T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2014

Paper II – Code 0

PART I

1	2	3	4	5	6	7	8	9	10
B	**A**	**A**	**C**	**D**	**B**	**D**	**C**	**C**	**B**

11	12	13	14	15	16
C	**B**	**D**	**D**	**C**	**A**

17	18	19	20
C	**A**	**B**	**D**

Section I

1. $Rhc(Z-1)^2 \times \frac{3}{4} \propto \frac{1}{\lambda_{K_\alpha}}$

$\frac{\lambda_{Cu}}{\lambda_{M_0}} = \frac{(Z_{M_0}-1)^2}{(Z_{Cu}-1)^2} = \frac{41^2}{28^2}$

$= 2.14$

2. $\frac{hc}{\lambda} = W + \frac{1}{2}mu^2 = W + KE$

$\frac{hc}{\lambda_1} = \frac{1240}{248} = 5$

$\frac{hc}{\lambda_2} = \frac{1240}{310} = 4$

$5 = W + KE_1 = W + 4KE_2$

$4 = W + KE_2 = W + KE_2$

$(KE_2 = 4KE_1)$

$W = 4 - \frac{1}{3} = \frac{11}{3} = 3.7\ eV$

3. $\sigma A(T^4 - 300^4) = \pi R^2 \times 912$

$5.7 \times 10^{-8} \times 4\pi R^2(T^4 - 300^4) = 912\pi R^2$

$T^4 - 300^4 = \frac{912}{5.7 \times 4 \times 10^{-8}} = 40 \times 10^8$

$T^4 = 300^4 + 40 \times 10^8$

$= 1.21 \times 10^{10} = 121 \times 10^8$

$T \cong 330\ K$

4. $\frac{R}{90} = \frac{40}{60} \Rightarrow R = 60\ \Omega$

$40\ cm \equiv 60\ \Omega$

$400\ mm \equiv 60$

$dr = R\left[\frac{dx}{x} + \frac{dx}{1-x}\right]$

$\rightarrow 60 \times \left[\frac{1mm}{400mm} + \frac{1mm}{600\,mm}\right] = 0.25$

5. Initially $mg > \frac{mv^2}{r}$

$\frac{mv^2}{r}$ is centrifugal force acting radially outward

6. $g_e = \frac{GM_e}{R_e^2} = \frac{G.\frac{4}{3}\pi R_e^2}{R_e^2} = 10$

$g_P = G\frac{4}{3}\pi(0.1R_e) = 1$

$g_x = g\left(1 - \frac{x}{R}\right)$

$d(mg') = \rho\Delta x g\left(1 - \frac{x}{R}\right)$

$W = \int_0^{R/5} \rho g\left(\Delta x - \frac{1}{R}x.\Delta x\right)$

$= \rho g\left(\frac{R}{5} - \frac{R}{50}\right) = \rho g.\frac{9R}{50}$

$= 10^{-3} \times 1 \times \frac{9}{5} \times \frac{1}{10}(0.6 \times 10^6)$

$= 108\ N$

7. Angle between ST and the vertical is $\left(\theta + \frac{\alpha}{2}\right)$

$\pi b^2.h\rho g = 2\pi b s\cos\left(\theta + \frac{\alpha}{2}\right)$

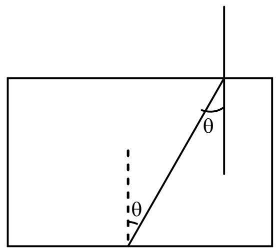
$h = \frac{2s}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$

8. $E_1 = \frac{KQ}{R^2}$; $E_2 = \frac{KQ.2}{R^2} = 2E_1$

$E_3 = \frac{K.4Q}{4R^2} \times \frac{1}{2} = \frac{KQ}{2R^2} = \frac{E_1}{2}$

$E_2 > E_1 > E_3$

9.



$\sin\theta = \frac{1}{{}_L\mu_B} = \frac{\mu_L}{\mu_B} = \frac{\mu_L}{2.72}$

$\tan\theta = \frac{5.77}{10}$

$\theta = 30°$

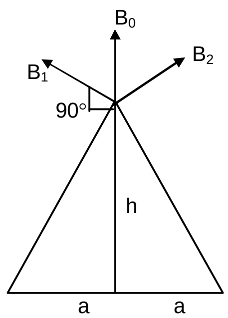
$\sin 30 = \frac{\mu_L}{2.72}$

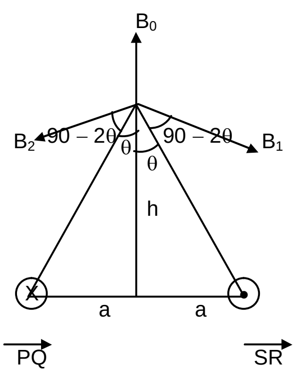
$\mu_L = 2.7L \times \frac{1}{2} = 1.36$

10. B

Section II

11.





$2B_1\cos(90 - \theta) = B_0 = 2B_1\sin\theta$

$B_{coil} = \frac{\mu_0\left(I\left(\pi r^2\right) \times 2\right)}{4\pi\left(r^2 + x^2\right)^{3/2}} = \frac{\mu_0 I r^2}{2\left(r^2 + x^2\right)^{3/2}}$

$B_0 = \frac{\mu_0 I a^2}{2\left(a^2 + h^2\right)^{3/2}}$

$= 2 \times \frac{\mu_0 I}{2\pi\left(a^2 + h^2\right)^{1/2}} \frac{a}{\left(a^2 + h^2\right)^{1/2}}$

$\frac{a}{\left(a^2 + h^2\right)} = \frac{2}{\pi}$

$\frac{a^2}{a^2 + h^2} = \frac{4}{\pi^2} = 0.4$

$a^2 = 0.4a^2 + 0.4h^2$

$h^2 = \frac{0.6a^2}{0.4} = 1.5a^2$

$h = \sqrt{1.5}a$

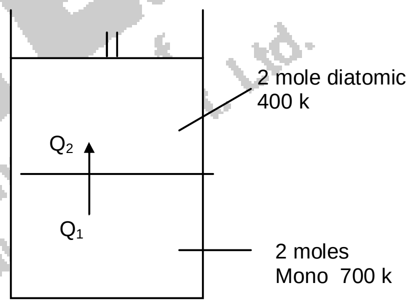
$\cong 1.2a$

12. $2 \times \frac{\mu_0 I}{2\pi d}(\sin 30°) I\pi a^2$

$2 \times \frac{\mu_0 I}{2\pi d} . \frac{1}{2}\pi a^2$

$= \frac{\mu_0 I^2 a^2}{2d}$

13.



$Q_1 = Q_2$

$mC_V\Delta T + 0$ work $= nC_v\,\Delta T + P\Delta v$

$= nC_p\Delta T$

$2 \times \frac{3R}{2} \times (700 - T) = 2 \times \frac{7R}{2} \times (T - 400)$

$2100 - 3T = 7T - 2800$

$4900 = 10\ T$

$T = 490$

14. When particle moves, both process become isobaric.

$Q_1 = Q_2$

$(nC_p\Delta T)_1 = (nC_p\Delta T)_2$

$2 \times \frac{5R}{2}(700 - T) = 2 \times \frac{7R}{2}(T - 400)$

$3550 - 5T = 7T - 2800$

$6300 = 13T \quad T = \frac{6300}{12}$

Taking both gases as one system

$Q = \Delta U + W = 0$

$W = -\Delta U$

$= n_1 C_{V_1} \Delta T + n_2 C_{V_2} \Delta T$

$= -\left[2 \times \frac{5R}{2}\left(\frac{6300}{12} - 400\right) + 2\frac{3R}{2}\right]\left[\frac{6300}{12} - 700\right]$

$= -100R$

15. av = constant
$r^2 v$ = constant
$r_1^2 v_1 = r_2^2 v_2$
$20^2 \times 5 = 1^2 v_2$
$v_2 = 2000$ mm s^{-1}
$= 2$ m s^{-1}

16. $w_{piston} = p_0 + \frac{1}{2}\rho_\ell v^2$

$\Rightarrow v \propto \sqrt{\frac{\rho_{air}}{\rho_\ell}}$

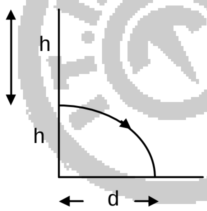
Volume = v × area of nozz;e

$\propto \sqrt{\frac{\rho_{air}}{\rho_\ell}}$

Section III

17. $R = \sqrt{2gh}\sqrt{\frac{2h'}{g}}$

$= 2\sqrt{hh'}$



$P \rightarrow 1$
$Q \rightarrow$
$R \rightarrow 1$

$S \rightarrow 4$

18. $P \rightarrow 3$
$Q \rightarrow 1$
$R \rightarrow 4$
$S \rightarrow 2$

19. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

20. For S the only match can be option C because normal component of the mass would be equal to ($m_1 + m_2 \cos\theta$). Hence friction need to be $\mu \times (m_1 + m_2)\cos\theta$.

PART II

21	22	23	24	25	26	27	28	29	30
D	**C**	**B**	**B**	**C**	**D**	**A**	**A**	**C**	**B**

31	32	33	34	35	36
C	**D**	**A**	**C**	**B**	**D**

37	38	39	40
B	**C**	**A**	**C**

Section I

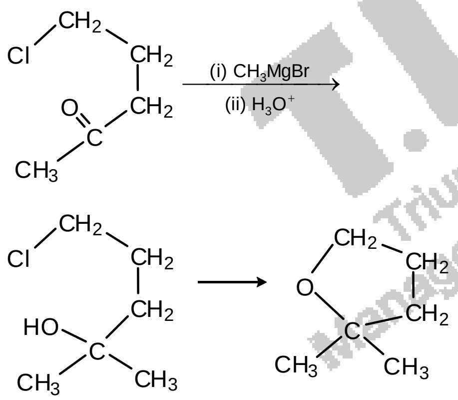
21. Coupling reaction of phenol is carried out in alkaline medium

22. $C_2 - \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2,$
$\sigma 2p_z^2, \pi 2p_x^1, \pi 2p_y^1$

23. Rate $\alpha\ [M^2]^3$

24. Boiling point decreases with increase in branching

25. Presence of para-methoxy phenyl group will stabilise the carbocation intermediate involved in the reaction

26. 

27. $3H_2O_2 \rightarrow 3H_2O + 3(O)$
$2NH_2OH + 3(O) \rightarrow 2NO + 3H_2O$
$\therefore 3H_2O_2 + 2NH_2OH \rightarrow 2NO + 6H_2O$
$KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O_2 + O_2$

28. $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

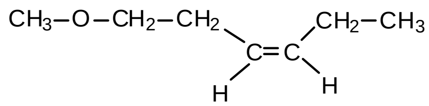
29. $XeF_6 \xrightarrow[\text{hydrolysis}]{\text{complete}} \underset{(P)}{XeO_3} + 6HF \xrightarrow{OH^- / H_2O}$
$HXeO_4^- \xrightarrow{OH^- / H_2O} XeO_6^{4-} + Xe + O_2 + 2H_2O$

30. The process is at equilibrium.
$\therefore \Delta S_{system} = \dfrac{\Delta H}{T}$
i.e., $\Delta S_{system} > 0$ and $\Delta S_{surroundings} < 0$

Section II

31. $\dfrac{d}{24 - d} = \sqrt{\dfrac{M_y}{M_x}}$
$\dfrac{d}{24 - d} = \sqrt{\dfrac{40}{10}}$
d = 16 cm

32. Increased collision frequency of X with the inert gas compared to that of Y decreases the speed of X

33. $HOCH_2–CH_2–C\equiv CH \xrightarrow{NaNH_2}$
$NaOCH_2–CH_2–C\equiv CNa \xrightarrow{CH_3CH_2I}$
$NaO–CH_2–CH_2–C\equiv C–CH_2–CH_3 \xrightarrow{CH_3I}$
$CH_3–O–CH_2–CH_2–C\equiv C–CH_2–CH_3 \xrightarrow[\text{Lindlar's catalyst}]{H_2}$


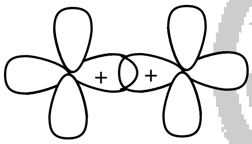
34. Y contains $CH_3–\overset{O}{\overset{\|}{C}}–$ group and so gives a positive iodoform test

35. $\underset{(M_1)}{Ni^{2+}} \xrightarrow[(Q)]{HCl} \underset{\text{Tetrahedral}}{[NiCl_4]^{2-}}$
$Ni^{2+} \xrightarrow[(R)]{KCN} \underset{\text{Square planar}}{[Ni(CN)_4]^{2-}}$

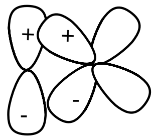
36. $\underset{(M_2)}{Zn^{2+}} \xrightarrow[(S)]{KOH} \underset{\text{(White ppt)}}{Zn(OH)_2} \xrightarrow[(S)]{\text{excess KOH}} \underset{\text{(Soluble)}}{K_2[Zn(OH)_4]}$

Section III

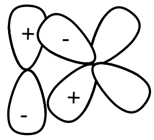
37. $[Cr(NH_3)_4Cl_2]Cl$ – Cr^{3+} – d^3 . Hence paramagnetic
It exhibits cis trans isomerism
$[Ti(H_2O)_5Cl](NO_3)_2$ – It exhibits ionisation isomerism
$Ti^{3+} \rightarrow d^1$ Hence paramagnetic
$[Pt(en)(NH_3)Cl]NO_3$
It exhibits ionisation isomerism
$[Pt^{+2} \rightarrow d^8$ – pairing takes place.
Hence diamagnetic
$[Co(NH_3)_4(NO_3)_2]NO_3$
It exhibits cis-trans isomerism
$Co^{3+} \rightarrow d^6 \rightarrow$ pairing takes place.
Hence diamagnetic

38. 

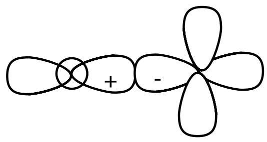
d-d σ bonding



p-d π bonding



p-d π antibonding



d-d σ antibonding

39. (a)

40. (c)

PART III

41	42	43	44	45	46	47	48	49	50
C	**B**	**D**	**A**	**D**	**D**	**A**	**C**	**B**	**B**
		51	52	53	54	55	56		
		B	**C**	**D**	**B**	**A**	**D**		
			57	58	59	60			
			D	**A**	**D**	**C**			

Section I

41. Cards 2 3 4 5 6
Envelops 1 3 4 5 6
If we assume C_1 is corresponding to C_1
No: of rearrangements

$$5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$= 44$

No of rearrangements = 9
Total = 44 +9 = 53

42. $(a + b)x = x^2$
$ab = y$
$x + c = a + b + c = 2s$ ——— (1)
$x^2 - y = c^2$
$y = x^2 - c^2$
$ab = (a + b)^2 - c^2$
$= a^2 + b^2 + 2ab - c^2$
$c^2 = a^2 + b^2 - ab$
$= a^2 + b^2 - 2ab\cos 60°$
Therefore, $\angle c = 60°$

$$\Delta = \frac{1}{2}ab\sin c$$

$$= \frac{1}{2}ab\sin 60°$$

$$= \frac{1}{2}ab\frac{\sqrt{3}}{2}$$

$$\Delta^2 = \frac{3a^2b^2}{16}$$

$$\frac{r}{R} = \frac{\Delta}{sR} = \frac{\Delta \times 4\Delta}{s\ abc}$$

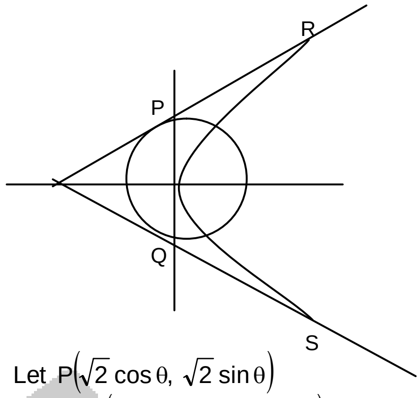
$$= \frac{4\Delta^2}{s\ abc}$$

$$= \frac{4 \times 3a^2b^2}{\left[\frac{16(x+c)}{2}\right]abc}$$

$$= \frac{12ab}{8c\,(x+c)}$$

$$= \frac{3y}{2c(x+c)}$$

43.



Let $P\left(\sqrt{2}\cos\theta,\ \sqrt{2}\sin\theta\right)$
and Q $P\left(\sqrt{2}\cos\theta,\ -\sqrt{2}\sin\theta\right)$
Tangent to the circle at P is

$$x\sqrt{2}\ \mathrm{cis}\theta + y\sqrt{2}\sin\theta = 2$$

$$x\cos\theta + y\sin\theta = \sqrt{2}$$

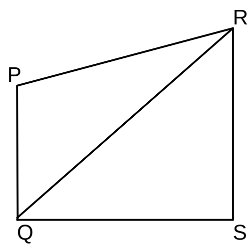
$$y = -x\cot\theta + \frac{\sqrt{2}}{\sin\theta} \text{ ——— (1)}$$

(1) is a tangent to $y^2 = 8x$

$$\Rightarrow \frac{\sqrt{2}}{\sin\theta} = \frac{-2}{\cos\theta}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$$

Points P and Q are (–1, 1) and (–1, 1) Respective.
A point on $y^2 = 8x$ is
$(2t^2, 4t)$
Here, $t = \tan 135° = -1$
Points R and S are (2, 4) and (2, –4)



Area of PQRS
= Area of ΔPQR + Area of ΔQRS
= 3 +12 = 15

44. Total no of ways we can arrange 2G and 3B is 5! = 120
We need to enumerate for each position. Possible positions are

B	B	B	B	B
G	B	B	G	B

B	G	B	B	G
G	B	G	G	B
B	G	G	B	G

In each case Boys and Girls may be permented amongst them $\Rightarrow 2! \times 3! = 12$

$\therefore$ Total no. of ways = $5 \times 12 = 60$

Probability = $\frac{60}{120} = \frac{1}{2}$

45. Let $p(x) = x^2 + 1 = 0$

$p(p(x)) = 0$

gives $(x^2 + 1)^2 + 1 = 0$

$x^4 + 2x^2 + 2 = 0$

$(x^2)^2 + 2(x^2) + 2 = 0$

Neither real nor complex

46. $\sin x + 2\sin 2x - \sin 3x = 3$

$\sin x + 2 \times 2\sin x\cos x - (3\sin x - 4\sin^3 x) = 3$

$-2\sin x + 4\sin x \cos x + 4\sin^3 x = 3$

$-2\cos 2x \sin x + 2\sin x 2x = 3$

$(2\sin x)\,[2\cos x - \cos 2x] = 3$

$(\sin x)(\cos x - \cos 2x) = 3$

max. $(\sin x)\ (0, \pi) = 1$

$\max(\cos x - \cos 2x)$ in $(0, \pi) = 2$

Their product cannot be 3. Therefore, no solution

47. $du = \frac{1}{\cot\frac{x}{2}} \times -\operatorname{cosec}^2\frac{x}{2} \cdot \frac{1}{2} . dx$

$= \frac{-1}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$

$= -\operatorname{cosec} x\, dx$

$du = \frac{-\left(e^4 + e^{-4}\right)}{2} dx$

$\frac{-2du}{e^4 + e^{-4}} = dx$

Put $u = \log\cot\left(\frac{x}{2}\right)$

$e^u = \cot\left(\frac{x}{2}\right)$

$e^{-u} = \tan\left(\frac{x}{2}\right)$

$\frac{e^4 + e^{-4}}{2} = \frac{\cot\left(\frac{x}{2}\right) + \tan\frac{x}{2}}{2}$

$= \frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{1}{\sin x}$

$= \operatorname{cosec} x$

$e^4 + e^{-4} = 2\operatorname{cosec} x$

$(e^4 + e^{-4})^{17} = (2\operatorname{cosec} x)^{174}$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\operatorname{cosec} x)^{17} dx = -\int_{\log(1+\sqrt{2})}^{0} \left(e^4 + e^{-4}\right)^{17} \frac{+2du}{\left(e^4 + e^{-4}\right)}$

$= \int_0^{\log(1+\sqrt{2})} 2\left(e^4 + e^{-4}\right)^{16} du$

48.

0	2	4	6	8						
0	3	6	9	12	15	18	21			
0	4	8	12	16	20	24	28	32	36	40
								44	48	52

= coefficient of x^{11} in $(1+4x^2 + 6x^4 + 4x^6 + x^8)$

$\times\left(1 + 7x^3 + \frac{7.6}{2}x^6 + \frac{7.65}{1.2.3}x^9\right)$

$\times\left(1 + 12x^4 + \frac{12.11}{2}x^8\right)$

$= 0 + 4 \times 7 \times 5 + 6 \times 7 \times 12 + 1 \times 7 + 7 \times 66$

$= 140 + 504 + 7 + 462$

$= 1113$

49. $f(0) = 1$ $F(x)2 \int_0^{x^2} f\left(\sqrt{t}\right)dt$

$F'(x) = f(x) \cdot 2x$

$\frac{dy}{dx} = y\ 2x = 2xy$

$\frac{dy}{y} = 2x dx$ $\qquad f(0) = 1$

$\log f(x) = x^2 + c$

$f(x) = e^{x^2}.c \Rightarrow c = 1$

$f(x) = e^{x^2}$

$F(x) = \int_0^{x^2} e^t dt$

$F(x) = \int_0^4 e^t dt = e^4 - 1$

50. $\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{x^4 + 2x}{\sqrt{1-x^2}}$

I. F $e^{\int \frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

Solution is

$y\sqrt{1-x^2} = \int \frac{x^4 + 2x}{\sqrt{1-x^2}}\sqrt{1-x^2}\,dx$

$\Rightarrow y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + c$

Given $f(0) = 0 \Rightarrow c = 0$

$y\sqrt{1-x^2} = \frac{x^5}{5} + x^2$

$\Rightarrow$ f(x) = y= $\dfrac{x^5}{5\sqrt{1-x^2}} + \dfrac{x^2}{\sqrt{1-x^2}}$

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$= 2\int_0^{\sqrt{3}/2} \frac{x^2dx}{\sqrt{1-x^2}}$$

$$2\int_0^{\pi/2} \sin^2\theta d\theta = \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)d\theta$$

$$\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{1}{2}.2.\frac{\sqrt{3}}{2}.\frac{1}{2}$$

$$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Section II

51.. P($x_1 + x_2 + x_3$ is odd)

For $\sum x_i$ to be odd $\Rightarrow$ 2 cases

Case 1: Only one x_1 odd, rest even

Probability = $\left(\frac{2}{3}\times\frac{2}{5}\times\frac{3}{7}\right)+\left(\frac{1}{3}\times\frac{3}{5}\times\frac{3}{7}\right)+\left(\frac{1}{3}\times\frac{2}{5}\times\frac{4}{7}\right)$

$= \frac{12+9+8}{105} = \frac{29}{105}$

Case 2: All three x_i are odd.

Probability = $\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7} = \frac{24}{105}$

Required probability $\frac{53}{105}$

52. P(x_1, x_2, x_3 are in AP)

Let d = 0 $\Rightarrow$ 3 options $\Rightarrow$ P = $3\left(\frac{1}{3}\times\frac{1}{5}\times\frac{1}{7}\right) = \frac{3}{105}$

Let d = 1 $\Rightarrow$ 3 options $\Rightarrow$ P = $3\left(\frac{1}{3}\times\frac{1}{5}\times\frac{1}{7}\right) = \frac{3}{105}$

Let d = 2 $\Rightarrow$ 3 options $\Rightarrow$ P = $\frac{3}{105}$

Let d = 3 $\Rightarrow$ 1 option $\Rightarrow$ P = $\frac{1}{105}$

Let d = –1 $\Rightarrow$ 1 option $\Rightarrow$ P = $\frac{1}{105}$

$\therefore$ Required Probability = $\frac{3+3+3+1+1}{105}$

$= \frac{11}{105}$

53. Since PQ is focal chord and P(at^2¸2at) $\Rightarrow$ Q is $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

R(ar^2, 2ar) and k(2a, 0)

QR || PK

$$\Rightarrow \frac{2ar + \frac{2a}{t}}{ar^2 - \frac{a}{t^2}} = \frac{-2at}{2a - at^2}$$

$$\Rightarrow \frac{r + \frac{1}{t}}{r^2 - \frac{1}{t^2}} = \frac{-t}{2 - t^2}$$

$$\left(r + \frac{1}{t}\right)\left(2 - t^2\right) = -t\left(r^2 - \frac{1}{t^2}\right)$$

$$r \neq \frac{-1}{t} \Rightarrow 2 - t^2 = -t\left(r - \frac{1}{t}\right)$$

$$\frac{t^2 - 2}{t} = r - \frac{1}{t}$$

$$\Rightarrow r = \frac{t^2 - 1}{t}$$

54. st = 1 $\Rightarrow$ s = $\frac{1}{t}$

yt = x + at^2 ——(1) sx + y = 2as + as^3 ——(1)

8tx + yt = 2ast + as^3t

x + yt = 2a+ as^2 ——(3)

2yt = 2a + a(t^2 + s^2)

= 2a + a $\left(t + \frac{1}{t^2}\right)$

$= \frac{a\left[2t^2 + t^4 + 1\right]}{t^2}$

$2yt^3$ = a $(t^2 + 1)^2$

y = $\frac{a\left(t^2+1\right)^2}{2t^3}$

55. g(a) = $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$

$g\left(\frac{1}{2}\right) = \lim_{h\to 0} \int_h^{1-h} t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}dt$

t = $\sin^2\theta$

At = 2sinθ cosθ dθ

Integrant = $\frac{1}{\sin\theta}\frac{1}{\cos\theta}\times 2\sin\theta\cos\theta d\theta$

= 2dθ

$\int_{\sin^{-1}(\sqrt{h})}^{\sin^{-1}\sqrt{1-h}} 2d\theta$

$= 2\left(\sin^{-1}\sqrt{1-h} - \sin^{-1}\left(\sqrt{h}\right)\right)$

$\lim_{h\to 0} \int_h^{1-h} = 2\left(\frac{\pi}{2} - 0\right) = \pi$

56. g(a) = $\lim_{h\to 0^+} \int_0^1 t^{-a}(1-e)^{a-1}dt$

$= g'\left(\frac{1}{2}\right) = \int_0^1 \left(\frac{1-t}{t}\right)^{\frac{1}{2}}\left(\frac{1}{1-t}\right)\log\left(\frac{1-t}{t}\right)dt$

$$\int_0^1 \frac{1}{t\sqrt{(1-t)}} \log\left(\frac{1-t}{t}\right) dt$$

$t = \sin^2\theta$

$$g'\left(\frac{1}{2}\right) = 2\int_0^{\frac{\pi}{2}} \log \cot\theta d\theta = 0$$

Clearly $g'\left(\frac{1}{2}\right) = 0$

Section III

57. (P) $f(x) = ax^2 + bx + c$

$f(0) = 0 \quad c = 0$
$f(x) = ax^2 + bx$

$$\int_0^1 ax^2 + bx = 1 \Rightarrow \frac{a}{3} + \frac{b}{2} = 1$$

$\Rightarrow 2a + 3b = 6$
$\Rightarrow a = 3 \; b = 0$ or
$b = 2 \; a = 0$
$\therefore$ two polynomial
$\therefore$ (P) $\to$ (2)

(Q) $f(x) = \sin(x^2) + \cos(x^2)$
$f'(x) = 2x (\tan(x^2) - \sin(x^2)) = 0$
$\Rightarrow \tan(x^2) = 1$

$\Rightarrow x^2 = \frac{\pi}{4} \; \therefore x = \pm\sqrt{\frac{\pi}{4}}, \pm\frac{\sqrt{3\pi}}{4}$

$\therefore$ 4 solution $\therefore$ Q $\to$ (3)

(R) $\int_{-2}^{2} \frac{3x^2}{1+e^x} dx \quad f(x) = \frac{3x^2}{1+e^x}$

$$\int_0^2 3x^2 dx = 3\left(\frac{x^3}{3}\right)_0^2 = 8$$

(S) $\int$ Now is an odd function

$$\therefore \int_{-a}^{a} f(x) dx = 0$$

S $\to$ (4)

58. (P) $y = \cos(3\cos^{7}x)$
$\cos^{-1}y = 3\cos^{-1}x$

$$\frac{1}{\sqrt{1-y^2}} y^1 = \frac{3}{\sqrt{1-x^2}}$$

$(1 - x^2)y_1^2 = 9(1 - y^2)$
$(1 - x^2)\, 2y_1y_2 + y_1^2(-2x) = 9 \; -2y^4,$
$(x^2 - 1)\, y_2 + xy_1 = 9y$

$$\frac{1}{y}\left(\left(x^2 - 1\right)y_2 + xy_1\right) = 9$$

(P) $\to$ (9) $\to$ (4)

(Q) $|\bar{a}_i| = a \quad \forall i \;\; \theta = \frac{2\pi}{x}$

$$\sum_{k=1}^{n-1} |\bar{a}_k \times \bar{a}_{k+}| = \sum_{k=1}^{1} |\bar{a}_k . \; \bar{a}_k|$$

$\Rightarrow (x - t) \; |\bar{a}_i|^2 \sin\theta = x|\bar{a}_1|^2 . \cos\theta$

$\Rightarrow \theta = \frac{\pi}{4} = \frac{2\pi}{x} \Rightarrow n = 8$

(Q) $\to$ 8 = (3)

(R) Equation of normal the (4, 1)
Which is perpendicular to
x + y = 8 is x – y = h – 1 ——(1)
Equation of normal at $\left(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta\right)$

i.e. $\frac{\sqrt{6}\,x}{\cos\theta} - \frac{\sqrt{3}\,y}{\sin\theta} = 3$

i.e. $\sqrt{6}\sin\theta, \sqrt{3}\cos\theta y = 3\sin\theta\cos\theta$ ——(2)

$$\frac{\sqrt{6}\sin\theta}{1} = \frac{\sqrt{3}\cos\theta}{1} = \frac{3\sin\theta\cos\theta}{x-1}$$

$\cos\theta = \frac{(x-1)\sqrt{6}}{3}$

$\sin\theta = \frac{(x-1)\sqrt{3}}{3}$

$\therefore 1 = (n - 1)^2 \left(\frac{6}{9} + \frac{3}{9}\right)$

$1 = (n - 1)^2 \Rightarrow n - 1 = \pm 1$
$\therefore n = 0$ or 2
Thus (R) $\to$ 2 $\to$ (2)

(S) $\tan^{-1}\frac{\left(\frac{1}{2x+1} + \frac{1}{4x+1}\right)}{1 - \frac{1}{(2x+1)(4x+1)}} = \tan^{-1}\frac{2}{x^2}$

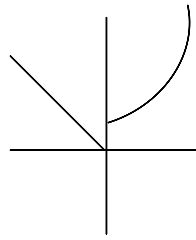
$$\frac{4x+1+2x+1}{8x^2+6x} = \frac{2}{x^2}$$

$$\frac{6x+2}{2x(4x+2)} = \frac{2}{x}$$

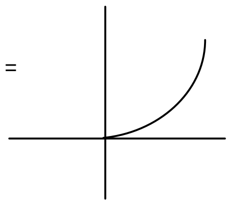
$6x^2 + 2x = 16x + 12$
$6x^2 - 14x - 12 = 0$
$3x^2 - 7x - 6 = 0$
$49 - 4. 3. - 6 > 0$
$\Rightarrow$ 2 solutions. Then only one is positive
$\therefore$ (S) $\to$ 1 $\to$ (2)

59.

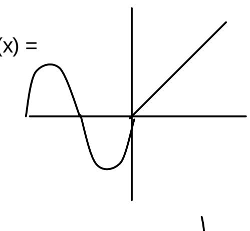
$f_1(x) =$



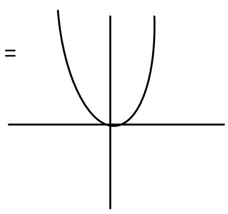
$f_2(x) =$



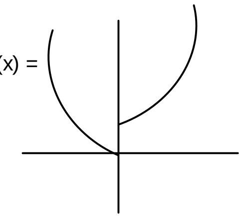
$f_3(x) =$



$f_4(x) =$



$f_2 0 \, f_1(x) =$



From the graphs we can see that f_2 is continuous and one- one

$\therefore S \rightarrow (4)$

F_4 is onto but not one-one

$\therefore P \rightarrow (1)$

f_2 0 f_1 is neither continuous nor one- one

$\therefore R \rightarrow (2)$

$\lim\limits_{x\to 0} \text{sum} = 0 = \lim\limits_{x\to 0^+} x = 0$

$\Rightarrow$ f_3 is differentiable. From the graph f_3 is continuous

$\therefore$ Option (D)

60. $z_k = \cos\frac{2kT}{10} + \text{is n}\left(\frac{2kT}{10}\right)$

$= e^{i\frac{2k\pi}{10}}$

(P) $\therefore$ $z = e^{i\frac{2\pi}{10}}, e^{i\frac{4\pi}{10}}..........e^{i\frac{18\pi}{10}}$

Clearly $z_k . z_j = 1$ for somke k and j

(P) $\rightarrow$ 1

(Q) $z_i . z_j = z_k$

$\Rightarrow e^{i\frac{2\pi}{10}}, z = e^{i\frac{\alpha\pi}{10}}$

$\Rightarrow z = e^{i\frac{2\pi}{10}(k-1)}$

has set for set of complex no.

$\therefore Q \rightarrow (2)$

(R) $(z^{10} - 1) = (z - 1)(1 + z^2 + ... + z^9)$

$(z - z_1)(z - z_2).....(z - z_9) = 1 + z + z^2 + ...+z^9$

Let z = 1

$(1 - z_1)(1 - z_2)........(1 - z_9) = 10$

$\therefore \frac{(1-z_1)(1-z_2).....(1-z_9)}{10} = 1$

(R) $\rightarrow$ (3)

(S) $\sum\limits_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right) = -1$ if k = ϕ, ------p

$\therefore 1 - \sum\cos\left(\frac{2k\pi}{10}\right) = 2$

(S) $\rightarrow$(4)