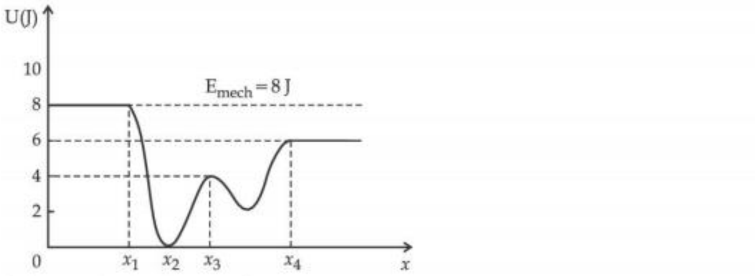
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
27th July Shift 2

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 Given below is the plot of a potential energy function $U(x)$ for a system, in which a particle is in one dimensional motion, while a conservative force $F(x)$ acts on it. Suppose that $E_{mech} = 8$ J, the incorrect statement for this system is :



[where K.E. = kinetic energy]

Options

1. at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.
2. at $x = x_3$, K.E. = 4 J.
3. at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.
4. at $x > x_4$, K.E. is constant throughout the region.

Ans: at $x < x_1$, KE is smallest and particle is moving at the slowest speed

Sol: $E_{mech} = 8$ J

At $x < x_1$, U = 8 J

KE = E_{mech} – U

= 8 – 8 = 0 J (particle is at rest)

∴ at $x = x_3$, U = 4 J

KE = E_{mech} – U

= 8 – 4 = 4 J

at $x = x_3$, U = 0

KE = E_{mech} – U

= 8 J

at $x > x_1$, U = 6 J

KE = E_{mech} – U

= 8 – 6

= 2 J

Q.2 The resistance of a conductor at 15°C is 16 Ω and at 100°C is 20 Ω. What will be the temperature coefficient of resistance of the conductor ?

Options

1. $0.033°C^{-1}$
2. $0.003°C^{-1}$
3. $0.010°C^{-1}$
4. $0.042°C^{-1}$

Ans: $0.003°C^{-1}$

Sol: $R_T = R_0 [1 + \alpha (T - T_0)]$

$16 = R_0 [1 + \alpha (15 - T_0)]$ ---------- (1)

$20 = R_0 [1 + \alpha (100 - T_0)]$ --------- (2) $T_0 \rightarrow$ reference temperature assuming $T_0 = 0°C$

Using equations (1) and (2)

$$\frac{16}{20} = \frac{1+\alpha\times 15}{1+\alpha\times 100}$$

$16 + \alpha \times 100 \times 16 = 20 + \alpha \times 15 \times 20$

Solving, $\alpha = 0.003°C^{-1}$

Q.3 A 100 Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz.

Options
1. 70.3 mH
2. 7.03×10^{-5} H
3. 0.70 H
4. 70.3 H

Ans: 70.3 H

Sol: C = 0.1 μF

$f_0 = 60$ Hz

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{4\pi^2 f_0^2 C}$$

$$L = \frac{1}{4\pi^2\times 60^2\times 0.1\times 10^{-6}} = 70.3 \text{ H}$$

Q.4 An object of mass 0.5 kg is executing simple harmonic motion. Its amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$s starting from mean position. Assume that the initial phase of the oscillation is zero.

Options
1. 6.2×10^{-3} J
2. 1.2×10^{3} J
3. 0.62 J
4. 6.2×10^{3} J

Ans: 0.62 J

Sol: $T = 2\pi\sqrt{\frac{m}{K}}$

$$0.2 = 2\pi\sqrt{\frac{0.5}{K}}$$

$K = 50\ \pi^2$

≈ 500

$x = A \sin (\omega t + \phi)$

$= 50 \text{ cm} \sin\left(\omega\frac{T}{4}+0\right)$

$= 50 \text{ cm} \sin\left(\frac{\pi}{2}\right) \quad \left(\Theta\ \omega = \frac{2\pi}{T}\right)$

= 5 cm

$PE = \frac{1}{2}Kx^2$

$= \frac{1}{2}(500)(5 \times 10^{-2})^2$

= 0.0255

Q.5 The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$\left\{ \text{Given } \frac{4\pi^2}{G} = 6 \times 10^{11} \text{ N}^{-1} \text{ m}^{-2} \text{ kg}^2 \right\}$

Options

1. 5.96×10^{19} kg
2. 7.02×10^{25} kg
3. 3.25×10^{21} kg
4. 6.00×10^{23} kg

Ans: 6.00×10^{23} kg

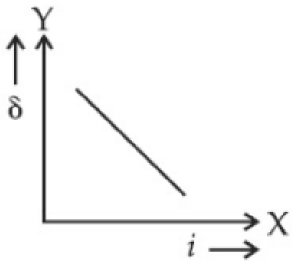
Sol: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

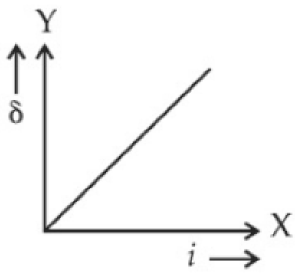
$M = \frac{4\pi^2 r^3}{GT^2}$

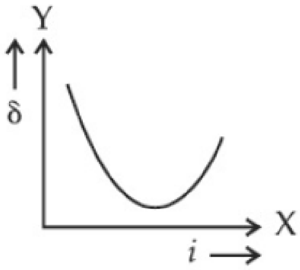
$M = \frac{6 \times 10^{11} \times (9 \times 10^3 \times 10^3)^3}{(729 \times 10^6)^2}$ 7 hrs, 30 min = 729 × 10^6 = 6 × 10^{23}

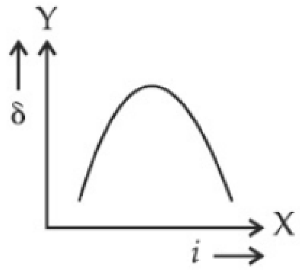
Q.6 The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence '*i*' in a prism is :

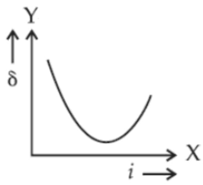
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: Basic concept

Q.7 An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electron and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron?

Options
1. 1.41 eV
2. 3.3 eV
3. 7.61 eV
4. No photoelectron would be emitted

Ans: 1.41 eV

Sol: Initial energy of electron = +3 eV

In 2nd excited state, energy of electron = $\frac{-13.6\ \text{eV}}{3^2}$ =-1.51 eV

Loss in energy is emitted as photon, photon energy = $\frac{hc}{\lambda} = 4.51\text{eV}$

Photoelectric equation, $\frac{hc}{\lambda} = \phi + KE_{max}$

4.51 eV = $\frac{hc}{\lambda_0} + KE_{max}$

KE_{max} = 4.51 eV - $\frac{12400\ \text{eV}\ \overset{0}{A}}{4000\ \overset{0}{A}}$ = 1.41 eV

Q.8 A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t - T}{T}\right)^2\right]$$

Where F_0 and T are constants. The force acts only for the time interval 2T. The velocity v of the particle after time 2T is :

Options
1. $4F_0T/3M$
2. $F_0T/3M$
3. $2F_0T/M$
4. $F_0T/2M$

Ans: $4F_0T$ / 3 M

Sol: Given $F = F_0\left[1 + \left(\frac{t-T}{T}\right)^2\right]$

Acceleration $a = \frac{F}{M} = \frac{F_0}{M} - \frac{F_0}{M}\left(\frac{t-T}{T}\right)^2$

$$a = \frac{dv}{dt}$$

$$\frac{dw}{dt} = \frac{F_0}{M} - \frac{F_0}{M}\left(\frac{t-T}{T}\right)^2$$

$$\int_0^v dv = \int_{t=0}^{2T}\left[\frac{F_0}{M} - \frac{F_0}{M}\frac{(t-T)^2}{T^2}\right]dt$$

$$= \left[\frac{F_0}{M}t\right]_0^{2T} - \frac{F_0}{MT^2}\left[\frac{t^3}{3} + T^2t - \frac{2t^2}{2}T\right]_0^{2T}$$

$$= \frac{2F_0T}{M} - \frac{F_0}{MT^2}\left[\frac{8T^3}{3} + 2T^3 - 4T^3\right]$$

$$= \frac{2F_0T}{M} - \frac{F_0T}{M}\left[\frac{8}{3} - 2\right]$$

$$= \frac{2F_0T}{M} - \frac{F_0T}{M}\left[\frac{2}{3}\right] = \frac{4F_0T}{3M}$$

Q.9 Match **List I** with **List II**.

	List I		**List II**
(a)	Capacitance, C	(i)	$M^1 L^1 T^{-3} A^{-1}$
(b)	Permittivity of free space, ε_0	(ii)	$M^{-1} L^{-3} T^4 A^2$
(c)	Permeability of free space, μ_0	(iii)	$M^{-1} L^{-2} T^4 A^2$
(d)	Electric field, E	(iv)	$M^1 L^1 T^{-2} A^{-2}$

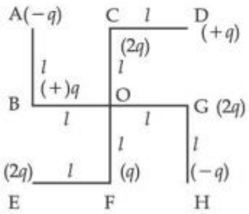
Choose the correct answer from the options given below :

Options
1. (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
2. (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)
3. (a) → (iv), (b) → (ii), (c) → (iii), (d) → (i)
4. (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

Ans: (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Sol: (a) Capacitance → $M^{-1} L^{-2} T^{4} A^{2}$ (iii)
(b) Permittivity of free space, ε_0 → $M^{-1}L^{-3}T^{4}A^{2}$ (ii)
(c) Permeability of free space, μ_0 → $M^{1}L^{1}T^{-2}A^{-2}$ (iv)
(d) Electric field, E → $M^{1}L^{1}T^{-3}A^{-1}$ (i)

Q.10 What will be the magnitude of electric field at point O as shown in figure ? Each side of the figure is l and perpendicular to each other ?



Options

1. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}(\sqrt{2})$

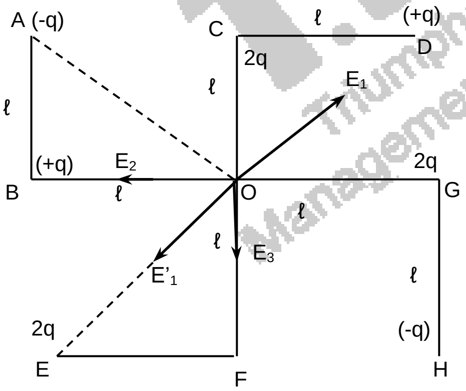
2. $\frac{q}{4\pi\varepsilon_0(2l)^2}$

3. $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}(2\sqrt{2}-1)$

4. $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

Ans: $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2\lambda^2)}\left(2\sqrt{2}-1\right)$

Sol:



Let E_1 is the resultant field at point O due to +q at D and 2q at E

$$E_1 = \frac{K2q}{2\lambda^2} - \frac{Kq}{2\lambda^2} = K\frac{q}{2\lambda^2}$$

E_2 is the resultant filed at point O due to 2q at G and q at B

$$E_2 = \frac{K2q}{\lambda^2} - \frac{Kq}{\lambda^2} = \frac{Kq}{\lambda^2} = E_3$$

Where E is the resultant field at point O due to 2q at C and q at F

Let E_1' be the resultant of $\vec{E}_2$ and $\vec{E}_3$

$$E_1' = \sqrt{\left(\frac{Kq}{\lambda^2}\right)^2 + \left(\frac{Kq}{\lambda^2}\right)^2} = \sqrt{2}\,\frac{Kq}{\lambda^2}$$

E_1 and E_1' are opposite to each other

E'' =resultant of $\vec{E}_1$ and E_1'

$$\sqrt{2}\,\frac{Kq}{\lambda^2} - \frac{Kq}{2\lambda^2} = \frac{Kq}{2\lambda^2}\left(2\sqrt{2}-1\right)$$

Field at point O due to –q at A and –q at H get cancelled

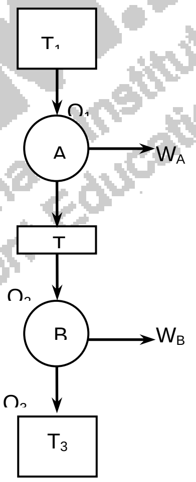
$$\therefore \text{Net field} = E_1'' = \frac{Kq}{2\lambda^2}\left(2\sqrt{2}-1\right)\ K = \frac{1}{4\pi\varepsilon_0}$$

Q.11 Two Carnot engines A and B operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at T_3. When workdone in both the cases is equal, the value of T is :

Options

1. $\frac{1}{3}T_1 + \frac{2}{3}T_3$
2. $\frac{2}{3}T_1 + \frac{3}{2}T_3$
3. $\frac{2}{3}T_1 + \frac{1}{3}T_3$
4. $\frac{3}{2}T_1 + \frac{1}{3}T_3$

Ans: $\frac{2}{3}T_1 + \frac{1}{3}T_3$



Sol: $W_A = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T}{T_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{T}{T_1}$

$W_B = 1 - \frac{Q_3}{\left(\frac{Q_2}{2}\right)} = 1 - \frac{T_3}{T} \Rightarrow \frac{2Q_3}{Q_2} = \frac{T_3}{T}$

$W_A = W_B$

$Q_1 - Q_2 = \frac{Q_2}{2} - Q_3$

$\Rightarrow \frac{2Q_1}{Q_2} + \frac{2Q_3}{Q_2} = 3$

$\Rightarrow \frac{2T_1}{T} + \frac{T_3}{T} = 3$

$\frac{2T_1}{3} + \frac{T_3}{3} = T$

Q.12 An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by :

Options

1. $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

2. $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

3. $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

4. $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

Ans: $\left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

Sol: Power, P = constant

P = FV = (ma) V

$$P = m\frac{dv}{dt}c$$

$$\int_0^t \frac{P}{m}dt = \int_0^v vdv$$

$$\frac{P}{m}t = \frac{v^2}{2}$$

$$v^2 = \frac{2P}{m}t \Rightarrow v = \left(\frac{2P}{m}t\right)^{1/2} = \left(\frac{2P}{m}\right)^{1/2} t^{1/2}$$

$$\frac{dx}{dt} = \left(\frac{2P}{m}\right)^{1/2} t^{1/2}$$

$$\int_0^x dx = \left(\frac{2P}{m}\right)^{1/2} \int_0^{1/2} t^{1/2}dt$$

$$x = \left(\frac{2P}{m}\right)^{1/2} \frac{t^{3/2}}{3/2} = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$$

$$x = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

Q.13 One mole of an ideal gas is taken through an adiabatic process where the temperature rises from 27°C to 37°C. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following is true ?
$[R = 8.314\ J\ mol^{-1}\ K^{-1}]$

Options
1. work done on the gas is close to 332 J
2. work done on the gas is close to 582 J
3. work done by the gas is close to 582 J
4. work done by the gas is close to 332 J

Ans: work done on the gas is close to 582

Sol: Each vibrational mode corresponds to two degrees of freedom
f = 3 (translation) + 3 (rotation) + 8 (vibration) = 14

$\gamma = 1 + \frac{2}{f}$

$\gamma = 1 + \frac{2}{14} = \frac{8}{7}$

Work done, W = $\frac{nR\Delta T}{\gamma - 1} = \frac{1 \times 8.314 \times 10}{\frac{8}{7} - 1}$ = -582

Q.14 A physical quantity 'y' is represented by the formula $y = m^2 r^{-4} g^x l^{-\frac{3}{2}}$

If the percentage errors found in y, m, r, l and g are 18, 1, 0.5, 4 and p respectively, then find the value of x and p.

Options
1. 4 and $\pm$ 3
2. $\frac{16}{3}$ and $\pm \frac{3}{2}$
3. 5 and $\pm$ 2
4. 8 and $\pm$ 2

Ans: $\frac{16}{3}$ and $\pm \frac{3}{2}$

Sol: Given $y = m^2 r^{-4} g^x \lambda^{-3/2}$

$\frac{\Delta y}{y} = \frac{2\Delta m}{m} + 4\frac{\Delta r}{r} + x\frac{\Delta g}{g} + \frac{3}{2}\frac{\Delta\lambda}{\lambda}$

18 = 2 ×1 + 4 × 0.5 + xP + $\frac{3}{2}$ × 4

xP = 8

From option, it is clear x = $\frac{16}{3}$ and P = $\pm \frac{3}{2}$

Q.15 Figure A and B show two long straight wires of circular cross-section (a and b with a < b), carrying current I which is uniformly distributed across the cross-section. The magnitude of magnetic field B varies with radius r and can be represented as :

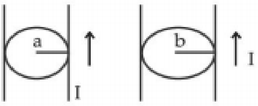
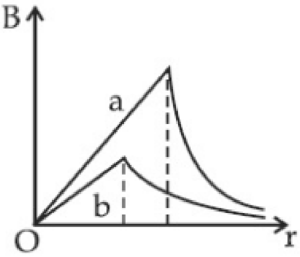
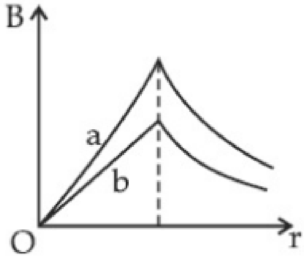
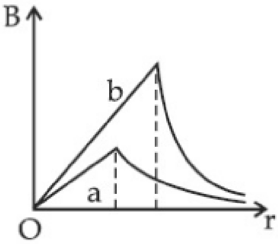


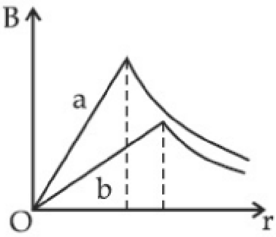
Fig. A Fig. B

Options

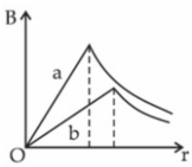
1. 

2. 

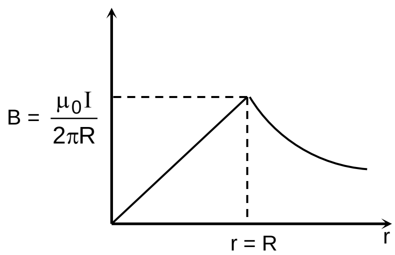
3. 

4. 

Ans:



Sol:

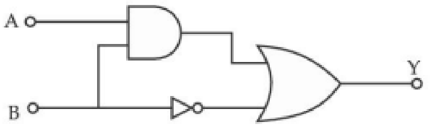


As b > a, $B_a > B_b$

$$B_a = \frac{\mu_0 I}{2\pi a}$$

$$B_b = \frac{\mu_0 I}{2\pi b}$$

Q.16 Find the truth table for the function Y of A and B represented in the following figure.



Options

1.

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

2.

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

3.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

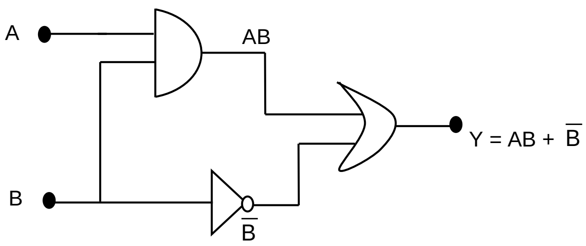
4.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Ans:

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Sol:



A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Q.17 Consider the following statements :

A. Atoms of each element emit characteristics spectrum.

B. According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.

C. The density of nuclear matter depends on the size of the nucleus.

D. A free neutron is stable but a free proton decay is possible.

E. Radioactivity is an indication of the instability of nuclei.

Choose the correct answer from the options given below :

Options

1. B and D only
2. A, B, C, D and E
3. A, B and E only
4. A, C and E only

Ans: A, B and E only

Sol: Basic concept

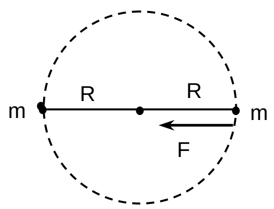
Q.18 Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is :

Options

1. $\frac{1}{2}\sqrt{\frac{G}{R^3}}$
2. $\frac{1}{2R}\sqrt{\frac{1}{G}}$
3. $\sqrt{\frac{2G}{R^3}}$
4. $\sqrt{\frac{G}{2R^3}}$

Ans: $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

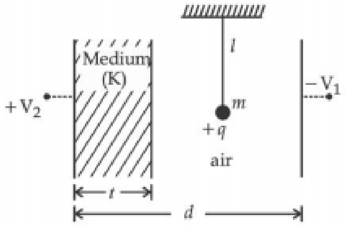
Sol:



$$F = \frac{Gm^2}{(2R)^2} = mR\omega^2$$

$$\therefore \omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

Q.19 A simple pendulum of mass 'm', length 'l' and charge '$+q$' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be :



Options

1. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

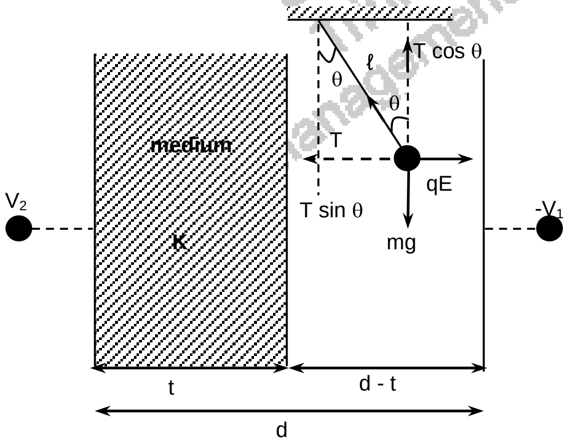
2. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

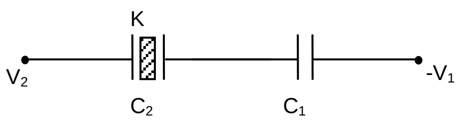
3. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

4. $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

Ans: $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

Sol:





Let E be the electric field in air

$T \sin \theta = qE$

$T \cos \theta = mg$

$\tan \theta = \dfrac{qE}{mg}$ – – – – – – – – (1)

$$Q = \left(\frac{C_1C_2}{C_1+C_2}\right)(V_1+V_2)$$

$$E = \frac{Q}{A\varepsilon_0} = \left(\frac{C_1C_2}{C_1+C_2}\right)\left(\frac{V_1+V_2}{A\varepsilon_0}\right)$$

$$C_1 = \frac{\varepsilon_0 A}{d-t}$$

$$\therefore E = \frac{\varepsilon_0 A}{(d-t)}\frac{C_2}{C_1+C_2}\frac{V_1+V_2}{A\varepsilon_0}$$

$$= \frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}$$

(1) $\theta = \tan^{-1}\left(\dfrac{qE}{mg}\right)$

$$\theta = \tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$$

Q.20 A raindrop with radius $R = 0.2$ mm falls from a cloud at a height $h = 2000$ m above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :

[Density of water $f_w = 1000$ kg m^{-3}

and Density of air $f_a = 1.2$ kg m^{-3},

$g = 10$ m/s^2

Coefficient of viscosity of air $= 1.8\times10^{-5}$ Nsm^{-2}]

Options

1. 14.4 ms^{-1}
2. 43.56 ms^{-1}
3. 250.6 ms^{-1}
4. 4.94 ms^{-1}

Ans: 4.94 ms^{-1}

Sol: At terminal speed

$a = 0$

$F_{net} = 0$

$mg = F_V = 6\pi\eta RV$

$$V = \frac{mg}{6\pi\eta R} = \frac{\rho_w \frac{4}{3}\pi R^3 g}{6\pi\eta R}$$

$$= \frac{2\rho_w R^2 g}{9\eta}$$

$$= \frac{2\times1000\times(0.2\times10^{-3})^2\times10}{9\times1.8\times10^{-5}} = 4.938 \text{ m/s}$$

Section B

Q.1 A particle executes simple harmonic motion represented by displacement function as

$x(t) = A \sin(\omega t + \phi)$

If the position and velocity of the particle at $t = 0$ s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is ________.

Given --
Answer :

Ans: 2.00

Sol: x(t) = A sin (wt + ϕ)

$V(t) = \frac{dx(t)}{dt} = Aw\cos(wt + \phi)$

2w = Aw cos ϕ --------(1)

2 = A sin ϕ -------------(2)

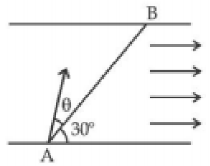
From (1) and (2)

tan ϕ = 1

ϕ = 45°

(2) $\Rightarrow$ 2 = A sin 45 $\Rightarrow$ A = $2\sqrt{2}$

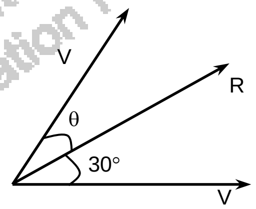
Q.2 A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ________ °, so that the swimmer reaches point B.



Given **30**
Answer :

Ans: 30.00

Sol: Both velocity vectors are of same magnitude. Therefore resultant would pass exactly midway through them θ = 30°

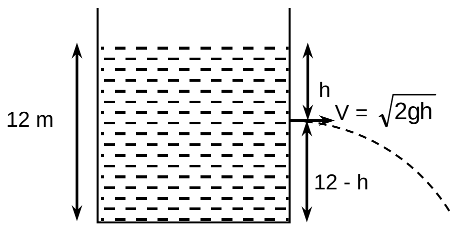


Q.3 The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging stream of water strikes the ground at the maximum range is ________ m.

Given --
Answer :

Ans: 6.00

Sol:



$$R = \sqrt{2gh} \times \sqrt{\frac{(12-h)\times 2}{g}}$$

$$R = \sqrt{4h(12-h)} = \left(48h - 4h^2\right)^{1/2}$$

For maximum R = $\frac{dR}{dh} = 0$

$$\frac{dR}{dh} = \frac{1}{2}\left(48h - 4h^2\right)^{-1/2}(48-8h)$$

$$\frac{dR}{dh} = \frac{48-8h}{2\left(48h-4h^2\right)^{1/2}}$$

48 – 8h = 0

$$h = \frac{48}{8} = 6\,m$$

Q.4 For the circuit shown, the value of current at time t = 3.2 s will be ________ A.

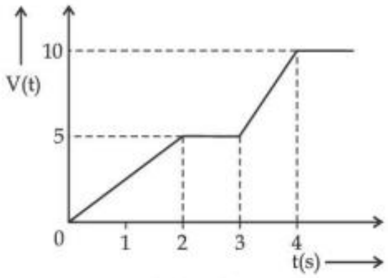


Figure 1

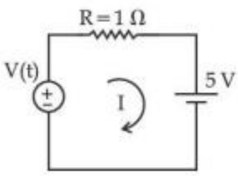


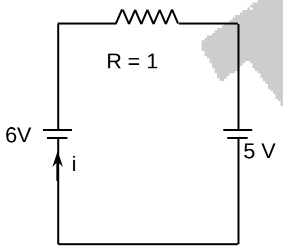
Figure 2

[Voltage distribution V(t) is shown by Fig. (1) and the circuit is shown in Fig. (2)]

Given --
Answer :

Ans: 1.00

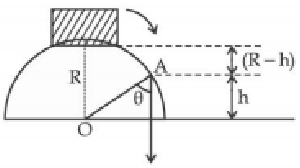
Sol: From graph, voltage at t = 3.2 sec is 6 volt



$$i = \frac{6-5}{1} = 1A$$

Q.5 A small block slides down from the top of hemisphere of radius R = 3 m as shown in the figure. The height 'h' at which the block will lose contact with the surface of the sphere is ________ m.

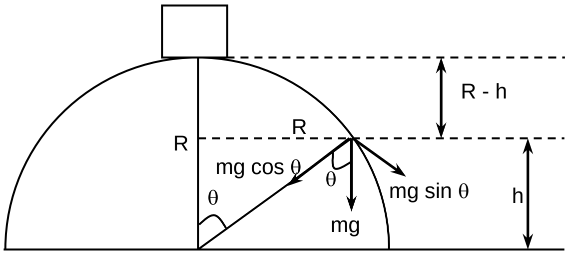
(Assume there is no friction between the block and the hemisphere)



Given --
Answer :

Ans: 2.00

Sol:



$$mg\cos\theta = \frac{mv^2}{R} - - - - - -(1)$$

$$\cos\theta = \frac{h}{R}$$

By bw of conservation of energy

$$Mg\{R - h\} = \frac{1}{2}mv^2 - - - - - - -(2)$$

Using (1) and (2) $mg\frac{h}{R} = \frac{2mg(R-h)}{R}$

h = 2 (R – h)

3h = 2R

$$h = \frac{2R}{3} = \frac{2\times3}{3} = 2m$$

Q.6 The K_α X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atom with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be __________ keV. (Round off to the nearest integer)

[$h = 4.14\times10^{-15}$ eVs, $c = 3\times10^8$ ms^{-1}]

Given --

Answer :

Ans: 10.00

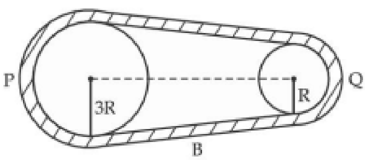
Sol: $E_{K\alpha} = E_K - E_L$

$$\frac{hc}{\lambda_{K\alpha}} = E_K - E_L \Rightarrow E_L = E_K - \frac{hc}{\lambda_{K\alpha}}$$

$$\therefore E_L = 27.5KeV - \frac{12.42\times10^{-7}eV_m}{0.071\times10^{-9}m}$$

$$E_L = (27.5 - 17.5)\times10^3 eV = 10\ K\ eV$$

Q.7 In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be __________.

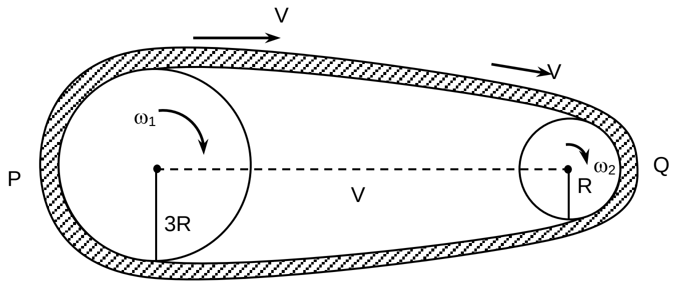


Given --

Answer :

Ans: 9.00

Sol:



$$\frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\omega_2^2$$

$$I_1\left(\frac{V}{3R}\right)^2 = I_2\left(\frac{V}{R}\right)^2$$

$$\frac{I_1}{I_2} = \frac{9}{1}$$

Q.8 The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is __________ mm.
[Refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000 Å]

Given --
Answer :

Ans: 2.00

Sol: Thickness t = $n\lambda$

$n\lambda_{vacuum} = (n + 1)\, \lambda_{air}$

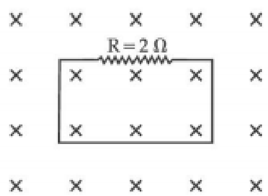
$$n\lambda = (n+1)\,\frac{\lambda}{\mu_{air}}$$

$$n = \frac{1}{\mu_{air}-1} = \frac{1}{1.0003-1} = \frac{1}{3\times10^{-4}}$$

t = $n\lambda$

$$= \frac{1}{3\times10^{-4}}\times 6000\,\overset{0}{A} = 2\text{ mm}$$

Q.9 In the given figure the magnetic flux through the loop increases according to the relation $\phi_\beta(t) = 10t^2 + 20t$, where ϕ_β is in milliwebers and t is in seconds.
The magnitude of current through R = 2 Ω resistor at $t = 5$ s is __________ mA.



Given --
Answer :

Ans: 60.00

Sol: $$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}\left(10t^2 + 20t\right)$$

= 20t + 20 mV

$$|i| = \frac{|\varepsilon|}{R} = \frac{20t+20}{2} = 10t + 10\text{ mA}$$

At t = 5 s

| i | = 10 × 5 + 10 = 60 mA

Q.10 The maximum amplitude for an amplitude modulated wave is found to be 12 V while the minimum amplitude is found to be 3 V. The modulation index is $0.6x$ where x is ________.

Given --
Answer :

Ans: 0.6

Sol: $A_{max} = A_c + A_n = 12$ -----(1)

$A_{min} = A_c - A_n = 3$ -------(2)

From (1) and (2) $A_c = \frac{15}{2}$

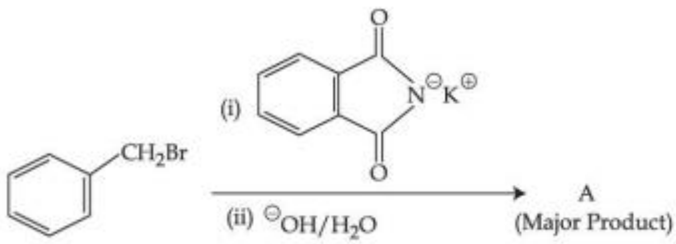
$A_n = \frac{9}{2}$

$$\text{Modulation index} = \frac{A_n}{A_c} = \frac{\frac{9}{2}}{\frac{15}{2}} = 0.6$$

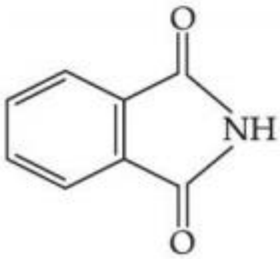
PART – B – CHEMISTRY

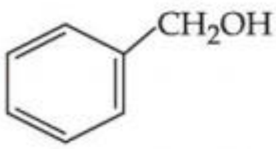
Section A

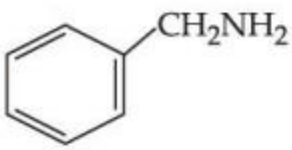
Q.1 What is A in the following reaction ?

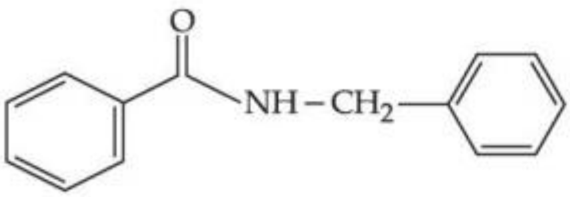


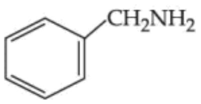
Options

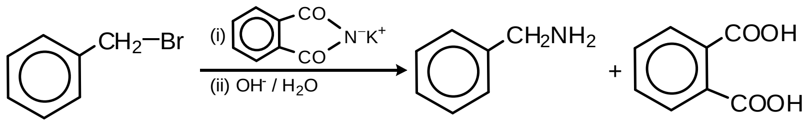
1. 

2. 

3. 

4. 

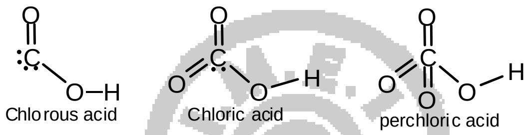
Ans: 

Sol: 

Q.2 Number of Cl=O bonds in chlorous acid, chloric acid and perchloric acid respectively are :

Options
1. 3, 1 and 1
2. 1, 1 and 3
3. 4, 1 and 0
4. 1, 2 and 3

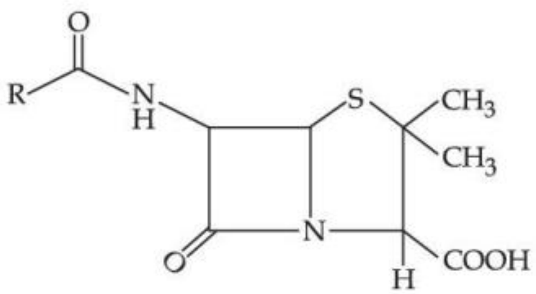
Ans: 1, 2 and 3

Sol: 

Q.3 Given below are two statements :

Statement I : Penicillin is a bacteriostatic type antibiotic.

Statement II : The general structure of Penicillin is :



Choose the **correct** option :

Options
1. Both **statement I** and **statement II** are true
2. **Statement I** is incorrect but **statement II** is true
3. **Statement I** is correct but **statement II** is false
4. Both **statement I** and **statement II** are false

Ans: Statement I is incorrect but statement II is true

Sol: Penicillin is a bactericidal antibiotic
The structure of penicillin gives is correct

Q.4 Given below are two statements :

Statement I : $[Mn(CN)_6]^{3-}$, $[Fe(CN)_6]^{3-}$ and $[Co(C_2O_4)_3]^{3-}$ are d^2sp^3 hybridised.

Statement II : $[MnCl_6]^{3-}$ and $[FeF_6]^{3-}$ are paramagnetic and have 4 and 5 unpaired electrons, respectively.

In the light of the above statements, choose the **correct** answer from the options given below :

Options
1. Both **statement I** and **statement II** are false
2. Both **statement I** and **statement II** are true
3. **Statement I** is correct but **statement II** is false
4. **Statement I** is incorrect but **statement II** is true

Ans: Both statement I and statement II are true

Sol: The ligands present in the given complexes are strong field therefore the splitting energy is large in all cases
i.e., statement I is true
In $[MnCl_6]^{3-}$, the central metal ion is Mn^{3+} with d^4 configuration. In presence of Cl^- d subshell splits as $t_{2g}^3 e_g^1$
In $[FeF_6]^{3-}$, the central meal ion is Fe^{3+} with d^5 configuration. In presence of F^- d subshell splits as $t_{2g}^3 e_g^2$
Hence both are paramagnetic

Q.5

$$R-CN \xrightarrow[\text{ii) } H_2O]{\text{i) DIBAL-H}} R-Y$$

Consider the above reaction and identify "Y".

Options
1. $-CH_2NH_2$
2. $-COOH$
3. $-CONH_2$
4. $-CHO$

Ans: $-CHO$

Sol: $R-C\equiv N \xrightarrow{\text{DIBAL-H}} R-CH=NH \xrightarrow{H_2O} R-CHO+NH_3$

Q.6 The number of neutrons and electrons, respectively, present in the radioactive isotope of hydrogen is :

Options
1. 1 and 1
2. 2 and 1
3. 2 and 2
4. 3 and 1

Ans: 2 and 1

Sol: Radioactive isotope of hydrogen is tritium ${}_{1}^{3}H$

Q.7 Match **List - I** with **List - II** :

	List - I		List - II
(a)	Li	(i)	photoelectric cell
(b)	Na	(ii)	absorbent of CO_2
(c)	K	(iii)	coolant in fast breeder nuclear reactor
(d)	Cs	(iv)	treatment of cancer
		(v)	bearings for motor engines

Choose the **correct** answer from the options given below :

Options
1. (a) - (v), (b) - (ii), (c) - (iv), (d) - (i)
2. (a) - (v), (b) - (i), (c) - (ii), (d) - (iv)
3. (a) - (v), (b) - (iii), (c) - (ii), (d) - (i)
4. (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

Ans: (a)-(v), (b)-(iii), (c)-(ii), (d)-(i)

Sol: White metal (Li-Pb alloy) used to make bearings for motor engines

Q.8 Select the correct statements.

(A) Crystalline solids have long range order.

(B) Crystalline solids are isotropic.

(C) Amorphous solids are sometimes called pseudo solids.

(D) Amorphous solids soften over a range of temperatures.

(E) Amorphous solids have a definite heat of fusion.

Choose the most appropriate answer from the options given below :

Options
1. (B), (D) only
2. (C), (D) only
3. (A), (B), (E) only
4. (A), (C), (D) only

Ans: (A), (C), (D) only

Sol: Crystalline solids have long range of order, they are anisotropic with sharp melting point
Amorphous solids have short range of order, they are isotropic and melts over a range of temperature

Q.9 The addition of silica during the extraction of copper from its sulphide ore

Options
1. converts iron oxide into iron silicate
2. reduces copper sulphide into metallic copper
3. converts copper sulphide into copper silicate
4. reduces the melting point of the reaction mixture

Ans: converts iron oxide into iron silicate

Sol: Silica is added as a flux to remove FeO impurity

$$FeO + SiO_2 \longrightarrow \underset{\text{slag}}{FeSiO_3} \text{ (iron silicate)}$$

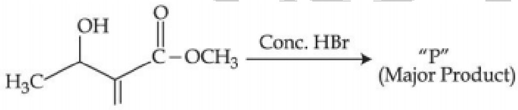
Q.10 Compound **A** gives D-Galactose and D-Glucose on hydrolysis. The compound **A** is :

Options
1. Amylose
2. Lactose
3. Maltose
4. Sucrose

Ans: Lactose

Sol: Lactose is a disaccharide which on hydrolysis gives D-galactose and D-glucose

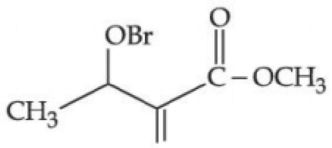
Q.11



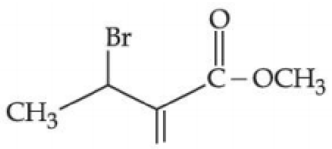
Consider the above reaction, the major product "P" formed is :

Options

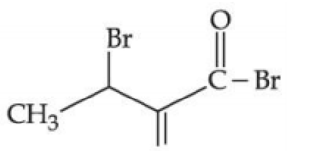
1.



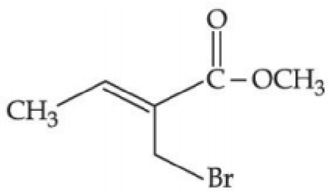
2.



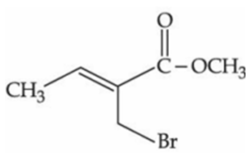
3.

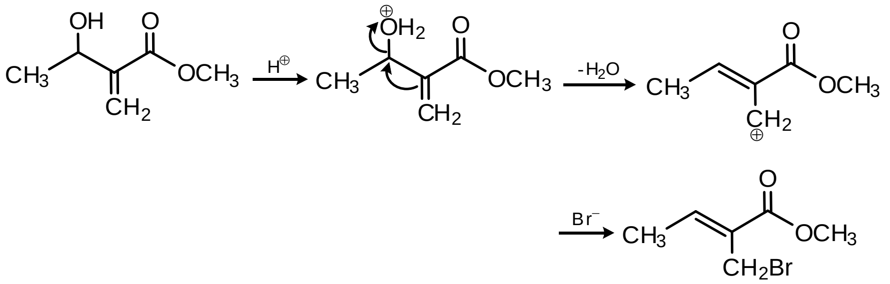


4.



Ans:



Sol: 

Q.12 The CORRECT order of first ionisation enthalpy is :

Options
1. Mg < Al < P < S
2. Al < Mg < S < P
3. Mg < Al < S < P
4. Mg < S < Al < P

Ans: Al < Mg < S < P

Sol: The first ionization enthalpy of Mg is greater than Al and that of P is greater than S

Q.13 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : $SO_2(g)$ is adsorbed to a larger extent than $H_2(g)$ on activated charcoal.

Reason R : $SO_2(g)$ has a higher critical temperature than $H_2(g)$.

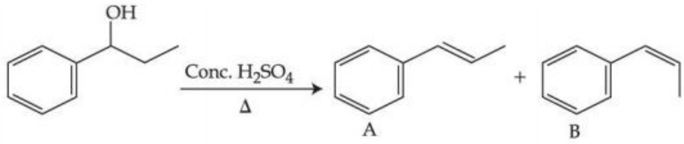
In the light of the above statements, choose the most appropriate answer from the options given below.

Options
1. **A** is not correct but **R** is correct.
2. Both **A** and **R** are correct but **R** is not the correct explanation of **A**.
3. Both **A** and **R** are correct and **R** is the correct explanation of **A**.
4. **A** is correct but **R** is not correct.

Ans: Both **A** and **R** are correct and **R** is the correct explanation of **A**

Sol: Gases having higher critical temperature adsorb to a greater extent. SO_2 adsorb more than H_2 Since critical temperature of SO_2 is greater than H_2

Q.14



Consider the above reaction, and choose the correct statement :

Options
1. Compound **B** will be the major product
2. Both compounds **A** and **B** are formed equally
3. Compound **A** will be the major product
4. The reaction is not possible in acidic medium

Ans: The compound **A** will be the major product

Sol: Dehydration results in an alkene C_6H_5–CH=CH–CH_3 that shows geometrical isomerism
Trans isomers is more stable than cis isomer

Q.15 Match **List - I** with **List - II** :

	List - I **(compound)**		**List - II** **(effect/affected species)**
(a)	Carbon monoxide	(i)	Carcinogenic
(b)	Sulphur dioxide	(ii)	Metabolized by pyrus plants
(c)	Polychlorinated biphenyls	(iii)	Haemoglobin
(d)	Oxides of nitrogen	(iv)	Stiffness of flower buds

Choose the **correct** answer from the options given below :

Options
1. (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)
2. (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)
3. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
4. (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Ans: (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Sol: (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Q.16 To an aqueous solution containing ions such as Al^{3+}, Zn^{2+}, Ca^{2+}, Fe^{3+}, Ni^{2+}, Ba^{2+} and Cu^{2+} was added conc. HCl, followed by H_2S.

The total number of cations precipitated during this reaction is/are :

Options
1. 1
2. 4
3. 2
4. 3

Ans: 1

Sol: In presence of HCl, H_2S ionises to a lesser extent. Therefore, sulphides with very low solubility product will precipitate
In the given set only Cu^{2+} will precipitate

Q.17 If the Thompson model of the atom was correct, then the result of Rutherford's gold foil experiment would have been :

Options
1. All α-particles get bounced back by 180°.
2. All of the α-particles pass through the gold foil without decrease in speed.
3. α-Particles are deflected over a wide range of angles.
4. α-Particles pass through the gold foil deflected by small angles and with reduced speed.

Ans: α-particles pass through the gold foil deflected by small angles and with reduced speed

Sol: According to Thomson model the positive charge is not centered, it is diffused

Q.18 Given below are two statements :

Statement I : Hyperconjugation is a permanent effect.

Statement II : Hyperconjugation in ethyl cation $\left(CH_3-\overset{+}{C}H_2\right)$ involves the overlapping of $C_{sp^2}-H_{1s}$ bond with empty 2p orbital of other carbon.

Choose the **correct** option :

Options
1. Both **statement I** and **statement II** are true
2. **Statement I** is correct but **statement II** is false
3. Both **statement I** and **statement II** are false
4. **Statement I** is incorrect but **statement II** is true

Ans: Statement I is correct but statement II is false

Sol: In $CH_3-CH_2^+$ hyperconjugation involved overlapping of sp^3–H_{1s} bond with empty 2p orbital of positive charged carbon

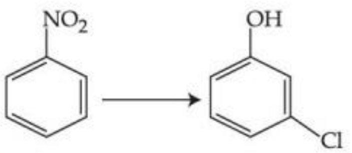
Q.19 Which one of the following set of elements can be detected using sodium fusion extract ?

Options
1. Sulfur, Nitrogen, Phosphorous, Halogens
2. Phosphorous, Oxygen, Nitrogen, Halogens
3. Halogens, Nitrogen, Oxygen, Sulfur
4. Nitrogen, Phosphorous, Carbon, Sulfur

Ans: Sulfur, Nitrogen, Phosphorous, Halogens

Sol: Sulphur, nitrogen, phosphorous and halogens can be detected by sodium fusion extract

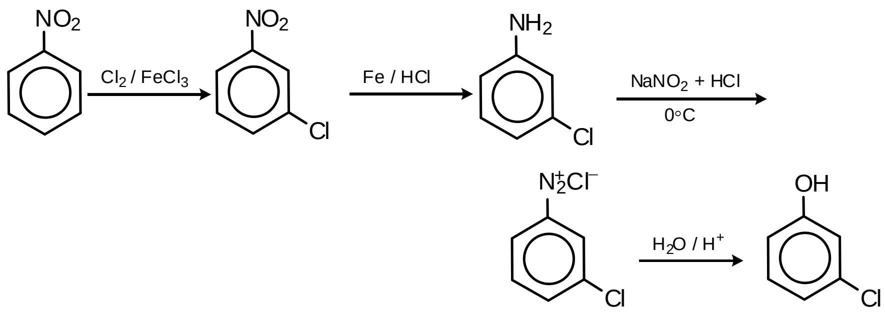
Q.20 The correct sequence of correct reagents for the following transformation is :



Options
1. (i) Fe, HCl (ii) $NaNO_2$, HCl, 0°C (iii) H_2O/H^+ (iv) Cl_2, $FeCl_3$
2. (i) Cl_2, $FeCl_3$ (ii) Fe, HCl (iii) $NaNO_2$, HCl, 0°C (iv) H_2O/H^+
3. (i) Cl_2, $FeCl_3$ (ii) $NaNO_2$, HCl, 0°C (iii) Fe, HCl (iv) H_2O/H^+
4. (i) Fe, HCl (ii) Cl_2, HCl (iii) $NaNO_2$, HCl, 0°C (iv) H_2O/H^+

Ans: (i) Cl_2, $FeCl_3$ (ii) Fe, HCl (iii) $NaNO_2$, HCl, 0°C (iv) H_2O / H^+

Sol:



Section B

Q.1 For the first order reaction $A \rightarrow 2B$, 1 mole of reactant A gives 0.2 moles of B after 100 minutes. The half life of the reaction is ________ min. (Round off to the Nearest Integer).

[Use : $\ln 2 = 0.69$, $\ln 10 = 2.3$

Properties of logarithms : $\ln x^y = y \ln x$;

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y]$$

Given **2**
Answer :

Ans: 6.00

Sol:

	$A \longrightarrow B$	
Start	1	-
t = 100 min	1-x	2x
2x = 0.2	(1-x) = 0.9	

$$k = \frac{2.303}{t_{1/2}} \times \log 2$$

$$t = \frac{2.303}{k} \log \frac{[R_0]}{[R_t]} = \frac{2.303 \times t_{1/2}}{2.303 \times \log 2} \times \log \frac{1}{0.9}$$

$$t_{1/2} = \frac{100 \times 0.3010}{1 - 0.95} = 600$$

Q.2 10.0 mL of 0.05 M $KMnO_4$ solution was consumed in a titration with 10.0 mL of given oxalic acid dihydrate solution. The strength of given oxalic acid solution is ________ $\times 10^{-2}$ g/L. (Round off to the Nearest Integer).

Given --
Answer :

Ans: 1575

Sol: Milli equivalents of $KMnO_4$ = milli equivalents of oxalic acid
$10 \times 0.05 \times 5 = 10 \times M \times 2$
Molariy of oxalic acid solution = 0.125 mol / L
Strength of oxalic acid (in g / L) = $0.125 \times 126 = 15.75 = 1575 \times 10^{-2}$

Q.3 The equilibrium constant for the reaction

$$A(s) \rightleftharpoons M(s) + \frac{1}{2} O_2(g)$$

is $K_p = 4$. At equilibrium, the partial pressure of O_2 is ________ atm. (Round off to the Nearest Integer).

Given --
Answer :

Ans: 16

Sol: $K_p = (p_{CO_2})^{1/2} = 4$

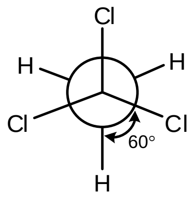
$\therefore p_{CO_2} = 16$ atm

Q.4 The dihedral angle in staggered form of Newman projection of 1,1,1-Trichloro ethane is ________ degree. (Round off to the Nearest Integer).

Given **112**
Answer :

Ans: 60

Sol:



Q.5 When 400 mL of 0.2 M H_2SO_4 solution is mixed with 600 mL of 0.1 M NaOH solution, the increase in temperature of the final solution is ________ $\times 10^{-2}$ K. (Round off to the Nearest Integer).

[Use : $H^+(aq) + OH^-(aq) \rightarrow H_2O : \Delta_y H = -57.1$ kJ mol^{-1}

Specific heat of H_2O = 4.18 J K^{-1} g^{-1}

density of H_2O = 1.0 g cm^{-3}

Assume no change in volume of solution on mixing.]

Given **2**
Answer :

Ans: 82

Sol: Number of moles $H^+ = \dfrac{400 \times 0.4}{1000} = 0.16$

Number of moles $OH^- = \dfrac{600 \times 0.1}{1000} = 0.06$

OH^- is the limiting reagent

Heat liberated = $0.06 \times 57.1 \times 10^3 = 3426$ J

Temperature rise $= \dfrac{3426}{4.18 \times 1000} = 0.819$K

Q.6 The total number of electrons in all bonding molecular orbitals of O_2^{2-} is ________. (Round off to the Nearest Integer).

Given **1**
Answer :

Ans: 10

Sol: Total number of basicity electron in O_2^{2-} is 10

Q.7 $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

The above reaction is carried out in a vessel starting with partial pressures $P_{SO_2} = 250$ m bar, $P_{O_2} = 750$ m bar and $P_{SO_3} = 0$ bar. When the reaction is complete, the total pressure in the reaction vessel is ________ m bar. (Round off to the Nearest Integer).

Given **--**
Answer :

Ans: 875

Sol: $2SO_{2(g)} + O_{2(g)} \rightarrow 2SO_{3(g)}$

	$2SO_{2(g)}$	$O_{2(g)}$	$2SO_{3(g)}$
Initial	250 m. bar	750 m. bar	
Final	–	625 m . bar	250 m. bar

Final pressure = 625 + 250 = 875 m. bar

Q.8 In a solvent 50% of an acid HA dimerizes and the rest dissociates. The van't Hoff factor of the acid is _________ $\times 10^{-2}$. (Round off to the Nearest Integer).

Given --
Answer :

Ans: 125

Sol: Let the no. of moles of HA be 1

$HA \rightleftharpoons (HA)_2$

	HA	$(HA)_2$
Initial	0.5	–
Final	–	0.25

$HA \rightleftharpoons H^+ + A^-$

	HA	H^+	A^-
Initial	0.5	–	–
Final	–	0.5	0.5

$\therefore$ Total no. of moles particles = 1.25

Vant Hoff factor $= \dfrac{1.25}{1} = 1.25$

Q.9 3 moles of metal complex with formula $Co(en)_2Cl_3$ gives 3 moles of silver chloride on treatment with excess of silver nitrate. The secondary valency of Co in the complex is _________. (Round off to the Nearest Integer).

Given --
Answer :

Ans: 6

Sol: Since 3 moles of the complex gives 3 moles of AgCl, the number of chloride ion associated with a formula unit should be one

$\therefore$ Complex is $[Co(en)_2Cl_2]Cl$

Secondary valency of Co = 6

Q.10 For the cell $Cu(s)|Cu^{2+}(aq)\ (0.1\,M)||Ag^+(aq)\ (0.01\,M)|Ag(s)$

the cell potential $E_1 = 0.3095$ V

For the cell $Cu(s)|Cu^{2+}(aq)\ (0.01\,M)||Ag^+(aq)\ (0.001\,M)|Ag(s)$

the cell potential = _________ $\times 10^{-2}$ V. (Round off to the Nearest Integer).

[Use : $\dfrac{2.303\,RT}{F} = 0.059$]

Given --
Answer :

Ans: 28

Sol: $Cu_{(s)} + 2Ag^+_{(aq)} \longrightarrow Cu^{2+}_{(aq)} + 2Ag$

$$E_{cell} = E^o_{cell} - \frac{0.059}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$E_1 = 0.3095 = E^o_{cell} - \frac{0.059}{2}\log\frac{(0.1)}{(0.01)^2}$$

$$E_2 = E^o_{cell} - \frac{0.059}{2}\log\frac{(0.01)}{(0.001)^2}$$

$$E_2 - 0.3095 = \frac{0.059}{2}(3-4)$$

$$E_2 = 0.28 = 28 \times 10^{-2}\,V$$

PART – C – MATHEMATICS

Section A

Q.1

The value of $\lim_{x \to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

Options
1. 4
2. -4
3. 0
4. -1

Ans: –4

Sol: $\lim_{x\to 0}\left[\frac{x}{(1-\sin x)^{1/8}-(1+\sin x)^{1/8}}\right] = \lim_{x\to 0}\left[\frac{1}{\frac{1}{8}(1-\sin x)^{-7/8}(-\cos x)-\frac{1}{8}(1+\sin x)^{-7/8}.(\cos x)}\right]$

$= 8.\lim_{x\to 0}\left[\frac{1}{\cos x(1-\sin x)^{-7/8}+\cos x(1+\sin x)^{-7/8}.}\right] = -8\times\frac{1}{1(1-0)^{-7/8}+1.(1+0)^{-7/8}}$

$= -8\times\frac{1}{1+1} = -4$

Q.2 A student appeared in an examination consisting of 8 true - false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

Options
1. 3
2. 6
3. 4
4. 5

Ans: 5

Sol: $P(X\ge n)<\frac{1}{2}$

$P(X\ge 6)=P(X=6)+P(X=7)+P(x=8)$

$=\left({}^8C_6+{}^8C_7+{}^8C_8\right)\frac{1}{2^8}=\frac{(1+8+28)}{28}=\frac{37}{256}\approx 0.14$

$P(X\ge 5)=P(X=5)+\frac{37}{256}={}^8C_5\left(\frac{1}{2}\right)^8+\frac{37}{256}=\frac{93}{256}\approx 0.36$

$P(X\ge 4)=P(X=4)+\frac{93}{256}={}^8C_4\left(\frac{1}{2}\right)^8+\frac{93}{256}=\frac{163}{256}\approx 0.64>0.5$

$\therefore n=5$

Q.3 Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z-2| \leq 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \geqslant 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

Options

1. $\dfrac{3+2\sqrt{2}}{2}$

2. $\dfrac{5+2\sqrt{2}}{2}$

3. $\dfrac{3+2\sqrt{2}}{4}$

4. $\dfrac{5+2\sqrt{2}}{4}$

Ans: $\dfrac{5+2\sqrt{2}}{4}$

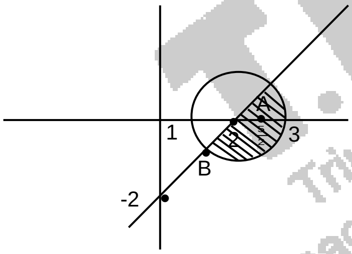
Sol: Let $z = x + iy$

$z(1+i) + \bar{z}(1-i) = (x+iy)(1+i) + (x-iy)(1-i)$

$= (x-y) + i(x+y) + (x-y) - (x+y)i = 2(x-y) \geq 4(\text{given})$

$\Rightarrow x - y \geq 2$

$S_1 : (x-2)^2 + y^2 \leq 1$



Solving x-y = 2 and $(x-2)^2 + y^2 = 1$, $\quad 2y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

If $y = -\frac{1}{\sqrt{2}}, x = 2 - \frac{1}{\sqrt{2}} \Rightarrow B\left(2-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$A\left(\frac{5}{2}, 0\right)$

$$AB = \sqrt{\left(\frac{5}{2} - 2 + \frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{(1+\sqrt{2})^2}{4} + \frac{1}{2}} = \sqrt{\frac{1+2+2\sqrt{2}+2}{4}} = \frac{\sqrt{5+2\sqrt{2}}}{2}$$

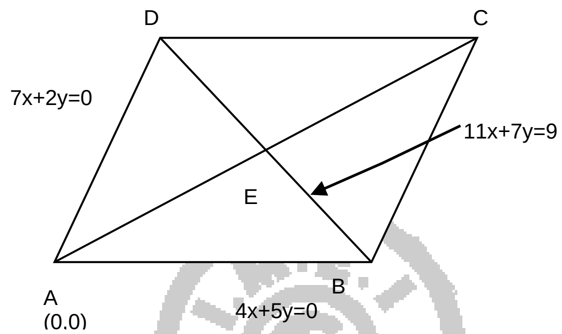
$\therefore \left|z - \frac{5}{2}\right|^2 = \frac{5+2\sqrt{2}}{4}$

Q.4 Two sides of a parallelogram are along the lines $4x+5y=0$ and $7x+2y=0$. If the equation of one of the diagonals of the parallelogram is $11x+7y=9$, then other diagonal passes through the point :

Options
1. $(1,\ 2)$
2. $(2,\ 1)$
3. $(2,\ 2)$
4. $(1,\ 3)$

Ans: (2, 2)

Sol:



Solve: $4x+5y=0$ ..(1)
$11x+7y=9$ …(2)

$x=\frac{5}{3}, y=-\frac{4}{3}$

$\therefore B\left(\frac{5}{3},-\frac{4}{3}\right)$

Solve: $7x+2y=0$ ..(1)
$11x+7y=9$ …(2)

$A\left(-\frac{2}{3},\frac{7}{3}\right)$

$$E=\left(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$$

AC passes through (0,0) and $\left(\frac{1}{2},\frac{1}{2}\right)$

$\therefore AE : y = x$

Clearly (2, 2) lies on y=x

Q.5 Let the mean and variance of the frequency distribution

$x:$	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
$f:$	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be :

Options
1. 4
2. 5
3. $\frac{16}{3}$
4. $\frac{17}{3}$

Ans: $\frac{17}{3}$

Sol: $\frac{8+24+8\alpha+9\beta}{8+\alpha+\beta} = 6 \Rightarrow 32+8\alpha+9\beta = 48+6\alpha+6\beta$

$\Rightarrow 2\alpha + 3\beta = 16 \qquad (1)$

$\sigma^2(\omega) = \frac{4(2-6)^2+4(6-6)^2+\alpha(8-6)^2+\beta(9-6)^2}{8+\alpha+\beta}$

$= \frac{64+4\alpha+9\beta}{8+\alpha+\beta} = 6.8 = \frac{34}{5}$

$\Rightarrow 320 + 20\alpha + 45\beta = 272 + 34\alpha + 34\beta$

$14\alpha - 11\beta = 48 \qquad (2)$

From (1) and (2), $\alpha = 5, \beta = 2$

$\bar{x}_{(c)} = \frac{8+24+35+18}{15} = \frac{85}{15} = \frac{17}{3}$

Q.6 A possible value of 'x', for which the ninth term in the expansion of

$\left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}$

is equal to 180, is :

Options
1. 2
2. 1
3. -1
4. 0

Ans: 1

Sol: $3^{\log_3\left(\sqrt{25^{x-1}+7}\right)} = \sqrt{25^{x-1}+7} = a$

$3^{-\frac{1}{8}\log_3\left(5^{x-1}+1\right)} = \left[3^{\log_3\left(5^{x-1}+1\right)}\right]^{-\frac{1}{8}} = \left(5^{x-1}+1\right)^{-\frac{1}{8}} = b$

$T_9 = {}^{10}C_8 a^2 b^8 = \frac{10.9}{1.2}(2.5^{x-1}+7)(5^{x-1}+1)^{-1}$

$$= \frac{45\left[\left(5^{x-1}\right)^2 + 7\right]}{\left(5^{x-1}+1\right)} = 180 \Rightarrow \frac{\left(5^{x-1}\right)^2 + 7}{5^{x-1}+1} = 4$$

Let $5^{x-1} = t \Rightarrow t^2 + 7 = 4t + 4 \Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow (t-1)(t-3) = 0 \Rightarrow t = 1$ or $t = 3 \Rightarrow 5^{x-1} = 1$ or $5^{x-1} = 3$

If $5^{x-1} = 1 \Rightarrow x = 1$

Q.7 Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :

Options
1. 0
2. 1
3. 2
4. 4

Ans: 0

Sol: $A^5 = B^5$ ....(1)

$A^2B^3 = A^3B^2$...(2)

$(1) + (2) \Rightarrow A^5 + A^2B^3 = B^5 + A^3B^2$

$\Rightarrow A^2\left(A^3 + B^3\right) = B^2\left(B^3 + A^3\right)$

$\Rightarrow A^2\left(A^3 + B^3\right) - B^2\left(A^3 + B^3\right) = 0$

$\Rightarrow \left(A^2 - B^2\right)\left(A^3 + B^3\right) = 0$

$\Rightarrow \left|A^2 - B^2\right| \left|A^3 + B^3\right| = 0$

Given that $\left|A^2 - B^2\right| \neq 0$

$\therefore \left|A^3 + B^3\right| = 0$

Q.8 Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x+y) + f(x-y) = 2f(x)\,f(y),\ f\left(\frac{1}{2}\right) = -1$. Then, the value of

$\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k + f(k))}$ is equal to :

Options
1. $\sec^2(21)\ \sin(20)\ \sin(2)$
2. $\operatorname{cosec}^2(21)\ \cos(20)\ \cos(2)$
3. $\sec^2(1)\ \sec(21)\ \cos(20)$
4. $\operatorname{cosec}^2(1)\ \operatorname{cosec}(21)\ \sin(20)$

Ans: $\operatorname{cosec}^2 1 . \operatorname{cosec} 21 . \sin 20$

Sol: $x = \frac{1}{2}, y = 0 \Rightarrow f\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right) = 2f\left(\frac{1}{2}\right)f(0)$

$\Rightarrow -1 + -1 = 2(-1)f(0) \Rightarrow f(0) = 1$

$x = \frac{1}{2}, y = \frac{1}{2} \Rightarrow f(1) + f(0) = 2f\left(\frac{1}{2}\right)f\left(\frac{1}{2}\right) \Rightarrow f(1) + 1 = 2(-1)(-1)$

$\Rightarrow f(1) = 2 - 1 \Rightarrow f(1) = 1$

$x = 1, y = 1 \Rightarrow f(2) + f(0) = 2f(1).f(1) \Rightarrow f(2) + 1 = 2$

$\Rightarrow f(2) = 1$

$x = 2, y = 1 \Rightarrow f(3) + f(1) = 2f(2).f(1) \Rightarrow f(3) + 1 = 2(1)(1)$

$\Rightarrow f(3) = 1$

Hence f(1) = f(2)=f(3)……=1 $\Rightarrow f(k) = 1, k \in N$

$\therefore S = \sum_{k=1}^{20} \frac{1}{\sin k.\sin(k+1)} = \sum_{k=1}^{20} \frac{1}{\sin 1} \frac{\sin[(k+1)-1]}{\sin k.\sin(k+1)}$

$= \frac{1}{\sin 1} \sum_{k=1}^{20} \frac{\sin[(k+1)\cos k - \cos(k+1)\sin k]}{\sin k.\sin(k+1)}$

$= \frac{1}{\sin 1} \sum_{k=1}^{20} [\cot k - \cot(k+1)] = \frac{-1}{\sin 1} \sum_{k=1}^{20} [\cot(k+1) - \cot(k)]$ .....(1)

I^{st} : $\cot 2 - \cot 1$

2^{nd} : $\cot 3 - \cot 2$

3^{rd} : $\cot 4 - \cot 3$

.
.
.
.

20^{th} : $\cot 21 - \cot 20$

Sum = cot 21- cot1

$\therefore (1) \Rightarrow \frac{-1}{\sin 1}\left[\frac{\cos 21}{\sin 21} - \frac{\cos 1}{\sin 1}\right]$

$= \frac{(\sin 21 \cos 21 - \cos 21 \sin 1)}{\sin^2 1.\sin 21}$

$= \frac{\sin(21-1)}{\sin^2 1.\sin 21} = \operatorname{cosec}^2 1.\operatorname{cosec} 21.\sin 20$

Q.9 Consider a circle C which touches the *y*-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the *x*-axis. Then the radius of the circle C is equal to :

Options
1. $\sqrt{53}$
2. 9
3. $\sqrt{82}$
4. 8

Ans: 9

Sol: Touch y axis $\Rightarrow f^2 = C$

$\Rightarrow x^2 + y^2 + 2gx + 2fy + f^2 = 0$ passes through (0,6)

$\Rightarrow 0 + 36 + 0 + 12f + f^2 = 0$

$\Rightarrow f^2 + 12f + 36 = 0 \Rightarrow f = -6$

x-intercept= $2\sqrt{g^2 - c} = 2\sqrt{g^2 - 36} = 6\sqrt{5}$

$\sqrt{g^2 - 36} = 3\sqrt{5} \Rightarrow g^2 - 36 = 45 \Rightarrow g^2 = 81 \Rightarrow g = \pm 9$

$$r=\sqrt{g^2+f^2-c}=\sqrt{g^2+f^2-f^2}=g=9$$

Q.10 Let $\alpha=\max\limits_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$ and $\beta=\min\limits_{x\in\mathbf{R}}\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\}$.

If $8x^2+bx+c=0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c-b$ is equal to :

Options
1. 50
2. 43
3. 42
4. 47

Ans: 42

Sol: $\alpha=\max\left[2^{6\sin 3x}.2^{8\cos 3x}\right]=\max\left[2^{2(3\sin 3x+4\cos 3x)}\right]$

$=2^{2.\sqrt{3^2+4^2}}=2^{2\times 5}=2^{10}$

$\beta=2^{2.\left(-\sqrt{3^2+4^2}\right)}=2^{2(-5)}=2^{-10}$

$\alpha^{1/5}=\left(2^{10}\right)^{1/5}=4$

$\beta^{1/5}=\left(2^{-10}\right)^{1/5}=2^{-2}=\frac{1}{4}$

$4+\frac{1}{4}=\frac{-b}{8}\Rightarrow -b=8\left(4+\frac{1}{4}\right)=32+2=34\Rightarrow b=-34$

$(4)\left(\frac{1}{4}\right)=\frac{c}{8}\Rightarrow c=8$

$\therefore c-b=42$

Q.11 Let $y=y(x)$ be the solution of the differential equation $(x-x^3)dy=(y+yx^2-3x^4)dx$, $x>2$. If $y(3)=3$, then $y(4)$ is equal to :

Options
1. 12
2. 4
3. 16
4. 8

Ans: 12

$\frac{dy}{dx}=\frac{y+yx^2-3x^4}{x-3^2}=\frac{y\left(1+x^2\right)}{x-x^3}-\frac{3x^4}{\left(x-x^3\right)}$

$\frac{dy}{dx}+\frac{\left(x^2+1\right)}{\left(x^3-x\right)}y=-\frac{3x^4}{x\left(1-x^2\right)}$

$\Rightarrow \frac{dy}{dx}+\frac{(x^2+1)}{(x^3-x)}y=\frac{3x^3}{x^2-1}$

I.F $=e^{\int Pdx}=e^{\int\frac{x^2+1}{x^3-x}dx}=e^{\int\frac{x^2+1}{x(x^2-1)}dx}=e^{\int\frac{(x^2+1)}{x(x+1)(x-1)}dx}$

I $=\frac{x^2+1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$

$\Rightarrow A(x+1)(x+1)+Bx(x-1)+cx(x+1)=x^2+1$

$x=-1\Rightarrow B(-1)(-2)=2\Rightarrow B=2$

$x=1\Rightarrow C(1)(2)=2\Rightarrow C=2$

$x=0\Rightarrow A(-1)=1\Rightarrow A=-1$

$\therefore I = -\log x + 2\log(x+1) + 2\log(x-1)$

$= \log\left[\frac{x^2-1}{x}\right]$

$\therefore IF = \frac{x^2-1}{x}$

$y\frac{(x^2-1)}{x} = \int \frac{3x^3}{(x^2-1)} \cdot \frac{x^2-1}{x} dx = \int 3x^2 dx = x^3 + C$

$\Rightarrow y = \frac{x}{x^2-1}(x^3+C)$

$y(3) = 3 \Rightarrow 3 = \frac{3}{8}(27+C) = 8 \Rightarrow C = -19$

$\therefore y = \frac{x(x^3-19)}{x^2-1}$

$y(4) = \frac{4.(64-19)}{16-1} = \frac{4 \times 45}{15} = 12$

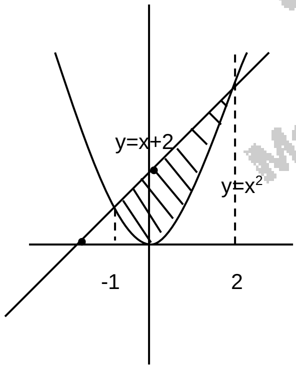
Q.12 The area of the region bounded by $y-x=2$ and $x^2=y$ is equal to :

Options

1. $\frac{16}{3}$
2. $\frac{4}{3}$
3. $\frac{2}{3}$
4. $\frac{9}{2}$

Ans: $\frac{9}{2}$

Sol: $y = x+2, y = x^2$



Solving,

$x^2 = x+2$

$\Rightarrow x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ or $x = 2$

$$\text{Area} = \int_{-1}^{2}\left(x+2-x^2\right)dx \quad = \left[\frac{x^2}{2}\right]_{-1}^{2} + 2[x]_{-1}^{2} - \frac{1}{3}\left[x^3\right]_{-1}^{2}$$

$$= \frac{1}{2}(4-1) + 2(2+1) - \frac{1}{3}(8+1) \qquad = \frac{3}{2} + 6 - 3 = 3 + \frac{3}{2} = \frac{9}{2}$$

Q.13 If $\tan\left(\frac{\pi}{9}\right)$, x, $\tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right)$, y, $\tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to :

Options
1. 4
2. 0
3. 1
4. 3

Ans: 0

Sol: $2x = \tan\frac{\pi}{9} + \tan\frac{7\pi}{18} = \tan\frac{\pi}{9} + \cot\frac{\pi}{9} = \frac{1}{\sin\frac{\pi}{9}\cos\frac{\pi}{9}} = \frac{2}{\sin\frac{2\pi}{9}}$

$\Rightarrow x = \frac{1}{\sin\frac{2\pi}{9}}$

$2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18} = \frac{\sin\frac{\pi}{9}}{\cos\frac{\pi}{9}} + \frac{\sin\frac{5\pi}{18}}{\cos\frac{5\pi}{18}} = \frac{\sin\left(\frac{\pi}{9}+\frac{5\pi}{18}\right)}{\cos\frac{\pi}{9}.\cos\frac{5\pi}{18}}$

$= \frac{\sin\frac{7\pi}{18}}{\cos\frac{\pi}{9}.\cos\frac{5\pi}{18}} = \frac{\cos\frac{\pi}{9}}{\cos\frac{\pi}{9}.\cos\frac{5\pi}{18}} = \frac{1}{\cos\frac{5\pi}{18}}$

$x - 2y = \frac{1}{\sin\frac{2\pi}{9}} - \frac{1}{\cos\frac{5\pi}{18}} = \frac{1}{\sin\frac{2\pi}{9}} - \frac{1}{\sin\frac{2\pi}{9}} = 0$

Q.14 Let $f: (a, b) \to \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x)=0$ has exactly five distinct roots in (a, b), then $g(x)g'(x)=0$ has at least :

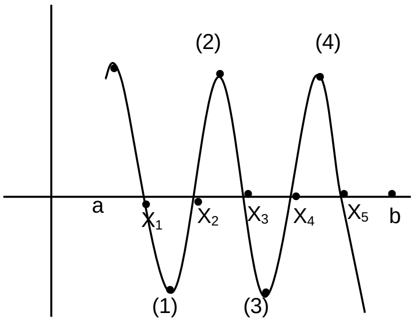
Options
1. three roots in (a, b)
2. seven roots in (a, b)
3. twelve roots in (a, b)
4. five roots in (a, b)

Ans: seven roots in (a, b)

Sol: $f(x) = \int_a^x g(t)dt$

$f'(x) = \frac{d}{dx}\left[\int_a^x g(t)dt\right] = g(x).x' - g(a).a' = g(x)$

$\therefore f'(x) = g(x)$



g(x)=0 ⇒f'(x)=0 has 4 roots

$f'(y_1) = 0,\ f'(y_2) = 0$

For the function f'(x)

$\Rightarrow f''(C) = 0$ for some $C \in (y_1, y_2)$

ie; $g'(C) = 0$ for some $C \in (y_1 y_2)$

$\Rightarrow g'(x) = 0$ has 3 roots

Hence total = 4+3=7

Q.15 Which of the following is the negation of the statement "for all M > 0, there exists $x \epsilon S$ such that $x \geq M$" ?

Options 1. there exists M > 0, there exists $x \epsilon S$ such that $x \geq M$

2. there exists M > 0, such that $x \geq M$ for all $x \epsilon S$

3. there exists M > 0, such that $x < M$ for all $x \epsilon S$

4. there exists M > 0, there exists $x \epsilon S$ such that $x < M$

Ans: there exist M >0, such that x<M for all x ε S

Sol: there exist M >0, such that x<M for all x ε S

Q.16 The point P (a, b) undergoes the following three transformations successively :

(a) reflection about the line $y = x$.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a + b$ is equal to :

Options 1. 5

2. 13

3. 9

4. 7

Ans: 9

Sol: After reflection , (a,b) $\rightarrow$ (b,a)

After translation , (b,a) $\rightarrow$ (b+2,a)

After rotation , (b+2,a) $\rightarrow$ (x,y) such that

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} b+2 \\ a \end{bmatrix} = \begin{bmatrix} \frac{b+2}{\sqrt{2}} - \frac{a}{\sqrt{2}} \\ \frac{b+2}{\sqrt{2}} + \frac{a}{\sqrt{2}} \end{bmatrix}$$

$$\therefore (x,y) = \left(\frac{b+2-a}{\sqrt{2}}, \frac{b+2+a}{\sqrt{2}}\right) = \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

$\Rightarrow b+2-a=-1 \Rightarrow a-b=3$ (1)

$\Rightarrow b+2+a=7 \Rightarrow a+b=5$ (2)

$\Rightarrow a=4, b=1$

$\therefore 2a+b=9$

Q.17 For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines

$$\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3} \text{ and } \frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3},$$

lies on the plane $x+2y-z=8$, then $\alpha-\beta$ is equal to :

Options
1. 9
2. 3
3. 5
4. 7

Ans: 7

Sol: $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ .....(A)

$\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$ .....(B)

$\Rightarrow 3y-3=2z-2 \Rightarrow 3y-2z=1$ ....(1)

$y-6=z-7 \Rightarrow y-z=-1$ ....(2)

(1) $\Rightarrow 3y-2z=1$

$2\times(2) \Rightarrow 2y-2z=-2$

y=3,z=4

$(A) \Rightarrow \frac{x-\alpha}{1} = 1 \Rightarrow x = \alpha+1$

$(B) \Rightarrow \frac{x-4}{\beta} = -1 \Rightarrow x = -\beta+4$

$\Rightarrow \alpha+1 = -\beta+4$

$\Rightarrow \alpha+\beta = 3$ ....(3)

Point of intersection: $(\alpha+1, 3, 4)$

Substituting in x+2y-z = 8

$\Rightarrow \alpha+1+6-4=8$

$\alpha+3=8$ $\Rightarrow \alpha=5$

$\Rightarrow \beta=-2$ (from(3))

$\alpha-\beta=7$

Q.18 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

Options

1. $\frac{\sqrt{3}+1}{\sqrt{3}}$
2. 2
3. 1
4. $\sqrt{3}+1$

Ans: 2

Sol: $|\vec{a}| = \sqrt{2}, |\vec{b}| = 1, |\vec{c}| = 2$

$\vec{a} = \vec{b} \times (\vec{b} \times \vec{c}) = (\vec{b}.\vec{c})\vec{b} - \vec{b}^2.\vec{c} = (|\vec{b}||\vec{c}|\cos\theta)\vec{b} - |\vec{b}|^2.\vec{c}$

$\Rightarrow \vec{a} = (2\cos\theta)\vec{b} - \vec{c}$

$\Rightarrow \vec{a}^2 = 4\cos^2\theta.\vec{b}^2 - 4\cos\theta.\vec{b}.\vec{c} + \vec{c}^2$

$\Rightarrow |\vec{a}|^2 = 4\cos^2\theta.|\vec{b}|^2 - 4\cos\theta.|\vec{b}|.|\vec{c}|\cos\theta + |\vec{c}|^2$

$2 = 4\cos^2\theta - 4\cos\theta(1)(2)\cos\theta + 4$

$\Rightarrow 2 = 4\cos^2\theta - 8\cos^2\theta + 4 \Rightarrow 4\cos^2\theta = 2 \Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$\therefore 1 + \tan\theta = 2$

Q.19 Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true ?

Options

1. f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$
2. f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$
3. f is differentiable everywhere in $(0, \infty)$
4. f is not continuous exactly at two points in $(0, \infty)$

Ans: f is differentiable everywhere in $(0,\infty)$

Sol: $f(x) = \begin{cases} \sin x & \text{if } 0 \le x \le \frac{\pi}{2} \\ 1 & \text{if } \frac{\pi}{2} < x \le \pi \\ 2 + \cos x & \text{if } x > \pi \end{cases}$

Clearly $f(\pi/2^-) = f(\pi/2^+) = f(\pi/2) \Rightarrow \text{continuous at } x = \frac{\pi}{2}$

$f(\pi^-) = 1, f(\pi^+) = 2 - 1 = 1, f(\pi = 1) \Rightarrow \text{continuous at } x = \pi$

$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}^-} = \cos\frac{\pi}{2} = 0, \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}^+} = 0 \Rightarrow \text{Differentiable at} x = \frac{\pi}{2}$

$\left(\frac{dy}{dx}\right)_{x=\pi^-} = 0, \left(\frac{dy}{dx}\right)_{x=\pi^+} = -\sin 0 = 0 \Rightarrow \text{Differentiable at} x = \pi$

Q.20 Let **N** be the set of natural numbers and a relation R on **N** be defined by $R = \{(x, y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is :

Options 1. symmetric but neither reflexive nor transitive

2. reflexive but neither symmetric nor transitive

3. an equivalence relation

4. reflexive and symmetric, but not transitive

Ans: reflexive but neither symmetric not transitive

Sol: $x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$x^2(x - 3y) - y^2(x - 3y) = 0$

$\Rightarrow (x - 3y)(x + y)(x - y) = 0$

$\Rightarrow x = 3y \text{ or } x = -y \text{ or } x = y$

Since x=y is a possibility, $(x, y) \in R \Rightarrow$ Re flexive

$(3,1) \in R \quad (\because 3 = 3(1))$ but $(1,3) \notin R$

$(9,3), (3,1) \in R$ but $(9,1) \notin R$

Section B

Q.1 Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to ________.

Given --
Answer :

Ans: 9.00

Sol: $\vec{a}.\vec{b} = 3 - \alpha\beta - \alpha\beta = -1$

$\Rightarrow 2\alpha\beta = 4$

$\Rightarrow \alpha\beta = 2 \qquad (1)$

$\vec{b}.\vec{c} = -3\alpha - 2\beta - \alpha = 10$

$\Rightarrow -4\alpha - 2\beta = 10 \Rightarrow 2\alpha + \beta = -5$

$\alpha = -2, \beta = -1$

$\therefore (\vec{a}\times\vec{b})\cdot\vec{c} = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 1(-1+4)-2(3-4)-1(-6+2) = 3+2+4 = 9$

Q.2 If the real part of the complex number $z = \dfrac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \dfrac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to ________.

Given --
Answer :

Ans: 1.00

Sol: $\dfrac{a+bi}{c+di} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{bc-ad}{c^2+d^2}i$

$\therefore \text{Re}\left(\dfrac{3+2i\cos\theta}{1-3i\cos\theta}\right) = 0 \Rightarrow (3)(1) + (2\cos\theta)(-3\cos\theta) = 0$

$\Rightarrow 3 - 6\cos^2\theta = 0 \Rightarrow \cos^2\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{4}$

$\sin^2 3\theta + \cos^2\theta = \sin^2\dfrac{3\pi}{4} + \cos^2\dfrac{\pi}{4} = \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 = 1$

Q.3 Let $A = \{n \in \mathbf{N} \mid n^2 \le n + 10{,}000\}$, $B = \{3k+1 \mid k \in \mathbf{N}\}$ and $C = \{2k \mid k \in \mathbf{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ________.

Given --
Answer :

Ans: 832.00

Sol: $A = \{n \in N : n^2 \le n + 10000\}$

$\Rightarrow n(n-1) \le 10000, n \in N$

$\Rightarrow n = 1,2,3,.........100$

$\therefore A = \{1,2,3,.........100\}$

$B - C = \{4,7,10,13,16,19,22,...\} - \{2,4,6,8,10,12,......\}$

$= \{7,13,19,25,31,........\}$

$A \cap (B-C) = \{7,13,19,25,31,........97\} \Rightarrow n = \dfrac{97-7}{6} + 1 = 16$

$\text{sum} = 7 + 13 + 19 + ... + 97 = \dfrac{16}{2}(7+97) = 832$

Q.4 The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points Q(3, −4, −5) and R(2, −3, 1) and the plane $2x+y+z=7$, is equal to ________.

Given 1
Answer :

Ans: 7.00

Sol: QR: $\dfrac{x-3}{-1} = \dfrac{y+4}{1} = \dfrac{z+5}{6} = \lambda$ (1)

Plane: 2x+y+z=7 (2)

$(1) \Rightarrow x = -\lambda + 3, y = \lambda - 4, z = 6\lambda - 5$

$(2) \Rightarrow -2\lambda + 6 + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$

$\therefore$ Point of intersection, $S = (1,-2,7)$ and $P = (3,4,4)$

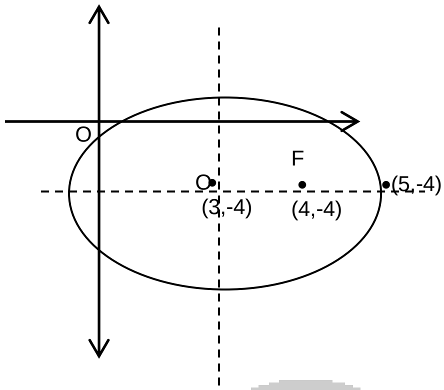
$\therefore PS = \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2} = \sqrt{4+36+9} = 7$

Q.5 Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3,-4)$, one focus at $(4,-4)$ and one vertex at $(5,-4)$. If $mx-y=4$, $m>0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to __________.

Given --
Answer :

Ans: 3.00

Sol:



$C = O'F = 1, a = O'A = 2$

$C = \sqrt{a^2 - b^2} \Rightarrow 1 = \sqrt{4 - b^2} \Rightarrow b^2 = 3$

$\therefore$ Ellipse : $\frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$ ....(1)

y = mx – 4 touches (1)

Solving,

$\frac{(x-3)^2}{4} + \frac{m^2x^2}{3} = 1$

$\Rightarrow 3(x^2 - 6x + 9) + 4m^2x^2 = 12$

$\Rightarrow (3 + 4m^2)x^2 - 18x + 15 = 0$

$b^2 - 4ac = 0$

$\Rightarrow 324 - 60(3 + 4m^2) = 0$

$\Rightarrow 3 + 4m^2 = \frac{27}{5}$

$4m^2 = \frac{27}{5} - 3 = \frac{12}{5}$

$\Rightarrow m^2 = \frac{3}{5} \Rightarrow 5m^2 = 3$

Q.6 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \dots + A^{20}$, then the sum of all the elements of the matrix M is equal to __________.

Given **6**
Answer :

Ans: 2115.00

Sol: $M = A + A^2 + A^3 + \dots\dots + A^{20}$

$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^3 = A^2.A = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = A^3.A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^5 = A^4.A = \begin{bmatrix} 1 & 5 & 15 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 20 & (1+2+3+....+20) & (1+3+6+10+.....+20^{th}\text{term}) \\ 0 & 20 & (1+2+3+....+20) \\ 0 & 0 & 20 \end{bmatrix}$$

$$a_{12} = a_{21}(\text{of M}) = \frac{20(20+1)}{2} = 210$$

To find a_{13}

$$a_n = 1 + (2+3+4+......\text{to}(n-1)\text{terms})$$

$$= 1 + \frac{(n-1)}{2}[4+(n-1)]$$

$$= 1 + \frac{(n-1)}{2}(n+3) = \frac{n^2+2n-3+2}{2} = \frac{1}{2}(n^2+2n-1)$$

$$S_n = \sum \frac{1}{2}(n^2+2n-1) = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} - n\right]$$

$$\text{putting } n = 20, S_{20} = \frac{1}{2}\left[\frac{20\times 21\times 41}{6} + (20\times 21) - 20\right] = 1635$$

$$\therefore M = \begin{bmatrix} 20 & 210 & 1635 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\text{sum} = (20\times 3) + (210\times 2) + 1635 = 2115$$

Q.7 Let n be a non-negative integer. Then the number of divisors of the form "4n+1" of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to __________.

Given --
Answer :

Ans: 924.00

Sol: $10^{10} \times 11^{11} \times 13^{13} = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$

Let us denote the set of numbers which can be written in the form 4n+1 by {4n+1}

$\{5^0, 5^1, 5^2, 5^3, ...5^{10}\}, \{13^0, 13^1, 13^2, ...13^{13}\}, \{11^0, 11^2, 11^4, ...11^{10}\} \subset \{4n+1\}$

ie; $5^a.13^b.11^{2c} \subset \{4n+1\}$ where $a \in \{0,1,2,.....10\}$

$b \in \{0,1,2,.....13\}$

$c \in \{0,1,2,3,4,5\}$

Number of ways = $11\times 14\times 6 = 924$

Q.8 Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y}dx$; $\alpha \in \mathbb{N}$.

If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to __________.

Given --
Answer :

Ans: 2.00

Sol: $\frac{dy}{dx} = e^{\alpha x}.e^{y} \Rightarrow \int e^{-y}dy = \int e^{\alpha x}dx$

$\Rightarrow \frac{e^{-y}}{-1} = \frac{e^{\alpha x}}{\alpha} + C \Rightarrow e^{-y} = -\left[\frac{e^{\alpha x}}{\alpha} + C\right]$

If $x = \log_e 2, y = \log_e 2 \Rightarrow \frac{1}{2} = -\left[\frac{2^{\alpha}}{\alpha} + C\right]$

$\Rightarrow \frac{2^{\alpha}}{\alpha} + C = \frac{-1}{2}$ .....(1)

If $x = 0,\ y = \log\left(\frac{1}{2}\right) \Rightarrow 2 = -\left[\frac{1}{\alpha} + C\right] \Rightarrow \frac{1}{\alpha} + C = -2$ ...(2)

$(1) - (2) \Rightarrow \frac{2^{\alpha}}{\alpha} - \frac{1}{\alpha} = -\frac{1}{2} + 2 \Rightarrow \frac{2^{\alpha} - 1}{\alpha} = \frac{3}{2} \Rightarrow \alpha = 2$

Q.9

If $\int_0^{\pi} (\sin^3 x)\, e^{-\sin^2 x}\, dx = \alpha - \frac{\beta}{e}\int_0^1 \sqrt{t}\, e^t dt$, then $\alpha + \beta$ is equal to _________.

Given --
Answer :

Ans: 5.00

Sol: $I = 2\int_0^{\pi/2} \sin^3 x.e^{-\sin^2 x}dx$

$\sin^2 x = t$

$2\sin x \cos x dx = dt$

$\sin x dx = \frac{1}{2\sqrt{1-t}}dt$

$\therefore I = 2\int_0^1 t.\frac{1}{2\sqrt{1-t}}e^{-t}.dt$

$= \int_0^1 \frac{te^{-t}}{\sqrt{1-t}}dt = \int_0^1 \frac{(1-t)e^{-(1-t)}}{\sqrt{1-(1-t)}}dt$

$= \int_0^1 \frac{(1-t)e^{t-1}}{\sqrt{t}}dt = \frac{1}{e}\int_0^1 \left(\frac{1}{\sqrt{t}} - \sqrt{t}\right)e^t dt$

$= \frac{1}{e}\left[\int_0^1 \frac{1}{\sqrt{t}}e^t dt - \int_0^1 \sqrt{t}\, e^t dt\right]$ ....(1)

Let $I_1 = \int_0^1 \frac{1}{\sqrt{t}}e^t dt$

$= \left[e^t \int \frac{1}{\sqrt{t}}dt - \int\left[e^t.\int \frac{1}{\sqrt{t}}dt\right]dt\right]_0^1$

$= \left[e^t.2\sqrt{t}\right]_0^1 - 2\int_0^1 \sqrt{t}.e^t dt$

$= 2e - 2\int_0^1 \sqrt{t}e^t dt$

$\therefore (1) \Rightarrow \frac{1}{e}\left[2e - 3\int_0^1 \sqrt{t}.e^t dt\right]$

$$= 2 - \frac{3}{e}\int_0^1 \sqrt{t}.e^t dt$$

$$\Rightarrow \alpha = 2, \beta = 3 \Rightarrow \alpha + \beta = 5$$

Q.10 The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to __________.

Given --
Answer :

Ans: 2.00

Sol: $e^x = t$

$t^4 - t^3 - 4t^2 - t + 1 = 0$

$\Rightarrow (t+1)^2(t^2 - 3t + 1) = 0$

$t = -1$ or $t^2 - 3t + 1 = 0$

But $e^x \neq -1$

$t^2 - 3t + 1 = 0 \Rightarrow t = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{3}}{2} > 0$

$\therefore$ Two Solutions