Types of Differential Equations

Familiarity with various methods used in evaluating indefinite integrals or finding anti- derivatives of functions [or, in other words, evaluating $\int f(x)\,dx$] is a pre-requisite.

Differential Equations

An equation involving derivatives of a dependent variable with respect to one or more independent variables is called a differential equation. The equation may also contain the variables and / or their functions and constants. If there is only one independent variable, the corresponding equation is called an ordinary differential equation. If the number of independent variables is more than one, the corresponding equation is called a partial differential equation.

Examples:

1. $\frac{dy}{dx} = x^4 + e^{-x} + y$

2. $x^2\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 3y^4x = \sin x + 6$

3. $\frac{dy}{dx} + 5y = x^3 - \tan x$

4. $\frac{d^2y}{dx^2} + 4y = 0$

5. $\left(\frac{d^3y}{dx^3}\right)^2 + 5\left(\frac{dy}{dx}\right)^4 + e^{2xy} = 6$

6. $\frac{d^3y}{dx^3}+8\frac{d^2y}{dx^2}+\frac{3dy}{dx}+9y=16x^2$

7. $x\ \frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=8u$

8. $\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}=10$

9. $\frac{\partial^2 u}{\partial y^2}=25\frac{\partial^2 u}{\partial x^2}$

10. $\frac{\partial^4 u}{\partial x^4}+6\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial y^4}=e^{3xy}$

We note that in the examples above, equations (1) to (6) are ordinary differential equations while equations (7) to (10) are partial differential equations. We refer to these examples later on in this chapter.