SOLUTION & ANSWER FOR COMED-K-2009
VERSION – A

[MATHEMATICS]

1. If $\bar{u} = \bar{a} - \bar{b}.\bar{v} = \bar{a} + \bar{b}$ and $|\bar{a}| = |\bar{b}| = 2$.

then $|\bar{u} \times \bar{v}|$ is..

Ans: $2\sqrt{16 - (\bar{a}.\bar{b})^2}$

Sol. : $\left|(\bar{a} - \bar{b}) \times (\bar{a} + \bar{b})\right| = 2\left|(\bar{a} \times \bar{b})\right|$

$= 2\sqrt{\bar{a}^2 \bar{b}^{2-(\bar{a}.\bar{b})^2}}$

$= 2\sqrt{16 - (\bar{a}.\bar{b})^2}$.

2. The volume of the tetrahedron formed by the …

Ans: $\frac{5}{6}$

Sol. : Volume = $\frac{1}{6}\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} = \frac{1}{6} \times 5 = \frac{5}{6}$

3. Unit vector perpendicular to $\hat{i} - 2\hat{j} + 2\hat{k}$ …..

Ans: $\frac{1}{\sqrt{234}}\left(8\hat{i} - 7\hat{j} - 11\hat{k}\right)$.

Sol.: By trial, the only unit vector is in (d) and it is perpendicular to $\hat{i} - 2\hat{j} + 2\hat{k}$.

4. In the group Q – {–1} under the binary operation * defined by …….

Ans: $\frac{-10}{11}$ **.**

Sol: 0 is the identity element

$10 + b + 10b = 0 \Rightarrow b = \frac{-10}{11}$.

5. In the group {1, 2, 3, 4, 5, 6} under multiplication ……

Ans: 2

Sol: $2^{-1} = 4$, $4 \times 4 = 2$.

6. The group (Z, +) has

Ans: Infinitely many subgroups.

Sol: {m n : for any given integer m and n $\in$ Z} is a subgroup.

so infinite.

7. If $3x \equiv 5 \pmod 7$, then

Ans: $x \equiv 4$ (mod7)

Sol: Back substitution

8. The argument of the complex …..

Ans: $\frac{9\pi}{10}$

Sol: $\sin\frac{6\pi}{5} + i\left(1 + \cos\frac{6\pi}{5}\right)$

$= 2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5} + i\cos^2\frac{3\pi}{5}$

$= 2\cos\frac{3\pi}{5}\left(\cos\left(\frac{-\pi}{10}\right) + i\sin\left(\frac{-\pi}{10}\right)\right)$

$= 2\left|\cos\frac{3\pi}{5}\right|\left[\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}\right]$

so Arg. = $\frac{9\pi}{10}$.

9. The maximum value of n < 101 …..

Ans: 99

Sol: sum of 4 consecutive powers of i = 0.

10. The value of $\left(-1 + \sqrt{-3}\right)^{\omega}$ + …….

Ans: -2^{62}.

Sol: It is $(2\omega)^{62} + (2\omega^2)^{62}$

$= 2^{62}(\omega^2 + \omega)$

$= 2^{62}(-1) = -2^{62}$.

11. All complex numbers z which …….

Ans: real axis.

Sol: Locus of points equidistant from –6i & 6i
so, real axis.

12. The value of $\sin\left[\cot^{-1}\{\cos(\tan^{-1}x)\}\right]$ is

Ans: $\left(\sqrt{\frac{1+x^2}{2+x^2}}\right)$

Sol: $\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$

$\cos(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$

$\cot^{-1}\cos(\tan^{-1}x) = \tan^{-1}\sqrt{1+x^2}$

$= \sin^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

$\sin\cot^{-1}\cos(\tan^{-1}x) = \sqrt{\frac{1+x^2}{2+x^2}}$.

13. The value of α ($\neq 0$) for which the function ….

Ans: $\alpha = -1$

Sol: $1 + \alpha x = \frac{x-1}{\alpha}$

$\Rightarrow \alpha + \alpha^2 x = x - 1 \Rightarrow \alpha = -1$.

14. If x^r occurs is the expansion of $\left(x+\frac{1}{x}\right)^n$, ….

Ans: $\frac{n!}{\left(\frac{n-r}{2}\right)!\left(\frac{n+r}{2}\right)!}$

Sol: $T_{s+1} = {}^nC_s x^{n-s}.\frac{1}{x^s} = {}^nc_s x^{n-2s}$

$n - 2s = r \Rightarrow s = \frac{n-r}{2}$

coeffi. $= {}^nC_{\frac{n-r}{2}} = \frac{n!}{\left(\frac{n-r}{2}\right)!\left(\frac{n+r}{2}\right)!}$.

15. If tanA – tanB = x and cotB – cotA = y ….

Ans: $\frac{1}{x}+\frac{1}{y}$

Sol: $\cot(A-B) = \frac{1+\tan A\tan B}{\tan A-\tan B}$

$= \frac{1+x/y}{x} = \frac{1}{x}+\frac{1}{y}$.

16. $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12} =$

Ans: $\frac{3}{2}$.

Sol: $\frac{1}{2}\left[1+\cos\frac{\pi}{6}+1+\cos\frac{\pi}{2}+1+\cos\frac{5\pi}{6}\right]$

$= \frac{1}{2}(3+0+0) = \frac{3}{2}$

17. If $\sin\theta$, $\cos\theta$ and $\tan\theta$ are in GP then …

Ans: 1

Sol: $\cos^2\theta = \sin\theta\tan\theta$

$\Rightarrow \cos^3\theta = \sin^2\theta$

$\Rightarrow \cos^3\theta = \text{cosec}\,\theta$

$\Rightarrow \cot^6\theta = \text{cosec}^2\theta = \cot^2\theta + 1$

$\Rightarrow \cot^6\theta - \cot^2\theta = 1$.

18. If $\frac{3x^2-2x+4}{(x+1)^6} = \frac{A_1}{x+1}+\frac{A_2}{(x+1)^2}$.....,

Ans: (–8, 12).

Sol: $y = x + 1 \Rightarrow$

$\frac{3x^2-2x+4}{(x+1)^6} = \frac{3}{y^4}-\frac{8}{y^5}+\frac{9}{y^6}$

$\Rightarrow A_4 = 3, A_2 = -8, A_6 = 9$

$\Rightarrow (-8, 12)$.

19. If $\log_2(2^{x-1} + 6) + \log_2(4^{x-1}) = 5$, then x =

Ans: 2.

Sol: By back substitution, x = 2

20. If a, b, c, d are the roots of the equation ….

Ans: –1.

Sol: $1 + \sum a^2 = 1 + (\sum a^2) - 2\sum ab$

$= 1 + (-2)^2 - 2(3)$

$= 1 + 4 - 6 = -1$.

21. If $C_0, C_1, C_2, ……C_n$ are binomial coefficients of …….

Ans: $\frac{2^n-1}{n+1}$

Sol: $(1+x)^n = C_0 + C_1x + C_2x^2 + ...$

$\frac{(1+x)^{n+1}-1}{n+1} = C_0x + \frac{C_1}{2}x^2 + \frac{C_2}{3}x^3 + ..$

$\Rightarrow \frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_3}{3} + ...$

and $\frac{-1}{n+1} = -C_0 + \frac{C_1}{2} - \frac{C_2}{3} + ..$

$\Rightarrow \frac{C_1}{2} + \frac{C_3}{4} + ... = \frac{2^n-1}{n+1}$

22. The value of $(0.2)^{\log_{\sqrt5}\left(\frac14+\frac18+\frac1{16}....\text{to }\infty\right)}$ is

Ans: 4

Sol: $(.2)^{\log_{\sqrt5}\left(\frac14+\frac18+.....\text{to }\alpha\right)}$

$= \left(\frac{1}{5}\right)^{\log_{\sqrt5}1/2} = 5^{\log_5 4} = 4$

23. If n(A) = n(B) = m, then the number of possible……….

Ans: m!

Sol: (Conceptual)

24. $\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]=$

Ans: $\sin^{-1}x - \sin^{-1}\sqrt{x}$.

Sol: $x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}$

= "$\sin\theta\cos\phi - \cos\theta\sin\phi$"

= $\sin(\theta - \phi)$

$\therefore\ \sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$

= '$\theta - \phi$'

= $\sin^{-1}x - \sin^{-1}\sqrt{x}$.

25. If $\tan\theta + \tan4\theta + \tan7\theta = \tan\theta\tan4\theta\tan7\theta$, ….

Ans: $\frac{n\pi}{12}$

Sol: $\tan\theta + \tan4\theta = -\tan7\theta\,(1 - \tan\theta\tan4\theta)$

$\Rightarrow \tan5\theta = -\tan7\theta$

$\Rightarrow 5\theta = n\pi - 7\theta$

$\Rightarrow \theta = \frac{n\pi}{12}$

26. If a circle with the point (–1, 1) as its centre …..

Ans: (–3, –3)

Sol: (back substitution)

27. If the circles $x^2 + y^2 + 2gx + 2fy = 0$ and ….

Ans: gf' = g'f

Sol: If they touch, they touch at (0, 0)

$\therefore$ gx + fy = 0, g'x + f'y = 0 are identical.

(tangents at the origin)

$\therefore\ \frac{g}{g'} = \frac{f}{f'}$

gf' = g'f

28. The number of common tangents of the circles $x^2 + y^2 = 4$ and ……..

Ans: 2

Sol: Centre of 1st (0, 0), radius = 2

centre of 2nd (2, –1), radius = 3

distance between centres = $\sqrt{5}$

and $3 - 2 < \sqrt{5} < 3 + 2$

circles intersect; so 2 common tangents

No. of common tangents = 2

29. The length of the tangent drawn from any point on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ …..

Ans: 2

Sol: The length of tangent

$= \sqrt{x_1^2 + y_1^2 - 4x_1 + 6y_1}$

where $x_1^2 + y_1^2 - 4x_1 + 6y_1 = 4$

$= \sqrt{4} = 2$.

30. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$……

Ans: $b^2 = 16$.

Sol: Foci are the same

$\therefore\ \sqrt{25 - b^2} = \sqrt{\frac{144}{25} + \frac{81}{25}}$

$\Rightarrow b^2 = 16$.

31. The latus rectum of the ellipse is half the minor axis. Then …………

Ans: $e = \frac{\sqrt{3}}{2}$

Sol: $2\frac{b^2}{a} = b$

$\Rightarrow 4b^2 = a^2$

$\Rightarrow 4(1 - e^2) = 1 \Rightarrow e^2 = 3/4$

$e = \frac{\sqrt{3}}{2}$

32. The ends of the latus rectum of the ……

Ans: (3, 4) & (–13, 4)

Sol: Equation is $(x + 5)^2 = 16y$

& equation of L. R are

(8 – 5, 4) & (–8 –5, 4)

ie, (3, 4) & (–13, 4).

33. Which of the following functions is …..

Ans: $\sin|x| - |x|$

Sol: $\frac{\sin|x| - |x|}{x} = -\frac{1}{6}x|x| + ...$

$\rightarrow 0$ as $x \rightarrow 0$

$= \sin|x| - |x|$.

34. If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, y = a….

Ans: tan t

Sol: $\frac{dy}{dx} = \frac{a\cos t}{a[-\sin t + \text{cosect}]} = \frac{\cos t.\sin t}{1 - \sin^2 t}$

= tan t

35. If $\begin{bmatrix} 1 & -\tan\theta \\ -\tan\theta & 1 \end{bmatrix}$.......

Ans: $a = \cos 2\theta \;\; b = \sin 2\theta$

Sol: $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1}$

$= \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ -\tan\theta & 1 \end{bmatrix}$

so the product =

$$\cos^2\theta\left[= \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ -\tan\theta & 1 \end{bmatrix}^2\right]$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$= a = \cos 2\theta \;\; b = \sin 2\theta$

36. If A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n is

Ans: $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

Sol: Back substitution .

37. If α, β, γ are the roots of the equation ….

Ans: 0

Sol: $\alpha + \beta + \gamma = 0$

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0.$$

38. The number of distinct real root of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$

Ans:

Sol: (Question is incomplete)

39. The sum of non-prime positive divisors of 450 is

Ans: 1199.

Sol: Sum of all divisors

$= (1 + 2)\,(1 + 3 + 3^2)\,(1 + 5 + 5^2)$

$= 1209$

Sum of prime divisors = 2 + 3 + 5 = 10

$\therefore$ required sum = 1209 – 10 = 1199.

40. The last digit of $\sum_{\substack{1<p<100 \\ p-\text{prime}}} p! - \sum_{n=1}^{50} (2n)!$ is

Ans: 2

Sol: $\sum p!$ = 2 + 6 + multiple of 10

$\sum (2n)!$ = 2 + 24 + multiple of 10

$\therefore \sum p! - \sum (2n!)$ = 2 + multiple of 10

Ans. : 2.

41. The interval I such that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} \in I$ is given by

Ans: $\left[\frac{1}{\sqrt{2}}, 1\right]$

Sol: $\frac{1}{\sqrt{1+x^4}} \le 1$ in [0, 1]

$\int_0^1 \frac{dx}{\sqrt{1+x^4}} \le 1$

The only relevant choice is (b)

Ans.: $\left[\frac{1}{\sqrt{2}}, 1\right]$

42. $\int_0^{\frac{\pi}{2}} \log(\tan x)\,dx =$

Ans: 0

Sol: $\int_0^{\pi/2} \log \tan x\,dx = \int_0^{\pi/2} \log \cot x\,dx$

$= \frac{1}{2}\int_0^{\pi/2} \log 1\,dx = 0.$

43. The value of $\int_{-2}^{2} \left(ax^3 + bx + c\right)dx$ …..

Ans: "value of C"

Sol: (Conceptual)

44. The area of the region bound by the ….

Ans: $\frac{8}{3}$

Sol: Where the curves meet x = 0 or 2

$\therefore$ area = $\int_0^2 \left(4x - x^2 - x^2\right)dx$

$= \left(2x^2 - \frac{2x^3}{3}\right)_0^2 = 8 - \frac{16}{3} = \frac{8}{3}.$

45. The particular solution of $\frac{y}{x}\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, …..

Ans: $5(1 + x^2) = 2(1 + y^2)$

Sol: $\frac{ydy}{1+y^2} = \frac{xdx}{1+x^2} \Rightarrow \log\frac{1+y^2}{1+x^2} = C$

$y(1) = 2 \Rightarrow \log 5/2 = C$

$\therefore$ solution is $\frac{1+y^2}{1+x^2} = 5/2$

Ans.: $5(1 + x^2) = 2(1 + y^2)$

46. The solution of the differential equation $\frac{dy}{dx} = (x+y)^2$ is

Ans: $\tan^{-1}(x + y) = x + c$.

Sol: $\frac{dz}{dx} = 1 + z^2$ if $z = x + y$

$\Rightarrow \tan^{-1}z = x + c$

$\Rightarrow \tan^{-1}(x + y) = x + c$.

47. The maximum value of $\left(\frac{1}{x}\right)^{2x^2}$ is

Ans: $\sqrt[e]{e}$

Sol: $f(x) = \left(\frac{1}{x}\right)^{2x^2}$

$\Rightarrow f'(x) = \left(\frac{1}{x}\right)^{2x^2}[-2x(2\log x + 1)]$

and f '' $\left(\frac{1}{\sqrt{e}}\right)$ is –ve.

$\therefore$max. f(x) =

$$\left[\frac{1}{\left(\frac{1}{\sqrt{e}}\right)}\right]^{\frac{2}{e}} = e^{1/e}$$

48. Let x be a number which exceeds its square by ………..

Ans: $\frac{1}{2}$

Sol: $f(x) = x - x^2 \Rightarrow f'(x) = 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$

$f''\left(\frac{1}{2}\right) < 0$.

$\therefore$ f is max. when $x = \frac{1}{2}$

49. The subtangent at $x = \pi/2$ on the curve …..

Ans: $\pi/2$

Sol: subtangent

$$= \left(\frac{y}{\frac{dy}{dx}}\right) = \frac{\frac{\pi}{2}\sin\pi/2}{\frac{\pi}{2}\cos\pi/2 + \sin\pi/2} = \pi/2\,..$$

50. The value of $\int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^{x}}}dx$ is

Ans: $\frac{1}{\log 10}\sin^{-1}(10^x) + C$

Sol: $\int \frac{10^{x/2}dx}{\sqrt{10^{-x} - 10^{x}}} =$

$$\frac{1}{\log 10}\int \frac{10^x \log 10 dx}{\sqrt{1-(10x)^2}}$$

$$= \frac{1}{\log 10}\sin^{-1}(10^x) + C$$

51. $\int e^x\left\{\frac{1+\sin x\cos x}{\cos^2 x}\right\}dx =$ The foot of the perpendicular from the point (2, 4) -----

Ans: $e^x\tan x + C$

Sol: $\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx$

$= \int e^x(\tan x + \sec^2 x)dx$

$= e^x\tan x + C$

52. $\int \frac{x^2+1}{x^4+1}dx$

Ans: $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{x\sqrt{2}} + C$

Sol: $\int \frac{(x^2+1)dx}{x^4+1} = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}}$

$$= \int \frac{d\left(x - \frac{1}{x}\right)}{\left(x - \frac{1}{x}\right)^2 + 2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x - 1/x}{\sqrt{2}}\right) + C$$

Ans.: $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{x\sqrt{2}} + C$

53. The locus of the mid point of the intercept of ….

Ans: $x^{-2} + y^{-2} = 4p^{-2}$.

Sol: If (x, y) is mid point

$x = \frac{p}{2\cos\alpha}, y = \frac{p}{2\sin\alpha}$

$\therefore \frac{4}{p^2}\left(\cos^2\alpha + \sin^2\alpha\right) = \frac{1}{x^2} + \frac{1}{y^2}$

Locus is $x^{-2} + y^{-2} = 4p^{-2}$.

54. If the line through A = (4, –5) is inclined ….

Ans: (7, –2), (1, –8)

Sol: The line is x – y = 9;
then by back substitution the choice is (c)

55. If the line px + qy = 0 coincides with …..

Ans: $aq^2 - 2hpq + bp^2 = 0$

Sol: $-p/q$ satisfies $bm^2 + 2hm + a = 0$

$\therefore bp^2 - 2hpq + aq^2 = 0$

56. The function f(x) = $\left(\frac{\log_e(1+ax) - \log_e(1-bx)}{x}\right)$ ….

Ans: a + b.

Sol: $f(0) = \lim_{x\to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$

$= a + b$.

57. $\lim_{n\to\infty} \frac{(1^2 + 2^2 + .. + n^2)\sqrt[n]{n}}{(n+1)(n+10)(n+100)} =$

Ans: $\frac{1}{3}$

Sol: $\underset{n\to\infty}{Lt} \frac{\frac{n}{6}(n+1)(2n+1)\sqrt[n]{n}}{(n+1)(n+10)(n+100)}$

$= \underset{n\to\infty}{Lt} \frac{\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\sqrt[n]{n}}{\left(1+\frac{1}{n}\right)\left(1+\frac{10}{n}\right)\left(1+\frac{100}{n}\right)} = \frac{1}{3}$.

58. The number of triangles ……

Ans: 120

Sol: ${}^{10}C_3 = 120$.

59. The angle between hands of a clock …

Ans: $17\frac{1}{2}^\circ$.

Sol: The minute hand moves through 6° and the hour hand moves through $\frac{1}{2}^\circ$ in 1 minute.

∴ Angle between the hands at 4.25 am is $30^\circ - 12\frac{1}{2}^\circ = 17\frac{1}{2}^\circ$.

60. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + ...}}}}$ then $\frac{dy}{dx} =$

Ans: $\frac{y^2 - x}{2y^3 - 2xy - 1}$

Sol: $y^2 = x + \sqrt{2y}$

$(y^2 - x)^2 = 2y \Rightarrow 2(y^2 - x)(2yy' - 1)$
$= 2y'$

$y'[2y^3 - 2xy - 1] = y^2 - x$

$y' = \frac{y^2 - x}{2y^3 - 2xy - 1}$.