ y = 4 by putting x = 0

$\therefore m = \left(\dfrac{dy}{dx}\right)_{(0,4)}$

$-1 = 4e^0 \times \dfrac{1}{4}$

$\therefore$ Equation of tangent is $y - 4 = -x$

$\Rightarrow x + y = 4$.

**101.** Ans. : $10\sqrt{2}$ cm²/sec.

Sol. : $D = \sqrt{2}x$

$\dfrac{d(D)}{dt} = \sqrt{2}.\dfrac{dx}{dt}$

$= \sqrt{2} \times .5$

$A = x^2$; $x^2 = 400$, $x = 20$

$\dfrac{dA}{dt} = 2x.\dfrac{dx}{dt}$

$\left(\dfrac{dA}{dt}\right)_{x=20} = 2 \times 20 \times \dfrac{.5}{\sqrt{2}} = 10\sqrt{2}$ .

**102.** Ans. : 2x + y – 6 = 0.

Sol. : $2x - 2y - 2xy_1 + 2yy_1 + 2 + y_1 = 0$

$y_1(1 + 2y - 2x) = 2y - 2x - 2$

$y_1 = \dfrac{2(y - x - 1)}{1 + 2y - 2x}$

$m = \dfrac{2(-1)}{1} = -2$

$y - 2 = -2(x - 2)$

$2x + y - 6 = 0$.

**103.** Ans. : $\tan^{-1}\left|\dfrac{\log - \log b}{1 + \log a \log b}\right|$.

Sol. : $a^x = b^x$ for x = 0

∴ Intersecting point (0, 1)

$m_1 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log a$

$m_2 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log b$

$\tan\theta = \left|\frac{\log a - \log b}{1+\log a \log b}\right|.$

$\therefore \theta = \tan^{-1}\left|\frac{\log - \log b}{1+\log a \log b}\right|.$

**104.** Ans. : $\frac{53}{9}$

Sol. : $f'(x) = 2(x-7)(x-2)^7 + 7(x-7)^2(x-6)^6$

$= (x-7)(x-2)^6[2(x-2)+7(x-7)]$

$= (x-7)(x-2)^6(9x-53)$

$f'(\theta) = 0$

$\therefore \theta = \frac{53}{9}.$

**105.** Ans. : $\frac{5}{8}\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}} + C$

Sol. : $\int x^{\frac{1}{3}}.\left(1+x^{\frac{4}{3}}\right)^{\frac{1}{5}} dx,$

$1+x^{\frac{4}{3}} = t \qquad \therefore\ x^{\frac{1}{3}}dx = \frac{3}{4}dt$

$\frac{3}{4}\int t^{\frac{1}{5}}dt = \frac{3}{4}.\frac{t^{\frac{6}{5}}}{\frac{6}{5}} + C$

$= \frac{5}{8}\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}} + C$

**106.** Ans. : f(θ) + f''(θ) + C.

Sol. : $\frac{du}{d\theta} = -f''(\theta)\cos\theta - \sin\theta f'''(\theta)$

$-f'(\theta)\sin\theta + f''(\theta)\cos\theta$

$= -\sin\theta\,(f'(\theta) + f'''(\theta))$

$\frac{dv}{d\theta} = -f''(\theta)\sin\theta + f'''(\theta)\cos\theta$

$+f''(\theta)\sin\theta + f''(\theta)\cos\theta$

$= \cos\theta(f'(\theta) + f'''(\theta))$

squaring and adding and integrating we get the answer.

**107.** Ans. : $\frac{x^3}{3} + C$

Sol. : $\int \frac{x^6 - x^5}{x^4 - x^3}dx = \int \frac{x^5(x-1)}{x^3(x-1)}dx$

$= \int x^2 dx$

$= \frac{x^3}{3} + C$

**108.** Ans. : $e^x\left(\frac{1}{1+x^2}\right) + C$

Sol. : $\int e^x.\frac{1+x^2-2x}{(1+x^2)^2}dx$

$= \int e^x\left(\frac{1+x^2}{(1+x^2)^2} - \frac{2x}{(1+x^{2^2})}\right)dx$

$= e^x\left(\frac{1}{1+x^2}\right) + C$

**109.** Ans. : $\frac{\sqrt{x^4+x^2+1}}{x} + C$

Sol. : Substituting for $\frac{\sqrt{x^4+x^2+1}}{x} = t$

$\frac{x.\frac{1}{2\sqrt{x^4+x^2+1}}4x^3 + 2x - \sqrt{x^4+x^2+1}}{x^2}dx = dt$

$\frac{4x^4 + 2x^2 - 2x^4 - 2x^2 - 2}{2x^2\sqrt{x^4+x^2+1}}dx = dt$

$\frac{2x^4 - 2}{2x\left(x\sqrt{x^4+x^2+1}\right)}dx = dt$

$\frac{x^4 - 1}{x^2\sqrt{x^4+x^2+1}}dx = dt$

$\int dt = t + C$

$= \frac{\sqrt{x^4+x^2+1}}{x} + C.$

**110.** Ans. : $\frac{-1}{\sin x + \cos x} + C$

Sol. : $\int \frac{\cos x - \sin x}{(\cos x + \sin x)^2}dx$

put cosx + sinx = u

$\therefore \int \frac{1}{u^2}du = \frac{-1}{u} + C = \frac{-1}{\cos x + \sin x} + C$

**111.** Ans. : $\log_e(1+\log_e x)+C$

Sol. : $\int \frac{1}{x}(\log_{ex} e)dx = \int \frac{1}{x}.\frac{\log_e e}{\log_e ex}dx$

$= \int \frac{1}{x}\frac{1}{(1+\log x)}dx$

$= \int \frac{du}{u}$ (put $1+\log x = u$)

$= \log(1+\log x)+C$.

**112.** Ans. : $\frac{10e}{\log_e 10e}$

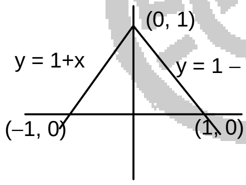
Sol. : $\int_1^e 10^{\log_{10} x.\log_e 10}dx = \int x^{\log_e 10}dx$

$= \left[\frac{x^{\log_e 10+1}}{\log_e 10+1}\right]_1^e = \frac{e^{\log_e 10+\log_e e}}{1+\log_e 10}$

$= \frac{e^{\log_e 10e}}{\log_e 10e} = \frac{10e}{\log_e 10e}$.

**113.** Ans. : 1 sq. unit

Sol. :




Area = $\frac{1}{2}\times 2\times 1 = 1$

**114.** Ans. : 1

Sol. : $\int_{\frac{1}{e}}^{e}\left(\frac{1}{t}-\frac{1}{t+1}\right)dt = [\log t-\log(t+1)]_{\frac{1}{e}}^{e}$

$= \log e-\log(e+1)-\log\frac{1}{e}+\log\left(\frac{1}{e}+1\right)$

= 1.

**115.** Ans. : $\frac{3}{4}$.

Sol. : Area of $\Delta AOB = \frac{1}{2}\times 2a\times a^2 = a^3$

Area of parabola = $2\int_0^{a^2}\sqrt{y}dy$

$= 2\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_0^{a^2} = \frac{4a^3}{3}$

Ratio is $a^3 : \frac{4}{3}a^3$

ie., 3 : 4.

**116.** Ans. : 13.

Sol. : $\int_{-2}^{-1} -(x+1)dx + \int_{-1}^{4}(x+1)dx$

$= \left(\frac{-x^2}{2}-x\right)_{-2}^{-1} + \left(\frac{x^2}{2}+x\right)_{-1}^{4}$

$= \left(-\frac{1}{2}+1\right)+\left(\frac{4}{2}-2\right)+\left(\frac{16}{2}+4\right)-\left(\frac{1}{2}-1\right)$

= 13.

**117.** Ans. : $e^x+e^{-\sin y}+\frac{x^3}{3}=C$

Sol. : $\cos y\frac{dy}{dx} = e^{x+\sin y}+x^2e^{\sin y}$

$= e^{\sin y}(e^x+x^2)$

$\int e^{-\sin y}\cos y dy = \int (e^x+x^2)dx$

$-e^{-\sin y} = e^x+\frac{x^3}{3}+C$.

**118.** Ans. : (2, 4)

Sol. : $(1+y_1^2)^{\frac{3}{4}} = (y_2)^{\frac{1}{3}}$

Taking 12th power

$(1+y_1^2)^9 = y_2^4$

$\therefore$ Order 2 degree 4.

**119.** Ans. : logy

Sol. : $y\log y\frac{dx}{dy} = \log y - x$

$\frac{dx}{dy}+\frac{x}{y\log y} = \frac{1}{y}$

$\int pdy = \int \frac{1}{y\log y}dy$ = log(logy)

$\therefore$ IF = logy

**120.** Ans. : $x = -y^2-2y-2+Ce^y$

Sol. : $x+y^2 = \frac{dx}{dy}$

$\frac{dx}{dy}-x = y^2$

$e^{\int pdx} = e^{\int -dy} = e^{-y}$

$\therefore\ xe^{-y} = \int e^{-y}.y^2dy$

$= -y^2e^{-y}-\int 2y\frac{e^{-y}}{-1}dy$

$$= -y^2e^{-y} + 2\left[\frac{ye^{-y}}{-1}\right] - 2\int \frac{e^{-y}}{-1}\,dy$$

$$= -y^2e^{-y} - 2ye^{-y} - 2e^{-y} + C$$

$$\therefore x = -y^2 - 2y - 2 + Ce^{y}$$