T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2016

Paper II – Code 0

PART I

1	2	3	4	5	6
C	**C**	**C**	**B**	**D**	**A**

7	8	9	10	11	12
C, D	**A, C**	**B, C**	**A, B, D**	**A, B, D**	**A, B**

13	14
A, B, C, D	**D**

15	16	17	18
C	**B**	**D**	**A**

Section I

1. $A = \frac{A_0}{2^{t/\tau}} = 64\ A$

$2^{t/\tau} = 2^6$ $\quad t = 6 \times \tau$

$= 6 \times 18$

$= 108$ days

2. $(BE)_{^{15}_{7}N} = 7m_P + 8m_n - m_{^{15}_{7}N}$

$(BE)_{^{15}_{8}O} = 8m_P + 7m_n - m_{^{15}_{8}O}$

$\Rightarrow \Delta(BE) = (m_n + m_P) + \left(m_{^{15}_{8}O} - m_{^{15}_{7}N}\right)$

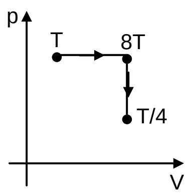
$= 0.00084 + 0.002956$

$= 0.003796$ u

$\Rightarrow \frac{3}{5} \times \frac{14 \times 1.44\ \text{MeV}\ f_m}{0.003796 \times 931.5\ \text{MeV}} = R$

$\Rightarrow R = 3.42\ f_m$

3.



$\gamma = \frac{5}{3}$

$C_p = \frac{5R}{2}$

$C_V = \frac{3R}{2}$

$Q = nC_p(8T - T) + nC_V\left(\frac{T}{4} - 8T\right)$

$= n \times \frac{5R}{2} \times 7T + n \times \frac{3R}{2}\left(-\frac{31}{4}\right)T$

$= nRT\left[\frac{35}{2} - \frac{93}{8}\right] = p_1V_1 \times \frac{(140 - 93)}{8}$

$= 10^5 \times 10^{-3} \times \frac{47}{8} = 588$ J

4. C_1 $\quad$ 10 VSD = 9 MSD

1 VSD = 0.9 MSD

(L.C) = 1 MSD – 1 VSD

= 0.1 MSD = 0.1 mm

= 0.01 cm

Measured value

= MSR + VCD $\times$ L.C

= 2.8 + 7 $\times$ 0.01

= 2.87 cm

C_2 $\quad$ 10 VSD = 11 MSD

1 VSD = 1.1 MSD = 1.1 mm

= 0.11 cm

VCD 7

Measured value

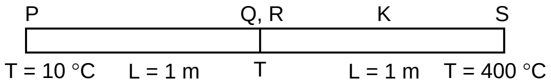
= MSR + (8 MSD – 7 VSD)

= 2.8 + (8 $\times$ 0.1 – 7 $\times$ 0.11)

= 2.83 cm

5. Ans. D

6.



Let temperature of junction = T

Rate of heat transfer =

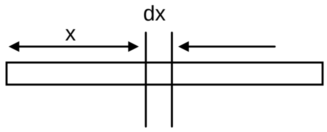
$$\frac{dQ}{dt} = \frac{2KA(T-10)}{L} = \frac{KA(400-T)}{L}$$

$\Rightarrow 2(T - 10) = 400 - T$

$3T = 420$

$T = 140\ °C$

For wire PQ



$$\frac{\Delta T}{\Delta x} = \frac{140-10}{1} = 130$$

Temp. at distance x

$T = 10 + 130\,x$

$T - 30 = 130x$

Inc. in length of small element

$$\frac{dy}{dx} = \alpha \Delta T$$

$$\frac{dy}{dx} = \alpha(T - 10)$$

$$\frac{dy}{dx} = \alpha(130x)$$

$$\int_0^{\Delta L} dy = 130\alpha \int_0^{L} x\,dx$$

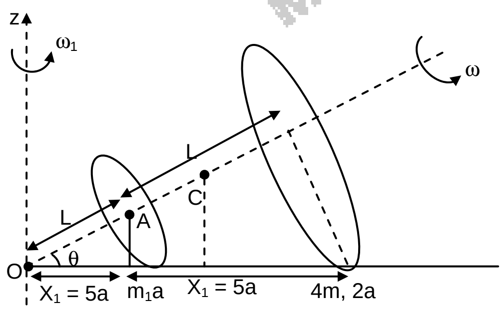
$$\Delta L = \frac{130\alpha x^2}{2}$$

$$\Delta L = \frac{130 \times 1.2 \times 10^{-5} \times 1}{2}$$

$\Delta L = 78 \times 10^{-5}$ m = 0.78 mm

Section II

7.



$$\tan\theta = \frac{a}{L} = \frac{1}{\sqrt{24}} \Rightarrow \sin\theta = \frac{1}{5}$$

(C) $X = \sqrt{L^2 + a^2}$

$X = a\sqrt{24+1}$

$X = 5a$

Velocity of disc A,

$v_A = \omega_z .\ x$

Disc performs pure rolling

$v_P = v_A - \omega a = 0$

$v_A = \omega a = \omega_z x$

$$\omega_z = \frac{\omega}{5}$$

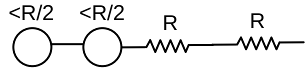
(D) Angular momentum

$L = I\omega$

$$L = \left[\frac{ma^2}{2} + \frac{4m(2a)^2}{2}\right]\omega$$

$$L = \frac{17ma^2}{2} \times \omega$$

8.

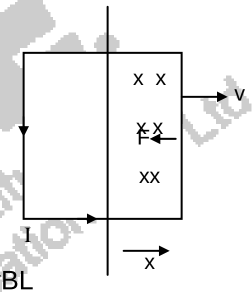


Maximum resistance all in series

So maximum voltage

Minimum resistors when all in parallel. Hence maximum current in this case.

9.



$\mathcal{E} = vBL$

$$I = \frac{vBL}{R}$$

$F = (-)ILB$

$$= \frac{vBL}{R} \times LB$$

$$m\frac{dv}{dt} = (-)\frac{vB^2L^2}{R}$$

$$m\frac{dv}{dr}v = (-)\frac{vB^2L^2}{R}$$

$$dv = -\frac{B^2L^2}{mR}.dx$$

$$v - v_0 = -\frac{B^2L^2}{mR} \times x$$

$$v = v_0 - \frac{B^2L^2}{mR} \times x$$

(C) correct

$I = \frac{vBL}{R}$, v is linear $\Rightarrow$ I is linear

ACW $\Rightarrow$ I +ve initially. I –ve when coming out
B correct
(D) wrong
Force is always retarding.
Hence (A) wrong.

10. Error in measurement of r

$\frac{\Delta r}{r} = \frac{1\ \text{mm}}{10\ \text{m}} \times 100 = 10\%$

(A) correct

Average of T = $\frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$

= 0.556 s = 0.56 s

Average of $|\Delta T|$ = $\frac{0.04 + 0 + 0.01 + 0.02 + 0.03}{5}$

= $\frac{0.1}{5}$ = 0.02 s

% error $\frac{|\Delta T|}{T} = \frac{0.02}{0.56} \times 100 = 3.57\%$

(B) correct
(C) wrong

$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta(R - r)}{R - r}$

$= 2 \times 3.57\% + \left(\frac{\Delta R + \Delta r}{R - r}\right) \times 100$

$= 7.14\% + \frac{2\ \text{mm}}{50\ \text{mm}} \times 100 = 11.14\%$

(D) correct

11. In case (i) there is loss of kinetic energy (perfectly inelastic collision). Hence amplitude will decrease. In the second case there is no loss of K.E, hence amplitude remains unchanged.

K.E = $\frac{p^2}{2M} = \frac{1}{2}kA^2$ (before m is attached)

K.E = $\frac{p^2}{2(M + m)} = \frac{1}{2}kA_1^2$ (after m is attached)

$\frac{A_1}{A} = \sqrt{\frac{M}{M + m}}$ (A) correct

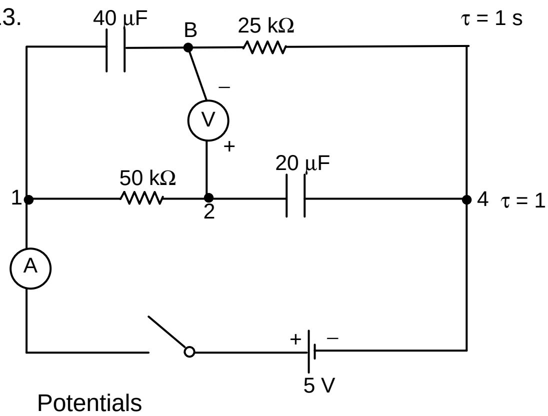
(B) correct in both cases final

$T = 2\pi\sqrt{\frac{M + m}{R}}$

(C) Wrong. Total energy decreases only in the first case.
(D) Instantaneous speed at mean position decreases only in both the cases.
D $\Rightarrow$ correct.

12. A. Hemispherical wavefronts intersecting with a plane give rise to semicircular bands, both bright and dark, on the screen both centered at O.

B. As wavefronts cannot reinforce at region at and very close to O, the region appears dark.

13.


Potentials
At t = 0, $V_1 = 5$ V $V_2 = 0$
$V_3 = 5$ V $V_4 = 0$
$V_3 - V_2 = 5$ V $\Rightarrow$ voltmeter reading –5 V
After long time, capacitors get charged
$V_1 = 5$ V, $V_3 = 0$
$V_2 = 5$ V, $V_4 = 0$
$V_3 - V_2 = -5$ V $\Rightarrow$ voltmeter reads +5 V
$\Rightarrow$ (A) correct
Voltmeter will read zero if $V_3 = V_2$
$V_{40} = V_0(1 - e^{-t/\tau})$ & $V_{20} = V_0(1 - e^{-t/\tau})$
Potential $V_3 = V_0 - V_{40}$
Potential $V_2 = V_{20}$
equating $V_2 = V_3$
$V_0 - V_{40} = V_{20} \Rightarrow V_0 = 2V_{20}$
$\Rightarrow V_0 = 2 \times V_0(1 - e^{-t/\tau})$
$e^{t/\tau} = 2$
$\Rightarrow t = \tau\ell n2 = 1 \times \ell n2 = \ell n2$ seconds
$\Rightarrow$ (B) correct
$q_1 = VC_1(1 - e^{-t/\tau})$

$i_1 = \frac{V}{R_1}e^{-t/\tau}$ $i_2 = \frac{V}{R_2}e^{-t/\tau}$

$i = i_1 + i_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)Ve^{-t/\tau} = i_0e^{-t/\tau}$

$\Rightarrow i = i_0e^{-1}$ at $t = \tau = 1$ s $\Rightarrow$ (C) correct.
Capacitors and voltmeter block the current after long time $\Rightarrow$ ammeter reads zero (D) correct.

14. A. $\lambda_e = \frac{h}{p} = \frac{h}{mv}$

$\lambda_{ph} = \frac{hc}{\lambda}$

As λ_{ph} increases λ decreases
B wrong
C. As ϕ increases λ_e decreases. So also about λ_{ph}
Hence (C) is wrong
D. $\lambda \propto \frac{1}{\sqrt{V}}$ $V \to 4$ V then $\lambda \to \frac{\lambda}{2}$

Section III

15. $2m(\bar{v}_{rot} \times \bar{\omega})$ is a pseudo force (coriolis force)

$= 2m(v_{rot}\hat{i}) \times \omega\hat{k}$

$= 2m\omega v_{rot}(-\hat{j})$

Hence normal reaction of side wall of groove on the block should nullify the above coriolis force in the reference frame of the rotating disc.

i.e. $N_{wall} = 2m\omega v_{rot}\left(+\hat{j}\right)$ ---(A)

$m(\overline{\omega}\times\overline{r})\times\overline{\omega}$ is also a pseudo force (centrifugal force) $\rightarrow$ radially outward

$$m\left(\omega\hat{k}\times r\hat{i}\right)\times\omega\hat{k} = m\omega^2 r\hat{j}\times\hat{k}$$

$$= m\omega^2 r\hat{i}$$

This force decides the radial distance r;

$$F = \frac{mdv_{rot}}{dt} = m\omega^2 r$$

$$\frac{dv_{rot}}{dt} = \omega^2 r \Rightarrow \frac{dv_{rot}}{dr}, \quad \frac{dr}{dt} = \omega^2 r$$

$$\frac{dv_{rot}}{dr}.v_{rot} = \omega^2 r \Rightarrow \int_0^v v_{rot}dv_{rot} = \int_{r_0}^r \omega^2 r dr$$

$$\Rightarrow \left.\frac{v_{rot}{}^2}{2}\right|_0^v = \left.\frac{\omega^2 r^2}{2}\right|_{r_0}^r$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - r_0{}^2\right)$$

$$v = \omega\sqrt{r^2 - r_0{}^2} = \frac{dr}{dt}$$

$$\int_0^t \omega dt = \int_{r_0}^r \frac{dr}{\sqrt{r^2 - r_0{}^2}} = \cosh^{-1}\left(\frac{r}{r_0}\right) - \cosh^{-1}\left(\frac{r_0}{r_0}\right)$$

$$\omega t = \cosh^{-1}\left(\frac{r}{r_0}\right) - 0 \Rightarrow \frac{r}{r_0} = \cosh(\omega t)$$

$$\Rightarrow r = r_0\left[\frac{e^{+\omega t} + e^{-\omega t}}{2}\right] = \frac{R}{2}\left[\frac{e^{+\omega t} + e^{-\omega t}}{2}\right]$$

$$\Rightarrow r = \frac{R}{4}\left[e^{+\omega t} + e^{-\omega t}\right]$$

16. $\overline{N}_{wall} = 2m\omega v_{rot}\left(+\hat{j}\right)$

(From equation (A) in previous question)

$$r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

$$\frac{dr}{dt} = v_{rot} = \frac{R}{4}\times\omega\left(e^{\omega t} - e^{-\omega t}\right)$$

$$\Rightarrow \overline{N}_{wall} = 2m\omega\times\frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

$$= \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

$$\overline{N}_{floor} = mg\hat{k}$$

The net reaction of the disc on the block

$$= \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

17. The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

The balls after sticking to the upper plate will lose their positive charge and get negatively charged. They will now be repelled downwards. The process will keep repeating.

18. The average current at steady state can be expressed as I = nq

q the charge on each ball and n the number of balls per second transisting space between plates.

$n\times\frac{1}{2}mv^2 = eV_0 \Rightarrow n \propto V_0$

$q = C.V_0 = 4\pi\varepsilon_0 rV_0 \Rightarrow q \propto V_0$

$I \propto V_0.V_0 = V_0{}^2$

PART II

19	20	21	22	23	24
D	**A**	**A**	**D**	**A**	**A**

25	26	27	28	29	30
B, C	**A, B**	**B, C, D**	**C, D**	**B, C, D**	**A, B, C**

31	32
A, C	**B, D**

33	34	35	36
B	**C**	**A**	**B**

Section I

19. Cell reaction is

$M^{4+} + H_2 \rightarrow M^{2+} + 2H^+$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[M^{2+}][H^+]^2}{[M^{4+}]}$$

$$0.092 = 0.151 - \frac{0.059}{2}\log\frac{[M^{2+}]}{[M^{4+}]}$$

$$\frac{0.059}{2}\log\frac{[M^{2+}]}{[M^{4+}]} = 0.059$$

$$\frac{[M^{2+}]}{[M^{4+}]} = 10^2$$

x = 2

20. The order of acid strength is
I > II > III > IV

21. $[Ni(NH_3)_6]^{2+}$ octahedral
$[Pt(NH_3)_4]^{2+}$ square planar
$[Zn(NH_3)_4]^{2+}$ tetrahedral

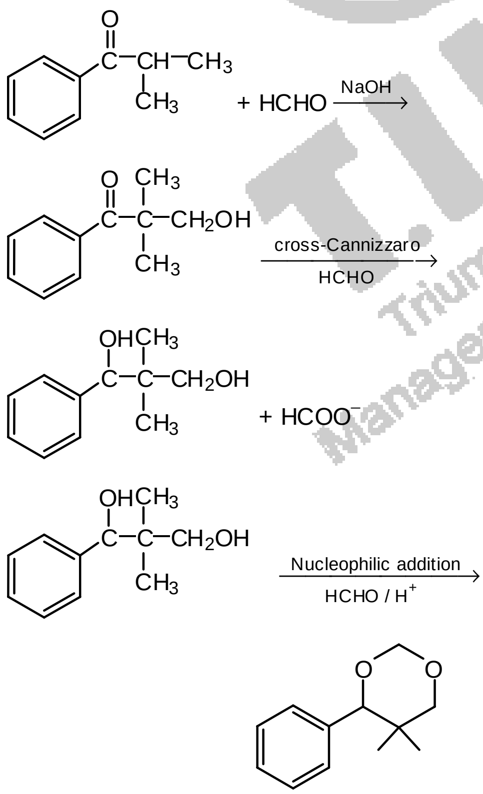
22. As the concentration of $CH_3OH(aq)$ increases, surface tension decreases slowly. Addition of an ionic compound increases the surface tension of aqueous solution. Addition of soap reduces the surface tension of water drastically.

23. $2Ag^+ + S_2O_3^{2-} \rightarrow Ag_2SO_3\downarrow$

$Ag_2S_2O_3 + S_2O_3^{2-} \rightarrow \underset{\text{Soluble}}{[Ag(S_2O_3)_2]^{3-}}$

$[Ag(S_2O_3)_2]^{3-} + 3Ag^+ \rightarrow \underset{\text{white}}{2Ag_2S_2O_3}\downarrow$

$Ag_2S_2O_3 + H_2O \rightarrow \underset{\text{Black}}{Ag_2S} + H_2SO_4$

24. 

Acetal formation

Section II

25. (B) and (C) are only correct statements.

26. $CCl_4 + CH_3OH$ – positive deviation
$CS_2 + CH_3COCH_3$ – positive deviation
Benzene + Toluene – ideal solution
Phenol + Aniline – negative deviation

27. The number of nearest neighbours for an atom in the top most layer of ccp arrangement is 9
Packing efficiency for ccp = 74%
The number of octahedral and tetrahedral voids per atom are 1 and 2 respectively.
Edge length, $a = 2\sqrt{2}\,r$

28. $NaBH_4$ and Raney Ni/H_2 cannot reduce carboxylic acid, ester or epoxide.

29. Reactions (B), (C) and (D) can give tert-butyl benzene.

30. Refining 'blister copper' is by
(a) Poling
(b) Electrolysis

31. C_2^{2-} = KK, $\sigma 2s^2$, $\sigma^* 2s^2$, $\pi 2p_x^2 = \pi 2p_y^2$, $\sigma 2p_z^2$

O_2 = KK, $\sigma 2s^2$, $\sigma^* 2s^2$, $\sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2$,
$\pi^* 2p_x^1 = \pi^* 2p_y^1$

B.O of O_2 = 2 B.O of $O_2^{2+} = 3$

N_2^+ has B.O = 2.5 (one Bonding e⁻ less) N_2^- has B.O = 2.5 (one e⁻ in ABMO)

He_2^+ molecular ion has $\frac{1}{2}$ bond order while 2 separate He atom have no bond order.

32. $2HNO_3 \xrightarrow[-H_2O]{P_2O_5} N_2O_5$
(A) P_4 reacts with HNO_3 to give H_3PO_4
(C) N_2O_5 has N–O–N bond
(D) $N_2O_5 + Na \rightarrow NaNO_3 + NO_2$ (brown gas)
(B) N_2O_5 is diamagnetic

Section III

33. $X_{2(g)} \rightleftharpoons 2X_{(g)}$ Total

$1-\frac{\beta}{2}$ $\frac{\beta}{2}$ $1+\frac{\beta}{2}$

$$p_{x_2} = \frac{(2-\beta)2}{2+\beta}$$

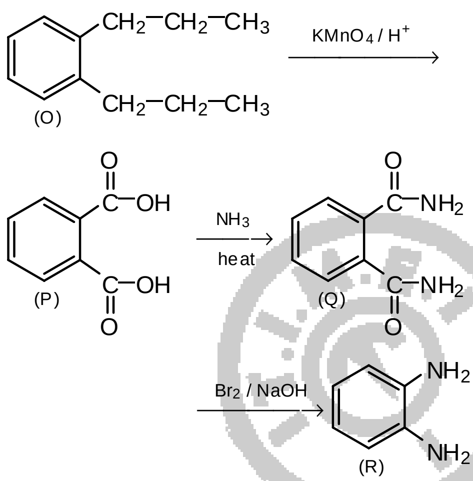
$$p_x = \frac{\beta\times 4}{2+\beta}$$

$$K_p = \frac{p_x^2}{p_{x_2}}$$

$$= \frac{16\ \beta^2}{(2+\beta)^2} \times \frac{2+\beta}{(2-\beta)2}$$

$$= \frac{8\beta^2}{4-\beta^2}$$

34. $\Delta G^\circ = -RT \ln K_p$
Given that ΔG° is positive, hence K_p is less than 1.
$K_p = K_c (RT)^{\Delta n}$
Since $\Delta n = 1$, K_c is also less than 1
If $\beta = 0.7$,
$$K_p = \frac{8\beta^2}{4-\beta^2} = 1.12$$
$\therefore\ \beta \neq 0.7$

35.


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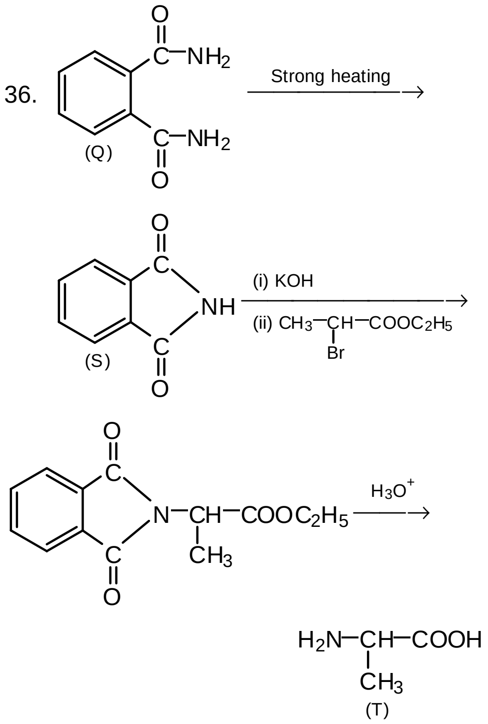


PART III

37	38	39	40	41	42
C	**C**	**B**	**C**	**B**	**A**

43	44	45	46	47	48
A, D	**A, C, D**	**A, B**	**B, C**	**B, C**	**B, C, D**

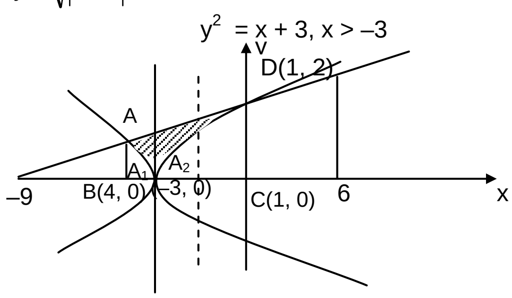
49	50
B, C	**A, C, D**

51	52	53	54
B	**C**	**A**	**C**

Section I

37. Image of (3, 1, 7) with respect to $x - y + z = 3$ is $(-1, 5, 3)$ Equation of plane passing through $(-1, 5, 3)$, $(0, 0, 0)$ and counting the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is $x - 4y + 7z = 0$

38. $y \geq \sqrt{|x+3|}$



$x \le 6$

Area of the trapezium ABCD

$$= \frac{1}{2}(1+2)5 = \frac{15}{2}$$

$$A_1 + A_2 = \int_{-4}^{-3} \sqrt{-3-x}\ dx + \int_{-3}^{1} \left[\frac{2(x+3)^{\frac{3}{2}}}{3}\right]_{-3}^{1}$$

$$= \frac{2}{3} + \frac{16}{3}$$

Required Area = Area ABCD – ($A_1 + A_2$)

$$= \frac{15}{2} - \left(\frac{2}{3} + \frac{16}{3}\right) = \frac{3}{2}$$

39. $b_1, b_2, b_3 \ldots\ldots b_{101}$ are G.P with C R = 2

$a_1, a_2 \ldots\ldots a_{101}$ are A.P

$a_1 = b_1 \quad a_{51} = b_{51}$

$b_1, 2b_1, 2^2 b_1 \ldots\ldots 2^{100} b_1$ G.P

$a_1 + 50d = 2^{50} b_1$

$a_1 + 50d = 2^{50} a_1$

$\therefore t = b_1(2^{51} - 1)$

$s = \frac{51}{2}(2a + 50)$

$= \frac{51}{2}\left(a + 2^{50} a_1\right)$

$\therefore\ s = \frac{51}{2} a_1 + \frac{51}{2} 2^{50} a_1$

$\therefore s > t$

also $a_{101} = 2^{51} a_1 - a_1$

$< 2^{51} a_1$

$\therefore b_{101} > a_{101}$

40. $$\frac{2}{2\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$

$$= 2\sum \frac{\sin\left(\left(\frac{k\pi}{6} + \frac{\pi}{4}\right) - \left((k-1)\frac{\pi}{6} + \frac{\pi}{4}\right)\right)}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right).\ \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$

$$= 2\sum \cot\left((k-1)\frac{\pi}{6} + \frac{\pi}{4}\right) - \cot\left(\frac{k\pi}{6} + \frac{\pi}{4}\right)$$

$$= 2\left(\cot\frac{\pi}{4} - \cot\left(\frac{13\pi}{6} + \frac{\pi}{4}\right)\right)$$

$$= 2\left(1 - \cot\frac{5\pi}{12}\right)$$

$$= 2\left(1 - 2 - \sqrt{3}\right)$$

$$= 2\sqrt{3} - 2$$

41. $$|P| = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{vmatrix} = 1$$

$P^{50} = I + R$

$$P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 16 & 1 & 0 \\ 160 & 16 & 1 \end{pmatrix}$$

16, 48, 96, 160, 240, ……

32, 48, 64, 80

16, 16, 16

$An^2 + Bn + C$

$A + B + C = 16$

$4A + 2B + C = 48$

$9A + 3B + C = 96$

$3A + B = 32$

$5A + B = 48$

$2A = 16$

$A = 8$

$B = 32 - 24 = 8$

$C = 16 - 8 - 8 = 0$

Hence, the 3rd row 1st column element of P^{50}

$= 8n^2 + 8n$ where $n = 50$

$= 8 \times 2500 + 400$

$= 20400$

3rd row 2nd column element of P^{50}

$= 4 + 49 \times 4$

$= 200$

$$\frac{q_{31} + q_{32}}{q_{21}} = \frac{20400 + 200}{4 \times 50}$$

$$= \frac{20600}{200} = 103$$

42. $$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^{-x}} dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x \; e^x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\left(1+e^x\right)x^2 \cos x}{1+e^x} dx$$

$$= 2\int_0^{\frac{\pi}{2}} x^2 \cos x \; dx$$

$$I = \left(x^2 \sin x\right)_0^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x \sin x \; dx$$

$$= \frac{\pi^2}{4} - 2\left(-x \; \cos x\right) + \int \cos x \; dx$$

$$= \frac{\pi^2}{4} - 2$$

43. f '(2) = 0 = g(2)

f " (2) $\neq$ 0

g '(2) $\neq$ 0

$$\lim_{x\to 2} \frac{f(x)\,g(x)}{f'(x)\;g'(x)} \to \left(\frac{0}{0} \;\; \text{form}\right)$$

$$= \lim_{x\to 2} \frac{f(x)\,g'(x) + f'(x)\;g(x)}{f'(x)\;g''(x) + f''(x)\;g'(x)}$$

$$= \frac{f(2)\times g'(2) + f'(2)\;g(2)}{f'(2)\;g''(2) + f''(2)\;g'(2)}$$

$$= \frac{f(2)\;g'(2)}{f'(2)\;g''(2) + f''(2)\;g'(2)} = 1$$

$$= \frac{f(2)\;g'(2)}{f'(2)\;g''(2) + g'(2)} = 1$$

$\Rightarrow$ f "(2) = f(2)

Since the range of f is (0, ∞)   2 = 4 – 8+

f(2) > 0

$\Rightarrow$ f "(2) > 0

$\Rightarrow$ f is minimum at x = 2 $\to$ (A) is true

Since f "(2) – f(2) = 0

$\Rightarrow$ (D) is true

44. $y^2 = 4x$

$P(t^2, 2t)$

Normal at P $\to$ $y + xt = 2t + t^3$

It passes through the centre S(2, 8) of the circle

$8 + 2t = 2t + t^3$

t = 2

P(4, 4)

$SP^2 = (4 - 2)^2 + (8 - 4)^2 = 4 + 16 = 2D$

SP = $2\sqrt{5}$ $\Rightarrow$ (A) is true

Q is the point (3, 6) $\Rightarrow$ (B) is false

Normal at P(4, 4)

y + 2x = 4 + 8 = 12

$\frac{x}{6} + \frac{y}{12} = 1$ $\Rightarrow$ (C) true

Circle is $(x - 2)^2 + (y - 8)^2 = 4$

$2(x - 2) + 2(y - 8)y' = 0$

$$y' = -\frac{(x-2)}{(y-8)}$$

$$= -\left(\frac{3-2}{6-8}\right) = \frac{1}{2}$$

45. $f(x) = a\cos|x^3 - x| + b|x| \sin(|x^3 + x|)$

$x^3 - x = x(x^2 - 1)$

= (x + 1) x (x –1)

$x^3 + x = (x^2 + 2)x$

$$f(x) = \begin{cases} a\cos\left(x - x^3\right) - bx\left[-\sin\left(x^3 + x\right)\right], & -\infty < x < -1 \\ a\cos\left(x^3 - x\right) - bx\left[-\sin\left(x^3 + x\right)\right] & -1 < x < 0 \\ a\cos\left(x - x^3\right) + bx\sin\left(x^3 + x\right), & 0 < x < 1 \\ a\cos\left(x^3 - x\right) + bx\sin\left(x^3 + x\right) & x > 1 \end{cases}$$

$f(0^-) = a = f(0^+)$

$f(1^-) = a + b\sin 2 = f(1^+)$

f(x) is continuous at x = 0, 1

$f'(x) = [-a\sin(x^3 - x)]\,[3x^2 - 1]$
$+b\{\sin(x^3 + x)$
$+3(x^2 + 1)x\cos(x^3 + x)\}$
$-1 < x < 0$

$= [-a\sin(x - x^3)]\,[1 - 3x^2]$
$+b\{\sin(x^3 + x) + x(3x^2 + 1)\cos(x^3 + x)\}$
$0 < x < 1$

$= [-a\sin(x^3 - x)]\;\;[3x^2 - 1]$
$+ b\{\sin(x^3 + x) + x(3x^2 + 1)\cos(x^3 + x)\}$ x > 1

$f'(0^-) = 0$

$f'(0^+) = 0$

$f'(0^+) = 0$

$f'(1^-) = b\{\sin 2 + 4\cos 2\}$

$f'(1^+) = b\{\sin 2 + 4\cos 2\}$

$\therefore$ A, B are true

46. $f(x) = [x^2 - 3]$

When $x \in \left[-\frac{1}{2}, 2\right]$

$x^2 - 3$ varies from –2.75 to 1

f(x) varies –2.75 to –2 (break point)

–2 to –1 (break point)

–1 to 0 (break point)

0 to 1 (break point)

f(x) is discontinuous at 4 points

option (B)

$g(x) = [x^2 - 3]\,\{|x| + |4x - 7|\}$

In $\left(\frac{-1}{2}, 2\right)$, $\left[x^2 - 3\right]$ discontinuous at 3 points

and |x| + |4x – 7| is discontinuous

at $x = 0, \frac{7}{4}$

But as x = 0 is already included

⇒ total 4 discontinuities

∴ (B), (C) true

47 $\log f(x) = \lim\limits_{n\to\infty} \frac{x}{n} \log\left(\frac{\pi\left(x + \frac{1}{r/n}\right)}{\pi\left(x^2 + \frac{1}{(r/n)^2}\right)\pi(r/n)}\right)$

$= x \lim\limits_{n\to\infty} \log\frac{\left(x\frac{r}{n} + 1\right)}{\left(x\frac{r}{n}\right)^2 + 1}$

$= x \int_0^1 \log\left(\frac{1+tx}{1+t^2x^2}\right)dx$

log f(x) is increasing from (0, 1) and decreasing from (1, ∞).

∴ B and C are true

48. $ax + 2y = \lambda$

$3x - 2y = \mu$

$\begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6$

$= -2(a + 3)$

(B) is true

When a = –3 $\begin{vmatrix} a & \lambda \\ 3 & \mu \end{vmatrix} = a\mu - 3\lambda$

$= -3\mu - 3\lambda$

$= -3(\lambda + \mu)$

$\begin{vmatrix} 2 & \lambda \\ -2 & \mu \end{vmatrix} = 2\mu + 2\lambda$

⇒ (C) is true

(D) is true

Section II

49. $|u|\,|v| \sin\theta = 1$

$1 \times |v| \sin\theta = 1$ ——— (1)

$w\,(u \times v) = 1$

$[w\ u\ v] = 1$

⇒ $|w|\,|v|\,|u| = 1$

$v\sin\theta = 1$

infinity many vectors $\bar{v}$

∴ u and v are $\perp^r$ and |u| = |v|-= 1

∴ (A) incorrect

∴ B is true

If $|u_1| = |u_2|$ then $|u| = 1$

$(u_1, u_2, 0)$

⇒ $\sqrt{u_1^2 + u_2^2} = 1$ only if $|u_1| = u_2|$ ∴ (C) correct

$(u_1, 0, u_3)$

⇒ $\sqrt{u_1^2 + u_3^3} \neq$ if $2|u_1| = |u_2|$

∴ D incorrect

50. $x + iy = \frac{1}{a + ibt}$

$= \frac{a - ibt}{a^2 + b^2t^2}$

⇒ $x = \frac{a}{a^2 + b^2t^2}$ and $y = \frac{-bt}{a^2 + b^2t^2}$

Consider option (A)

Given circle is

$\left(x - \frac{1}{2a}\right)^2 + y^2 = \frac{1}{4a^2}$

i.e., $a^2x^2 + a^2y^2 - x = 0$ ——— (1)

$x = \frac{a}{a^2 + b^2t^2}$ and $y = \frac{-bt}{a^2 + b^2t}$ satisfy (1)

∴ (x, y) lies on (1)

(A) correct

But x and y do not satisfy are circle given in option (B)

(B) incorrect

Now, for b = 0 and a ≠ 0,

$x + iy = \frac{1}{a}$

⇒ $x = \frac{1}{a}$ and y = 0

⇒ lies on x – axis

(C) correct

For a = 0 and b ≠ 0,

$x + iy = -\frac{i}{bt}$

⇒ x = 0 and $y = -\frac{1}{bt}$ ⇒ lies on y-axis

(D) correct

Section III

Possibilities are T_1T_1, T_1T_2, T_2T_2, T_1D, T_2D, DT_1, DT_2 with (x, y) values (6, 0) (3, 3), (3, 3), (0, 6), (4, 1), (1, 4), (4, 1), (1, 4), (D, D) (2, 2)

51. P(x > 4) = P((6, 0), (4, 1), (4, 1))

i.e. $P(T_1T_1, T_1D, DT_1)$

$= \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{2}$

$= \frac{5}{12}$

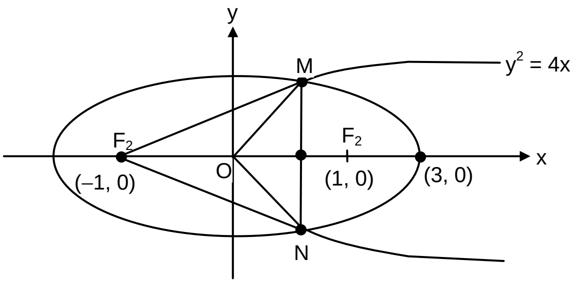
52. P(x = 4) = P((3, 3), (3, 3) (2, 2))

i.e. $P(T_1T_2, T_2T_1)$

$= \frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{6}=\frac{1}{6}+\frac{1}{6}+\frac{1}{36}$

$\frac{13}{36}$

53.



Equation of the parabola is $y^2 = 4c$

$\therefore\ \frac{x^2}{9}+\frac{y^2}{8}=1 \Rightarrow \frac{x^2}{9}+\frac{4x}{8}=1 \Rightarrow \frac{x^2}{9}+\frac{x}{2}=1$

$\Rightarrow 2x^2 + 9x - 18 = 0$

$\Rightarrow 2x^2 + 12x - 3x - 18 = 0$

$\Rightarrow 2x(x + 6) - 3(x + 6) = 0 \Rightarrow (x + 6)(2x - 3) = 0$

$\Rightarrow x = -6$ or $\frac{3}{2}$

Since $x > 0$, take $x = \frac{3}{2}$

$\Rightarrow y = +\sqrt{6}$

$\therefore$ M is $\left(\frac{3}{2}, \sqrt{6}\right)$ and N is $\left(\frac{3}{2}, -\sqrt{6}\right)$

Slope of $F_2N = \frac{-2\sqrt{6}}{5}$

$\therefore$ Slope of the altitude through M is $\frac{5}{2\sqrt{6}}$

$\therefore$ Equation of this altitude is

$y-\sqrt{6}=\frac{5}{2\sqrt{6}}\left(x-\frac{3}{2}\right)$ ——— (1)

Also x – axis (y = 0) is an altitude

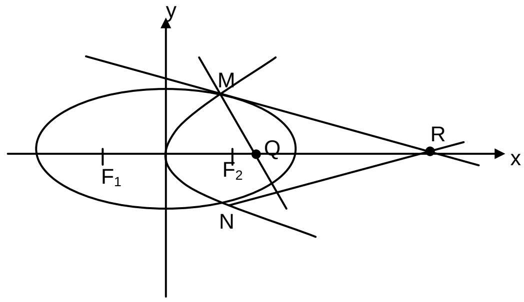
Put y = 0 in (1)

$\therefore\ -\sqrt{6}=\frac{5}{2\sqrt{6}}\left(x-\frac{3}{2}\right)$

$\Rightarrow x = \frac{-9}{10}$

$\therefore$ orthocenter is $\left(\frac{-9}{10}, 0\right)$

54.



Slope of the tangent at $M\left(\frac{3}{2}, \sqrt{6}\right)$ to the ellipse

$= \frac{dy}{dx}=\frac{-4}{3\sqrt{6}}$

$\therefore$ Equation of the tangent at M is

$y-\sqrt{6}=\frac{-4}{3\sqrt{6}}\left(x-\frac{3}{2}\right)$

Put y = 0. Then x = 6

$\therefore$ R is (6, 0)

Slope of tangent at $M\left(\frac{3}{2}, \sqrt{6}\right)$ to the parabola

$\frac{dy}{dx}=\frac{2}{\sqrt{6}}$

Equation of the normal at M is

$y-\sqrt{6}=\frac{-\sqrt{6}}{2}\left(x-\frac{3}{2}\right)$

Put y = 0. Then $x = \frac{7}{2}$

$\therefore$ Q is $\left(\frac{7}{2}, 0\right)$

Area of $\Delta MQR = \frac{5\sqrt{6}}{4}$

Area of quadrilateral MF_1NF_2

$= 2 \times$ Area of ΔF_1MF_2

$= 2\sqrt{6}$

$\therefore$ Ratio is $\frac{5\sqrt{6}}{4} : 2\sqrt{6} = 5 : 8$

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