

**SOLUTIONS & ANSWERS FOR KERALA ENGINEERING  
ENTRANCE EXAMINATION-2010 – PAPER II  
VERSION – B1**

**[MATHEMATICS]**

1. Ans:  $1 + \frac{1}{a}$

Sol:  $\frac{\alpha a^2}{a+1} = a$   
 $\Rightarrow \alpha = 1 + \frac{1}{a}$

2. Ans: -4

Sol:  $a * b = 5$   
 $\Rightarrow \frac{a}{a+b} = 5$   
 $a * b + b * a = \frac{a}{a+b} + \frac{b}{a+b} = 1$   
 $\Rightarrow b * a = 1 - a * b = 1 - 5 = -4$

3. Ans:  $\{(z, b), (y, b), (a, d)\}$

Sol: Conceptual. The first element of  $(a, d)$  is not from A.

4. Ans:  $[6, 24]$

Sol:  $-1 \leq \log_2 \left( \frac{x}{12} \right) \leq 1$   
 $\Rightarrow \frac{1}{2} \leq \frac{x}{12} \leq 2$   
 $\Rightarrow 6 \leq x \leq 24$   
 $\Rightarrow \text{domain} = [6, 24]$

5. Ans:  $x^4$

Sol:  $(\text{gof})(x) = [x^2 - 1 + 1]^2 = x^4$

6. Ans:  $(A - B) \cup (B - A)$

Sol: Conceptual. It is clearly  $(A - B) \cup (B - A)$ .

7. Ans: -120.

Sol:  $x + iy = (2 + 3i)^3$   
 $= -46 + 9i$   
 $\Rightarrow x = -46, y = 9$   
 $\Rightarrow 3x + 2y = -120$

8. Ans: 3

Sol:  $z = x + iy$   
 $\Rightarrow (x+3)^2 + (y-1)^2 = 1$  and  
 $\frac{y}{x} = 0$

$\Rightarrow Y = 0, X = -3$   
 $|z| = \sqrt{9+0} = 3$

9. Ans:  $\left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right|$

Sol:  $|z_1 + z_2 + \dots + z_n| = |\bar{z}_1 + \bar{z}_2 + \dots + \bar{z}_n|$

10. Ans: i

Sol:  $\frac{(\sqrt{3} + i)(1 + i\sqrt{3})}{(1 - i\sqrt{3})(1 + i\sqrt{3})} = i$

11. Ans:  $2\cos 2\theta$

Sol:  $(\cos\theta + i\sin\theta)^2 + (\cos\theta - i\sin\theta)^2 = 2\cos 2\theta$

12. Ans:  $\sqrt{6}$

Sol:  $|z_1 z_2| = |z_1| |z_2|$

13. Ans:  $a = 3$

Sol:  $(2\sqrt{6})^2 - 8a = 0$

14. Ans:  $a + b = 31$

Sol:  $\left( x - \left( \frac{3}{2} + \frac{7i}{2} \right) \right) \left( x - \left( \frac{3}{2} - \frac{7i}{2} \right) \right) = 0$   
 $\Rightarrow 2x^2 - 6x + 29 = 0$   
 $\Rightarrow a + b = 31$

15. Ans:  $b^2 - 4c = 1$

Sol: If  $\alpha$  and  $\alpha + 1$  are the roots,  $2\alpha + 1 = b$  and  $\alpha(\alpha + 1) = c$   
 $\left( \frac{b-1}{2} \right) \left( \frac{b+1}{2} \right) = c$   
 $\Rightarrow b^2 - 4c = 1$

16. Ans:  $acx^2 - bx + 1 = 0$

Sol:  $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$   
 $\text{Sum} = \frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$   
 $= \frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab(\alpha + \beta) + b^2}$

$$= \frac{b}{ac}$$

$$\text{Product} = \frac{1}{ac}$$

$$x^2 - \frac{b}{ac}x + \frac{1}{ac} = 0$$

$$\Rightarrow acx^2 - bx + 1 = 0.$$

17. Ans: 1 and -2

Sol: Let a and b be the roots  
 $\Rightarrow a + b = -a$   
 $ab = b$   
 $\Rightarrow a = 1$  and  $b = -2$  (from options)

18. Ans:  $p^2 - 4q$

Sol:  $\alpha + \beta = -p$ ;  $\alpha\beta = q$   
 $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$   
 $= p^2 - 4q.$

19. Ans: 15

Sol: Given,  $n(n+1) = 240 = 15 \times 16$   
 comparing;  $n = 15.$

20. Ans: -8

Sol: sum =  $(\sqrt{10} + \sqrt{9}) - (\sqrt{11} + \sqrt{10}) +$   
 $(\sqrt{12} + \sqrt{11}) + \dots +$   
 $(\sqrt{120} + \sqrt{119}) - (\sqrt{121} - \sqrt{120})$   
 $= \sqrt{9} - \sqrt{121} = 3 - 11 = -8.$

21. Ans: 3

Sol:  $a_{11} + a_{12} + a_{13} = 141$   
 and  $a_{21} + a_{22} + a_{23} = 261$   
 $\Rightarrow 3a + 33d = 141$  and  $3a + 63d = 261$   
 $\Rightarrow a = 3.$

22. Ans:  $\tan^{-1}\left(\frac{5n-5}{1+a_n a_1}\right)$

Sol:  $\tan^{-1}\left[\frac{a_2 - a_1}{1 + a_1 a_2}\right] + \tan^{-1}\left[\frac{a_3 - a_2}{1 + a_2 a_3}\right] +$   
 $\dots + \tan^{-1}\left[\frac{a_n - a_{n-1}}{1 + a_{n-1} a_n}\right]$   
 $= \tan^{-1} a_2 - \tan^{-1} a_1 + \dots + \tan^{-1} a_n -$   
 $\tan^{-1} a_{n-1}$   
 $= \tan^{-1} a_n - \tan^{-1} a_1$   
 $= \tan^{-1} \frac{a_n - a_1}{1 + a_1 a_n} = \tan^{-1}\left(\frac{5n-5}{1+a_n a_1}\right)$

23. Ans: 702.

Sol:  $12 + 19 + \dots + 96$   
 $= \frac{13}{2} [12 + 96] = 702.$

24. Ans: 8.

Sol:  $\frac{a+2}{2} - \sqrt{2a} = 1$   
 $\Rightarrow \frac{a+2}{2} - 1 = \sqrt{2a}$   
 $\Rightarrow a^2 = 8a$   
 $\Rightarrow a = 8.$

25. Ans:  $8 \cdot ({}^6C_3)$

Sol: The middle term is the fourth term.  
 $T_4 = {}^6C_3 \cdot x^3 \cdot (2y)^3$   
 $= 8 \cdot ({}^6C_3) \cdot x^3 y^3$   
 $\therefore \text{coefficient} = 8 \cdot ({}^6C_3)$

26. Ans: 7

Sol: Given  ${}^nC_1, {}^nC_2, {}^nC_3$  are in A.P  
 $\Rightarrow 2 \cdot \frac{n(n-1)}{2} = n + \frac{n(n-1)(n-2)}{6}$   
 $\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow (n-1)(n-2) = 0$   
 $\Rightarrow n = 7$  from options.

27. Ans: 72

Sol: There are 5 places, out of which the first can be filled in 3 ways. The remaining four places are filled in  $4! = 24$  ways  
 $\therefore$  Total number of integers =  $3 \times 24 = 72.$

28. Ans: 3

Sol:  $(a^2 - 6a + 11)^{10} = 2^{10}$   
 $\Rightarrow a^2 - 6a + 11 = 2$   
 $6a + 11 = 2$   
 $\Rightarrow a^2 - 6a + 9 = 0$   
 $\Rightarrow a = 3, \text{ or } a = 3.$

29. Ans: 41

Sol:  ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$   
 $\Rightarrow \frac{55 \times 56}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$   
 $\Rightarrow r = 41$

30. Ans: 132

Sol: Required difference =  ${}^{11}C_5 - {}^{11}C_4$   
 $= 462 - 330$   
 $= 132.$

31. Ans: 4

Sol:  $A^{-1} = \frac{1}{7x+6} \begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$   
 $\Rightarrow 7x + 6 = 34$   
 $\Rightarrow x = 4.$

32. Ans: 21.

Sol: To get  $a_0$ , put  $x = 0$   
 $a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = 21.$

33. Ans: 2(15!) (16!) (17!)

Sol:  $2n! (n+1)! (n+2)!$   
 $= 2(15!) (16!) (17!).$

34. Ans:  $|A|A$

Sol:  $\text{adj}(\text{adj } A) = |A|^{n-2}A$   
 $= |A|A.$

35. Ans:  $x = 1, y = -2, z = 3.$

Sol:  $x - y - z = 0, -y + z = 5$  and  $z = 3$   
 $\Rightarrow y = -2$  and  $x = 1.$

36. Ans:  $B^{-1} A^{-1}.$

Sol:  $(AB)^{-1} = B^{-1} A^{-1}.$

37. Ans:  $(-\infty, -11) \cup (3, \infty)$

Sol:  $x + 11 > 0$  and  $x - 3 > 0$   
or  $x + 11 < 0$  and  $x - 3 < 0$   
 $\Rightarrow x > -11$  and  $x > 3$   
or  $x < -11$  and  $x < 3 \Rightarrow x > 3$  or  $x < -11$   
 $\Rightarrow (-\infty, -11) \cup (3, \infty).$

38. Ans:  $8 \leq t + 1 \leq 13$

Sol:  $21 \leq 3t \leq 36$   
 $\Rightarrow 7 \leq t \leq 12$   
 $\Rightarrow 8 \leq t + 1 \leq 13.$

39. Ans: 7 is greater than 4 and Paris is not in France

Sol:  $\sim(p \vee q) \equiv \sim p \wedge \sim q$   
 $\Rightarrow 7$  is greater than 4 and Paris is not in France.

40. Ans:  $p \vee (q \vee r)$

Sol:  $S(p, q, r) \equiv \sim[p \wedge q \wedge r]$   
 $S(\sim p, \sim q, \sim r) \equiv \sim[\sim p \wedge \sim q \wedge \sim r]$   
 $\equiv [p \vee (q \vee r)]$

41. Ans:  $\sim p.$

Sol: Using truth table, the given expression is logically equivalent to  $\sim p.$

42. Ans:  $\frac{2 \sin \alpha}{\sqrt{\cos 2\alpha}}$

Sol:  $\frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a-b} + \sqrt{a+b}} = \frac{2b}{\sqrt{a^2 - b^2}}$   
 $= \frac{2 \frac{b}{a}}{\sqrt{1 - \left(\frac{b}{a}\right)^2}} = \frac{2 \tan \alpha}{\sqrt{1 - \tan^2 \alpha}}$

$$= \frac{2 \sin \alpha}{\sqrt{\cos^2 \alpha - \sin^2 \alpha}} = \frac{2 \sin \alpha}{\sqrt{\cos 2\alpha}}.$$

43. Ans:  $x = 1$

Sol:  $\tan^{-1}\left(\frac{x+2+x-2}{1-(x^2-4)}\right) = \tan^{-1} \frac{1}{2},$   
 $(x+2)(x-2) < 1$

$$\frac{2x}{5-x^2} = \frac{1}{2}, x^2 < 5$$
$$\Rightarrow x^2 + 4x - 5 = 0$$
$$\Rightarrow x = -5, 1$$

44. Ans:  $\frac{63}{65}$

Sol:  $\sin \alpha = \frac{4}{5} \Rightarrow \cos \alpha = \frac{3}{5}$

$$\cos(\alpha + \beta) = \frac{-12}{13} \Rightarrow \sin(\alpha + \beta) = \frac{5}{13}$$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta = \frac{-12}{13}$$

$$\sin \alpha \cos \beta - \cos \alpha \sin \beta = \frac{5}{13}$$

substituting the values and solving,

$$\sin \beta = \frac{63}{65}.$$

45. Ans: Number of solutions in  $(0, 2\pi)$  is 3.

Sol:  $1 - 2\sin^2 \theta = \sin \theta$   
 $\Rightarrow 2\sin^2 \theta + \sin \theta - 1 = 0$   
 $\Rightarrow \sin \theta = -1, \frac{1}{2}$   
 $\Rightarrow \theta = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}.$

46. Ans:  $\frac{\pi}{2}$

Sol:  $\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3}$   
 $= \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3}$   
 $= \tan^{-1} 3 + \tan^{-1} \frac{1}{3}$   
 $= \tan^{-1} \frac{3 + \frac{1}{3}}{1 - 1} = \frac{\pi}{2}.$

47. Ans:  $\sqrt{3}$

Sol:  $\tan 60 = \sqrt{3} = \tan(40 + 20)$   
 $= \frac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20}$   
 $\Rightarrow \tan 40 + \tan 20 + \sqrt{3} \tan 40 \tan 20 = \sqrt{3}.$

48. Ans:  $\frac{2\pi}{3}$

Sol:  $f(\theta) = 4 - \sin 3\theta$   
 period of  $\sin \theta = 2\pi$   
 $\Rightarrow$  period of  $\sin 3\theta = \frac{2\pi}{3}$ .

49. Ans:  $\frac{\pi}{12}$

Sol:  $\sin x \cos x = \frac{1}{4} \Rightarrow \sin 2x = \frac{1}{2} = \sin \frac{\pi}{6}$   
 $\Rightarrow 2x = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12}$ .

50. Ans:  $-\frac{\pi}{4}$

Sol:  $\cos 4095^\circ = \cos 135^\circ = -\frac{1}{\sqrt{2}}$   
 $\therefore \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ .

51. Ans: -6, 8

Sol:  $(x-1)^2 + (2-3)^2 = (2+5)^2 + (3-2)^2$   
 $\Rightarrow (x-1)^2 = 49$   
 $\Rightarrow x-1 = \pm 7$   
 $\Rightarrow x = 8, -6$ .

52. Ans: 8

Sol: The circle is  $(x-4)(x+2) + (y-7)(y+1) = 0$   
 $\Rightarrow x^2 + y^2 - 2x - 6y - 15 = 0$   
 The points A and B are given by putting  $y = 0$   
 $\Rightarrow x^2 - 2x - 15 = 0$   
 $\Rightarrow x = -3, 5$   
 $\Rightarrow AB = 8$ .

53. Ans:  $\sqrt{3}, -\sqrt{3}$

Sol:  $4 = x^2 + 1$   
 $\Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ .

54. Ans:  $\frac{3}{2}$

Sol: Area =  $\frac{1}{2} \begin{vmatrix} 2 & 2 & 1 \\ 5 & 5 & 1 \\ 6 & 7 & 1 \end{vmatrix} = \frac{3}{2}$  sq. units.

55. Ans:  $\frac{r(q-p)}{pq}$

Sol: Substituting (a, 0) and (0, b) in the line,  
 $a = \frac{r}{p}; b = \frac{-r}{q}$   
 $\therefore a + b = \frac{r}{p} - \frac{r}{q} = \frac{r(q-p)}{pq}$ .

56. Ans:  $y = x + 1$

Sol:  $\Delta ABC$  is right angled at B and BC is parallel to the x-axis. Also, bisector must pass through B. So only option which satisfies these conditions is  $y = x + 1$ .

57. Ans:  $x \cos \theta - y \sin \theta = a \cos 2\theta$ .

Sol:  $\frac{x}{\cos \theta} + \frac{y}{\sin \theta} = a$   
 $\Rightarrow x \sin \theta + y \cos \theta = a \sin \theta \cos \theta$   
 $\Rightarrow$  required line is  $x \cos \theta - y \sin \theta = k$   
 $\Rightarrow$  since it passes through  $(a \cos^3 \theta, a \sin^3 \theta)$ ,  
 $k = a \cos 2\theta \Rightarrow x \cos \theta - y \sin \theta = a \cos 2\theta$ .

58. Ans: -2 or  $\frac{1}{2}$ .

Sol: Slope of  $3x - y = -5$  is  $m_1 = 3$   
 $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} = 1$   
 $\Rightarrow \frac{m_2 - 3}{1 + 3m_2} = \pm 1$   
 $\Rightarrow m_2 = -2$  or  $\frac{1}{2}$ .

59. Ans:  $4x - 3y - 3 = 0$

Sol: Parallel to given lines only possible options are (C) or (D) or (E). Distance from  $(-1, -4) = 1$  unit is satisfied by  $4x - 3y - 3 = 0$ .

60. Ans:  $x^2 + y^2 - 2hx - 2ky + h^2 = 0$

Sol: radius = k, centre = (h, k)  
 equation is  $(x-h)^2 + (y-k)^2 = k^2$   
 $\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 = 0$ .

61. Ans:  $P\left(\frac{-19}{13}, \frac{9}{13}\right)$

Sol:  $C_1(-7, 3)$   $\overset{6}{\text{---}}$   $\overset{P}{\bullet}$   $\overset{7}{\text{---}}$   $C_2(5, -2)$   
 $P\left(\frac{-19}{13}, \frac{9}{13}\right)$

62. Ans:  $2x - 3y = -13$

Sol:  $xx_1 + yy_1 = x_1^2 + y_1^2$   
 $-2x + 3y = (-2)^2 + 3^2$   
 $2x - 3y = -13$

$$\left| \vec{a} \times \vec{b} \right| = 5\sqrt{11}$$

63. Ans:  $2\sqrt{2}$

Sol: Mid point of segment (4, 0) and (0, 4) is (2, 2). Distance from (0, 0) to (2, 2) =  $2\sqrt{2}$

64. Ans:  $(9, 6\sqrt{3})$

Sol:  $y^2 = 12x$   
 focus (3, 0)  
 focal distance = 12  
 Point  $(9, 6\sqrt{3})$

65. Ans:  $\frac{15}{17}$

Sol:  $2a = \frac{17}{8} \times 2b \Rightarrow b = \frac{8}{17}a$   
 $e^2 = \frac{a^2 - b^2}{a^2} = \frac{a^2 - \frac{64}{289}a^2}{a^2}$   
 $e = \frac{15}{17}$

66. Ans: 9

Sol: Maximum CP = 5  
 Minimum CP = 4  
 Total = 9

67. Ans: 8

Sol:  $7x^2 - 9y^2 = 63$   
 $\frac{x^2}{9} - \frac{y^2}{7} = 1, \quad a = 3$   
 Eccentricity =  $\frac{4}{3}$   
 Foci's are (4, 0), (-4, 0).  
 Distance = 8

68. Ans:  $5\sqrt{11}$

Sol:  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$   
 $\cos \theta = \frac{-5}{6}$   
 $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2 \theta)$   
 $= 25 \times 36 \left(1 - \frac{25}{36}\right)$   
 $= 25 \times 11$

69. Ans:  $5\sqrt{2}$

Sol: By given condition,  $\sum |\vec{p}|^2 = 50$   
 $|\vec{p} + \vec{q} + \vec{r}|^2 = \sum |\vec{p}|^2$   
 $= 50$   
 $\Rightarrow |\vec{p} + \vec{q} + \vec{r}| = 5\sqrt{2}$

70. Ans: 5a

Sol: Let the sides of cube be along the axes.  
 Diagonals have D.C's  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ ,  
 $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$  and  $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$   
 $\therefore$  Adding; resultant =  $a \cdot \sqrt{9 + \frac{16}{2} + \frac{28}{2}}$   
 $= 5a$

71. Ans:  $2(\hat{i} - \hat{j} - 2\hat{k})$

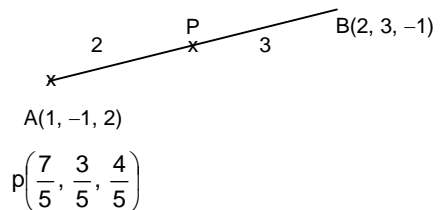
Sol: By back substitution.

72. Ans: 1

Sol:  $\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$   
 $2(2\lambda + 9) - 1(-4\lambda - 6) + 4(-12 + 4) = 0$   
 $4\lambda + 18 + 4\lambda + 6 - 32 = 0$   
 $8\lambda - 8 = 0; \lambda = 1$

73. Ans:  $\frac{1}{5}(\hat{i} + 3\hat{j} + 4\hat{k})$

Sol:



74. Ans: 6

Sol:  $(\hat{i} + \hat{j} + 2\hat{k}) \cdot \left(\frac{m\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{m^2 + 13}}\right) = 2$   
 $\frac{m + 8}{\sqrt{m^2 + 13}} = 2$   
 $\Rightarrow 3m^2 - 16m - 12 = 0$   
 $\Rightarrow m = \frac{-2}{3}, m = 6$

75. Ans:  $2x + 3y + 6z = 18$

Sol:  $\frac{a}{3} = 3, \frac{b}{3} = 2, \frac{c}{3} = 1$   
 $a = 9, b = 6, c = 3$   
 $\frac{x}{9} + \frac{y}{6} + \frac{z}{3} = 1$   
 $2x + 3y + 6z = 18$

76. Ans:  $\frac{3}{2}$

Sol:  $\frac{\left(\frac{x-1}{3}\right)}{\frac{2b}{3}} = \frac{y-3}{-1} = \frac{z-1}{a}$   
 $3 \times \frac{2b}{3} + 1 \times -1 + 2a = 6$   
 $2b + 2a = 1$   
 $a + b = \frac{1}{2}$   
 $3a + 3b = \frac{3}{2}$

77. Ans:  $\frac{x-3}{2} = \frac{-y}{3} = \frac{z+4}{5}$

Sol:  $\frac{x-3}{2} = \frac{y-0}{-3} = \frac{z+4}{5}$

78. Ans: The line parallel is  
 $\hat{r} = (-\hat{i} + \hat{j} - \hat{k}) + t(-\hat{i} - 2\hat{j} + 4\hat{k})$

Sol: The given plane is  
 $\hat{r} = 2\hat{i} - 7\hat{j} - 3\hat{k} + s(\hat{i} + 2\hat{j} - 4\hat{k}) + t(\hat{i} + 2\hat{j} - 4\hat{k})$   
 Vector along normal to plane  
 $= 42\hat{i} - 3\hat{j} + 9\hat{k}$

79. Ans: 2

Sol: Point on the line (2, 2, -1)  
 $\therefore \text{Distance} = \frac{|2+4-2-10|}{\sqrt{1+4+4}}$   
 $= \frac{6}{3} = 2$

80. Ans:  $20x + 23y + 26z = 69$

Sol:  $x + y + z - 6 + \lambda(2x + 3y + 4z + 5) = 0$   
 $14\lambda = 3; \lambda = \frac{3}{14}$   
 Equation of the plane is  
 $\left(1 + \frac{6}{14}\right)x + \left(1 + \frac{9}{14}\right)y + \left(1 + \frac{12}{14}\right)z$   
 $-6 + \frac{15}{14} = 0$

81. Ans:  $8x + y - 5z = 7$

Sol: Points (1, -1, 0) and (0, 2, -1) in the plane  
 $a(x-1) + b(y+1) + c \cdot 2 = 0$   
 $-a + 3b - c = 0$   
 $2a - b + 3c = 0$   
 $\frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$   
 Equation is  $8x + y - 5z = 7$

82. Ans:  $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - \hat{k})$

Sol:  $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-3}{-1}$

83. Ans:  $m + 4$

Sol: Given  $7a + 21 = 7m$   
 $a + 3 = m \Rightarrow a = m - 3$   
 $\Rightarrow \text{Required A.M} = m + 4$

84. Ans:  $\frac{1}{72}$

Sol:  $\Sigma p_i = 1$   
 $72p = 1$   
 $p = \frac{1}{72}$

85. Ans: 16

Sol: Mean = 5 Variance = 0  
 $\sigma^2 = \frac{1}{n} \Sigma x_i^2 - \left(\frac{\Sigma x_i}{n}\right)^2$   
 $0 = \frac{1}{n} \times 400 - 5^2 \Rightarrow 400 = 25n$   
 $n = 16$

86. Ans:  $\frac{2}{7}$

Sol:  $P(A \cup B) = \frac{13}{21}$   $P(B) = \frac{1}{3}$   
 $P(A) + P(B) = \frac{13}{21}$   
 $P(A) = \frac{2}{7}$

87. Ans: 500

Sol:  $f(x+y) = f(x) + f(y)$   
 $f(2) = f(1) + f(1) = 10$   
 $f(3) = f(2) + f(1) = 15$   
 etc.  
 $f(100) = 5 + 99 \times 5$   
 $= 5 \times 100 = 500$

88. Ans:  $a = 3$

$$\text{Sol: } \lim_{x \rightarrow 0} \frac{(e^x - 1)^2}{\sin\left(\frac{x}{a}\right) \log\left(1 + \frac{x}{4}\right)}$$

$$\lim_{x \rightarrow 0} \frac{(e^x - 1)^2 \cdot x^2}{x \cdot \sin\left(\frac{x}{a}\right) \log\left(1 + \frac{x}{4}\right) \cdot \frac{x}{a} \cdot \frac{x}{\left(\frac{x}{a}\right)} \cdot \frac{x}{\frac{x}{4}} \cdot \frac{x}{4}}$$

$$\Rightarrow 4a = 12$$

$$a = 3$$

89. Ans: 1

$$\text{Sol: } \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} = 1$$

90. Ans:  $\frac{2}{9}$

$$\text{Sol: } \lim_{x \rightarrow \infty} \frac{3x^4 + 2x^3 - 3x^4 + 4x^2}{(3x^2 - 4)(3x + 2)}$$

$$= \lim_{x \rightarrow \infty} \frac{2x^3 + 4x^2}{(3x^2 - 4)(3x + 2)}$$

$$= \lim_{x \rightarrow \infty} \frac{x^3 \left(2 + \frac{4}{x}\right)}{x^3 \left(3 - \frac{4}{x^2}\right) \left(3 + \frac{2}{x}\right)}$$

$$= \frac{2}{9}$$

91. Ans:  $\frac{2(\log x + 1)}{(\log x + 2)^2}$

$$\text{Sol: } x^y = e^{2(x-y)}$$

$$y \log x = 2(x - y)$$

$$y(\log x + 2) = 2x \Rightarrow y = \frac{2x}{\log x + 2}$$

$$\frac{dy}{dx} = \frac{2(\log x + 1)}{(\log x + 2)^2}$$

92. Ans:  $\frac{-1}{2\sqrt{x}\sqrt{1-x}}$

$$\text{Sol: } \sin y = \sqrt{1-x}$$

$$\cos y \frac{dy}{dx} = \frac{-1}{2\sqrt{1-x}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}\sqrt{1-x}}$$

93. Ans:  $\frac{2}{3}$

$$\text{Sol: } u = \sin^{-1}\left(2x\sqrt{1-x^2}\right); v = \sin^{-1}(3x - 4x^3)$$

$$x = \sin A \quad \frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}}$$

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{2}{3}$$

94. Ans: 0

$$\text{Sol: } y = \tan^{-1}x + \sec^{-1}x + \cot^{-1}x + \operatorname{cosec}^{-1}x$$

$$\frac{dy}{dx} = 0$$

95. Ans: -3

$$\text{Sol: } f(x) = |x-2| + |x+1| - x$$

$$x < -1 \quad f(x) = -(x-2) - (x+1) - x$$

$$= -3x + 1$$

$$f'(-10) = -3$$

96. Ans:  $-\frac{1}{a}$

$$\text{Sol: } x = a(1 + \cos\theta) \quad y = a(\theta + \sin\theta)$$

$$\frac{dx}{d\theta} = -a \sin\theta \quad \frac{dy}{d\theta} = a(1 + \cos\theta)$$

$$\frac{dy}{dx} = \frac{-\cot\theta}{2}$$

$$\frac{d^2y}{dx^2} = \frac{\operatorname{cosec}^2\theta}{2} \times \frac{1}{2} \times \frac{-1}{a \sin\theta}$$

$$\theta = \frac{\pi}{2} \quad \frac{d^2y}{dx^2} = \frac{-1}{a}$$

97. Ans:  $-\frac{1}{2}$

$$\text{Sol: } y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$$

$$= \tan^{-1}\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \tan^{-1}\frac{2 \sin\left(\frac{\pi}{2} - \frac{x}{2}\right) \cos\left(\frac{\pi}{2} - \frac{x}{2}\right)}{2 \cos^2\left(\frac{\pi}{2} - \frac{x}{2}\right)}$$

$$\tan^{-1} \tan\left(\frac{\pi}{2} - \frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$$

$$\frac{dy}{dx} = -1$$

98. Ans:  $\frac{2}{\sqrt{5}}$

Sol:  $y = e^{2x} + x^2, y' = 2e^{2x} + 2x$   
 $y'$  at  $x = 0$  is equal to 2

$\therefore$  slope of normal  $= -\frac{1}{2}$

$\therefore y - 1 = -\frac{1}{2}(x) \Rightarrow 2y - 2 = -x$

$x + 2y - 2 = 0$

$\therefore$  distance  $= \frac{2}{\sqrt{5}}$

99. Ans:  $\frac{4}{3}$

Sol:  $f(x) = x(x-1)^2 = x^3 - 2x^2 + x$

$f'(x) = 3x^2 - 4x + 1$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

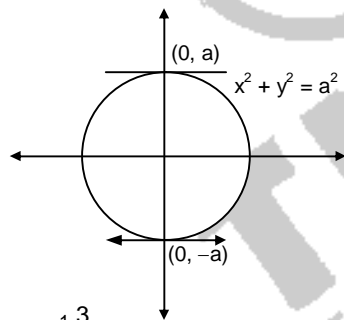
$3c^2 - 4c + 1 = \frac{f(2) - f(0)}{2 - 0}$

$= \frac{2 - 0}{2 - 0} = 1$

$3c^2 - 4c = 0 \Rightarrow c = \frac{4}{3}$

100. Ans: (0, a)

Sol:



101. Ans:  $\tan^{-1} \frac{3}{4}$

Sol:  $y = x^2, y^2 - x = 0$

$y' = 2x, 2yy' - 1 = 0, y' = \frac{1}{2y}$

At (1, 1)

$y' = 2$  and  $y' = \frac{1}{2}$

$\tan \theta = \left| \frac{2 - \frac{1}{2}}{1 + 1} \right| = \frac{3}{4}, \theta = \tan^{-1} \frac{3}{4}$

102. Ans: 750 cm<sup>3</sup>/sec.

Sol:  $\frac{da}{dt} = 10$  cm/sec  
 $v = a^3$

$\frac{dv}{dt} = 3a^2 \frac{da}{dt} = 3 \times 5^2 \times 10$   
 $= 750$  cm<sup>3</sup>/sec.

103. Ans: 7

Sol:  $f(x) = |3 - x| + 7$   
 Vertex = (3, 7)  
 Minimum value 7.

104. Ans: 0.1%

Sol:  $A = \pi r^2$   
 $\log A = \log \pi + 2 \log r$   
 $\frac{\Delta A}{A} \times 100 = 2 \frac{\Delta r}{r} \times 100$   
 $= 2 \times 0.05$   
 $= 0.1\%$

105. Ans:  $\frac{1}{4}$

Sol:  $x + 2 = A(4x + 6) + B$   
 $4A = 1 \rightarrow A = \frac{1}{4}$   
 $\therefore P = \frac{1}{4}$

106. Ans:  $\frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + C$

Sol:  $\int (x+1)(x+2)^7(x+3) dx$   
 $= \int [(x+2)^2 - 1]x+2]^7 dx$   
 $= \int [(x+2)^9 - (x+2)^7] dx$   
 $= \frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + C$

107. Ans:  $2 \left[ \frac{(x+1)^{\frac{7}{2}}}{7} - 2 \frac{(x+1)^{\frac{5}{2}}}{5} + 2 \frac{(x+1)^{\frac{3}{2}}}{3} \right] + C$

Sol:  $\int (x^2 + 1)\sqrt{x+1} dx$   
 $x + 1 = t^2, dx = 2t dt$   
 $x = t^2 - 1, x^2 + 1 = t^4 - 2t^2 + 2$   
 $2 \int (t^6 - 2t^4 + 2t^2) dt$   
 $= 2 \left[ \frac{t^7}{7} - 2 \frac{t^5}{5} + 2 \frac{t^3}{3} \right]$   
 $= 2 \left[ \frac{(x+1)^{\frac{7}{2}}}{7} - 2 \frac{(x+1)^{\frac{5}{2}}}{5} + 2 \frac{(x+1)^{\frac{3}{2}}}{3} \right] + C$

108. Ans:  $\log|1+xe^x|+C.$

Sol:  $\int \frac{1+x}{x+e^{-x}} dx = \int \frac{xe^x + e^x}{xe^x + 1} dx$   
 $= \log|1+xe^x|+C.$

109. Ans:  $\frac{1}{2} \left[ \log(x + \sqrt{1+x^2}) \right]^2 + C$

Sol: Put  $\log(x + \sqrt{1+x^2}) = t$   
 $dt = \frac{dx}{\sqrt{1+x^2}}$   
 $\int t dt = \frac{t^2}{2} + C = \frac{1}{2} \left[ \log(x + \sqrt{1+x^2}) \right]^2 + C.$

110. Ans:  $-\log[e^{-x} + \sqrt{e^{-2x} - 1}] + C$

Sol:  $\int \frac{dx}{\sqrt{1-e^{2x}}} = \int \frac{e^{-x} dx}{\sqrt{e^{-2x}}}$ , Put  $u = e^{-x}$   
 $du = -e^{-x} dx$   
 $\int -\frac{du}{\sqrt{u^2-1}} = -\log[e^{-x} + \sqrt{e^{-2x}-1}] + C$

111. Ans:  $\log \frac{x}{x + \cos x} + C$

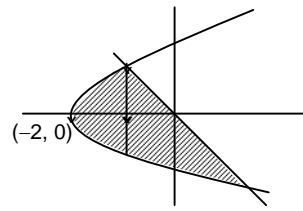
Sol:  $\int \frac{x + \cos x - x + x \sin x}{x(x + \cos x)} dx$   
 $\int \frac{x + \cos x}{x(x + \cos x)} - \frac{x(1 - \sin x)}{x(x + \cos x)} dx$   
 $= \int \left[ \frac{1}{x} - \left( \frac{1 - \sin x}{x + \cos x} \right) \right] dx$   
 $= \log x - \log(x + \cos x) + C$   
 $= \log \frac{x}{x + \cos x} + C.$

112. Ans:  $\int_0^{\frac{\pi}{6}} \frac{2x dx}{\tan x}.$

Sol:  $\sin^{-1} \frac{x}{2} = t \Rightarrow \sin t = \frac{x}{2}$   
 $\Rightarrow dx = 2 \cos t dt$   
 $\Rightarrow \int_0^{\frac{\pi}{6}} 2 \frac{\sin^{-1} \frac{x}{2} dx}{x} = \int_0^{\frac{\pi}{6}} 2t \cot t dt$   
 $= \int_0^{\frac{\pi}{6}} \frac{2t}{\tan t} dt$

113. Ans:  $\frac{9}{2}$

Sol:



$2 \int_{-2}^{-1} \sqrt{x+2} dx + \int_{-1}^2 (-x + \sqrt{x+2}) dx$   
 $= \frac{4}{3} \left[ (x+2)^{\frac{3}{2}} \right]_{-2}^{-1} - \frac{x^2}{2} + \frac{2}{3} \left[ (x+2)^{\frac{3}{2}} \right]_{-1}^2$   
 $= \frac{9}{2} \text{ sq. units.}$

114. Ans:  $m + n.$

Sol:  $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^{2a} f(2a - x) dx$   
 $= m + n$

115. Ans:  $\int_{-100}^{100} f(-x) dx$

Sol:  $\int_{-100}^{100} f(-x) dx = \int_{100}^{-100} f(y) (-dy)$   
 $= \int_{-100}^{100} f(y) dy$   
 $= \int_{-100}^{100} f(-x) dx.$

116. Ans: 0

Sol:  $(e^{x^3} + e^{-x^3})(e^x - e^{-x})$  is an odd function.  
 $\therefore \int_{-1}^1 (e^{x^3} + e^{-x^3})(e^x - e^{-x}) dx = 0.$

117. Ans:  $y \log y = \tan x \frac{dy}{dx}$

Sol:  $y = e^{a \sin x}$   
 $\log y = a \sin x \Rightarrow \frac{1}{y} \frac{dy}{dx} = a \cos x$

$$\Rightarrow \frac{dy}{dx} = \frac{\log y}{\sin x} \cos x$$

$$\Rightarrow y \log y = \frac{dy}{dx} \tan x .$$

118. Ans:  $e^x$

$$\text{Sol: } \frac{dy}{dx} + \frac{1+x}{x} y = 1$$

$$\Rightarrow 1 F = e^{\int \left(1 + \frac{1}{x}\right) dx} = e^{x + \log x} = x e^x .$$

$\therefore$  I. F of given differential equation is  $e^x$ .

119. Ans: 3, 1

$$\text{Sol: } \left(y - x \frac{dy}{dx}\right)^3 = a^2 \left(\frac{dy}{dx}\right)^2 + b^2$$

degree is 3 and order is 1.

120. Ans:  $x + e^{-(x+y)} = C$

$$\text{Sol: } x + y = z \Rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx}$$

substituting,

$$\Rightarrow e^{-z} dz = dx$$

$$\Rightarrow -e^{-z} = x + k$$

$$\Rightarrow -e^{-(x+y)} = x + k$$

$$\Rightarrow x + e^{-(x+y)} = C$$



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