T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2013

Paper I – Code 0

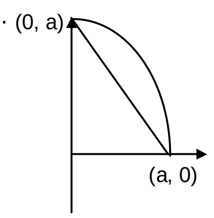
PART I

1	2	3	4	5	6	7	8	9	10
D	**A**	**D**	**A**	**B**	**B**	**C**	**C**	**A**	**B**

11	12	13	14	15
B, D	**A, C**	**B, C**	**A, D**	**B, D**

16	17	18	19	20
5	**5**	**1**	**4**	**8**

Section I

1. 

$x = a\cos\theta$
$y = a\sin\theta$
$dx = -a\sin\theta\, d\theta$
$dy = a\cos\theta\, d\theta$

$$\int_0^{\pi/2} \frac{a\cos\theta(-a)\sin\theta d\theta}{a^3} + \frac{a\sin\theta a\cos\theta d\theta}{a^3} = 0$$

2. $\frac{3\ell}{2kA} = 9$ -----(1)

$\frac{\ell}{3kA} = t$ -----(2)

$t = 2$ s

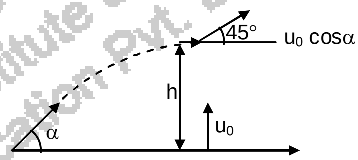
3. $\left(\frac{4}{7}p\right)V = n_1RT$

$\left(\frac{3}{7}p\right)V = n_2RT$, $\frac{n_1}{n_2} = \frac{4}{3}$

$n_1 = \frac{M_1}{2mN_A}$, $n_2 = \frac{M_2}{3mN_A}$

$\frac{n_1}{n_2} = \frac{M_1}{M_2}\frac{3}{2}$,

$\frac{M_1}{M_2} = \frac{8}{9}$, $d_1 : d_2 = \frac{M_1}{V} : \frac{M_2}{V} = \frac{M_1}{M_2} = \frac{8}{9}$

4. 

Energy conservation

(i) $\frac{1}{2}mu_0^2 - mgh = \frac{1}{2}mu_0^2\cos^2\alpha$

$\therefore \frac{1}{2}mu_0^2 - mgh = \frac{1}{2}mu_0^2\cos^2\alpha$

Vertical velocity of second particle is also $u_0\cos\alpha$

$\tan\theta = \frac{u_0\cos\alpha}{u_0\cos\alpha} \Rightarrow \theta = 45°$

5. $pt = \frac{nc}{\lambda}$

Momentum, $p = \frac{h}{\lambda} = \frac{pt}{c}$

$= \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^8}$

$= 1\times10^{-17}$ kg m/s

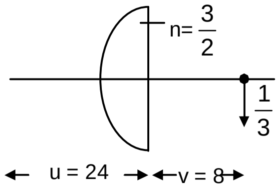
6. $I = I_0 \cos^2 \frac{\phi}{2}$

$\cos\frac{\phi}{2} = \frac{1}{\sqrt{2}} \Rightarrow \phi = \frac{\pi}{2}$

S = odd multiple of $\frac{\pi}{4}$

$= (2n+1)\frac{\lambda}{4}$

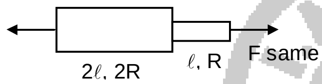
7.



$\frac{1}{f} = \frac{1}{8} - \frac{1}{-24} = \frac{0.5}{R}$

R = 3 m

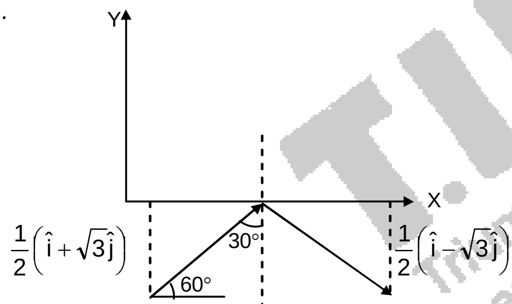
8.



$\Delta\ell = \frac{F\ell}{AY}$, F same

$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{\ell}{\pi R^2}\frac{\pi 4R^2}{2\ell} = 2$

9.



Ans. (A)

10. Each MSD = 0.05 cm

50 VSD = 2.45 cm = 49 MSD

$\therefore$ Least count = $\frac{1}{50}\text{MSD} = \frac{1}{50} \times 0.05$

= 0.001 cm

Reading = 5.10 + (24 × 0.001)

= 5.124 cm

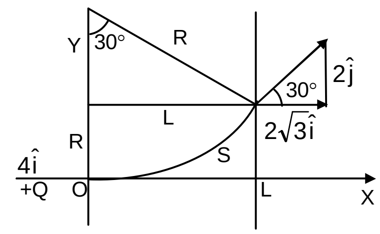
Section II

11. S_1 closed, $Q_1 = 2CV_0 \pm$

S_1 open, S_2 closed, $Q_1 = CV_0 \pm$, $Q_2 = CV_0 \pm$

S_2 open, S_3 closed, $Q_2 = CV_0 \mp$

12.



$S = 4 \times 10^{-2} = R\frac{\pi}{6}$

$R = \frac{24 \times 10^{-2}}{\pi}$, L = R sin30°, R = 2L

$R = \frac{4M}{QB}$, $B = \frac{4M}{QR} = \frac{4M\pi}{Q.24 \times 10^{-2}} = \frac{50\pi M}{3Q}$

13. Number of nodes = 6 A is wrong

4 = 2A sinkx cosωt

k = 62.8 ⇒, $\lambda = \frac{2\pi}{k} = 0.01$ m

$L = \frac{5\lambda}{2} = 0.25$ m $\therefore$ B correct

2 A = 0.01 m, C is correct

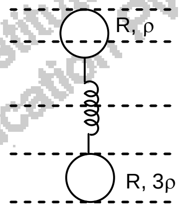
ω = 628, $\upsilon = \frac{\omega}{2\pi} = 100$ Hz

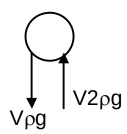
Fundamental frequency = $\frac{100}{5} = 20$ Hz

D is wrong

14. $V\rho g + V3\rho g = V'2\rho g$

V' = 2 V ⇒ just fully immersed

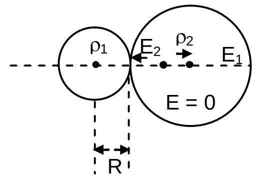




$\therefore kx = V2\rho g - V\rho g$

$x = \frac{V\rho g}{k} = \frac{4}{3}\frac{\pi R^3 \rho g}{k}$

15.



$E = \overline{E}_1 + \overline{E}_2 = 0$

$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4}{3}\frac{\pi R^3 \rho_1}{(2R)^2} = \frac{1}{3\varepsilon_0}\frac{\rho_1 R}{4}$

$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4}{3}\frac{\pi(2R)^3 R\rho_2}{(2R)^3} = \frac{1}{3\varepsilon_0}\rho_2 R$

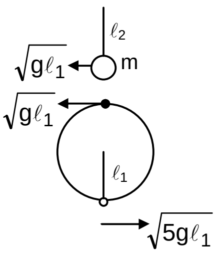
$\rho_1 = 4\rho_2$ ∴ Option D correct.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4}{3}\frac{\pi R^3\rho_1}{(2R)^2} = \frac{1}{3\varepsilon_0}\frac{\rho_1 R}{4}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4}{3}\frac{\pi(2R)^3\rho_2}{(5R)^2} = \frac{1}{3\varepsilon_0}\frac{8\rho_2 R}{25},$$

$$\frac{\rho_1}{4} = \frac{8\rho_2}{25}\ \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

Section III

16. 

$\therefore\ \sqrt{g\ell_1} = \sqrt{5g\ell_2}$

$\frac{\ell_1}{\ell_2} = 5$

17. $P \times t = \frac{1}{2}mv^2$

$0.5 \times 5 = \frac{1}{2}\times 0.2v^2$

$v = 5$

18. $h\upsilon = W + V_S$
$V_S = h\upsilon - W$
$= mx + C$
∴ Slope = h
∴ Ans. 1

19. Required: $\frac{(N_0 - N)}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t}$

$\lambda = \frac{\ell n2}{t_{1/2}} = \frac{0.693}{1386} = \frac{1}{2\times 10^3}$

$\lambda t = \frac{80}{2\times 10^3} = 0.04$

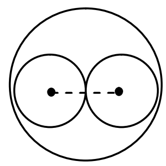
$\therefore\ 1 - e^{-\lambda t} = 1 - \frac{1}{e^{0.04}}$;

Expression $e^x = 1 + x + \frac{x^2}{2} + ... \cong 1 + x$

$\therefore\ e^{0.04} \cong 1.04$

$\therefore\ 1 - \frac{1}{e^{0.04}} = 1 - \frac{1}{1.04} \cong 1 - 0.96 = 0.04$

∴ Ans: 4%

20. 

$$\frac{MR^2}{2}\omega = \left[\frac{MR^2}{2} + 2mR^2 + 2mR^2\right]\omega'$$

$$\frac{mR^2}{2} = \frac{50\times 16\times 10^{-2}}{2} = 4$$

$MR^2 = 6.25 \times 4 \times 10^{-2} = 0.25$

$(4 \times 10) = (4 + 1)\omega' \Rightarrow \omega' = 8$

PART II

21	22	23	24	25	26	27	28	29	30
B	**A**	**A**	**C**	**D**	**B**	**B**	**D**	**B**	**D**

31	32	33	34	35
A	**A**	**B, D**	**A, B, C, D**	**B, C, D**

36	37	38	39	40
5	**8**	**2**	**4**	**6**

Section I

21. $Q = [V(H_2O)_6]^{2+} - d^3$
    3 unpaired electrons μ = 3.87 BM
    $R = [Fe(H_2O)_6]^{2+} - d^6$
    High spin complex 4 unpaired electrons
    μ = 4.90 BM
    $P = [FeF_6]^{3-} - d^5$
    High spin complex 5 unpaired electrons
    μ = 5.92 BM

22. $r_{A^+} = 0.414\ r_{X^-}$
    $= 0.414 \times 250$
    $= 104$ pm

23. These metals occur as Ag_2S silver glance or argentite
    Cu_2S, Chalcocite
    $Cu_2S.Fe_2S_3$ copper pyrites
    PbS – galena

24. $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$
    $\Delta H = 6 \times -400 + 6 \times -300 + 1300$
    $= -2900$ kJ mol^{-1}
    $= \frac{-2900}{180}$ kJ g^{-1}
    $= -16.11$ kJ g^{-1}

25. In ammoniacal (and neutral) medium Zn(II) is precipitated as ZnS (white ppt)
    Note : Fe^{3+} is reduced to Fe^{2+} by H_2S and the latter is precipitated as FeS

26. Adsorption is accompanied by decrease in enthalpy

27. The rate of S_N2 reaction is mainly decided by steric crowding in the transition state

28. Order of the reaction with respect to P is one ($t_{75\%} = 2 \times t_{50\%}$)
    Order of the reaction with respect to Q is zero $\left(t = \frac{Q_0}{k} - \frac{Q}{k}\right)$
    $\therefore \frac{dx}{dt} = k[P]^1\ [Q]^0$

29. HNO_3 decomposes into NO_2 on standing in presence of light
    $4HNO_3 \xrightarrow{h\upsilon} 4NO_2 + 2H_2O + O_2$

30. Carbolic acid (phenol) (pK_a = 10) is weaker than carbonic acid (pK_a = 6.38)

Section II

31. Rate = k[acid] [ester]^1
    $\frac{\text{Rate 1}}{\text{Rate 2}} = \frac{C^+_{H(HA)}}{C^+_{H(HX)}}$
    $\frac{1}{100} = \frac{C\alpha}{1}$
    $\alpha = \frac{1}{100}$
    $K_a = C\alpha^2$
    $= 10^{-4}$

32. Hyperconjugation involves
    $\sigma \rightarrow$ p(empty) and $\sigma \rightarrow \pi$ conjugation

33. (B) cis trans – cis trans
    (C) no isomerism – cis trans
    ($CoBr_2Cl_2$ is tetrahedral)
    (D) ionisation – ionisation

34. (P) involves cyclopropenyl cation, (Q) contains cyclopentadienyl anion (R) is 2,5-dimethyl pyrrole and (S) is hydroxytropylium chloride.

35. Dissolution of naphthalene in benzene is accompanied by an increase in entropy.
i.e., ΔS_{system} = positive
Since the solution is ideal,
$\Delta H = 0$ and $\Delta S_{surroundings} = 0$

Section III

36. $\lambda \propto \frac{1}{mv}$

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \frac{m_{Ne}.v_{Ne}}{m_{He}.v_{He}}$$

But $v \propto \sqrt{\frac{T}{M}}$

$$\frac{v_{Ne}}{v_{He}} = \sqrt{\frac{T_{Ne}}{T_{He}} \times \frac{M_{He}}{M_{Ne}}}$$

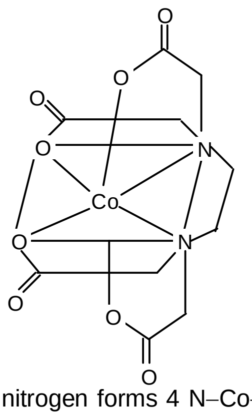
$$= \sqrt{\frac{1000}{200} \times \frac{4}{20}}$$

$$= 1$$

$$= \frac{\lambda_{He}}{\lambda_{Ne}} = \frac{20}{4}$$

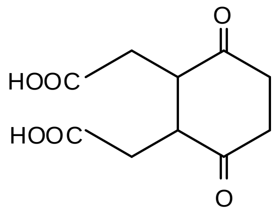
M = 5

37.



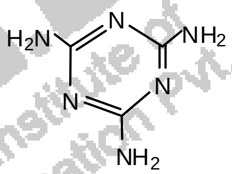
Each nitrogen forms 4 N–Co–O angles with four oxygens

38. Structure of (P) is



39. Tetrapeptide satisfying the given conditions are
Val – Phe – Gly – Ala
Phe – Val – Gly – Ala
Phe – Gly – Val – Ala
Val – Gly – Phe – Ala

40.



PART III

41	42	43	44	45	46	47	48	49	50
A*	**B**	**C**	**B**	**A**	**D**	**C**	**D**	**A**	**C**

51	52	53	54	55
B, D	**B, C**	**A, D**	**C, D**	**A,C**

56	57	58	59	60
5	**6**	**6**	**5**	**9**

Section I

41. Solving the two equation

A $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$ B (1, 1)

Then distance from $AB^2 = 8$

$(a + b + c)^2 < 4(a + b)^2$

$(a + b + c)^2 - 4(a + b)^2 < 0$

$((a + b + c) + 2(a + b))((a + b + c - 2(a + b))$

$\therefore\ a + b + c - 2a - 2b < 0$

$(a + b + c + 2(a + b) > 0)$

$-a - b + c < 0$

$a + b - c > 0$

Choice (A)

* JEE official key is A OR C OR A and C

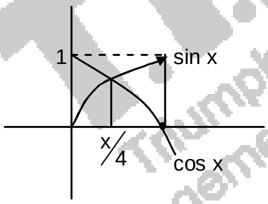
42. $y = \cos x + \sin x$

$= \frac{1}{\sqrt{2}}\sin\left(x + \frac{\pi}{4}\right)$

$y = |\cos x - \sin x|$

$y = \cos x - \sin x,\ \left(0, \frac{\pi}{4}\right)$

$\sin x - \cos x,\ \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$



$\int_0^{\pi/4}\left[(\cos x + \sin x) - (\cos x - \sin x)\right]dx$

$+ \int_{\pi/4}^{\pi/2}\left[(\cos x + \sin x) - (\sin x - \cos x)\right]dx$

$\int_0^{\pi/4} 2\sin x\,dx + \int_{\pi/4}^{\pi/2} 2\cos x\,dx$

$= (-2\cos x)_0^{\pi/4} + (2\sin x)_{\pi/4}^{\pi/2}$

$= \left(\frac{-2}{\sqrt{2}} + 2\right) + \left(2 - \frac{2}{\sqrt{2}}\right)$

$\frac{-2}{\sqrt{2}} + 2 + 2 - \frac{2}{\sqrt{2}}$

$= 4 - \frac{4}{\sqrt{2}} = 4 - 2\sqrt{2}$

43. $f(x) = x^2 - x\sin x - \cos x$

$f'(x) = (2 - \cos x)\,x > 0$ for $x > 0$

and $f'(x) < 0$ for $x < 0$

$f(-x) = f(x)$

$f(0) < 0,\ f\left(\frac{\pi}{2}\right) < 0$

But $f(\pi) > 0$

$\therefore$ f(x) has one root for $x > 0$

and another root in $x < 0$

$\therefore$ 2 Roots

44. $\sum_{k=1}^{m} 2k = n(n+1)$

$\cot^{-1}[1 + n(n+1)]$

$= \cot^{-1}[n^2 + n + 1]$

$= \tan^{-1}\left(\frac{1}{n^2 + n + 1}\right)$

$= \tan^{-1}\left[\frac{(n+1) - n}{1 + n(n+1)}\right]$

$= \tan^{-1}(n+1) - \tan^{-1}n$

$\sum_{n=1}^{23} = (\tan^{-1}2 - \tan^{-1}1)$

$+ (\tan^{-1}3 - \tan^{-1}2) +$-----------

$+ (\tan^{-1}24 - \tan^{-1}23)$

$= \tan^{-1}24 - \tan^{-1}(1)$

$= \tan^{-1}\frac{23}{25} = \cot^{-1}\left(\frac{25}{23}\right)$

Choice (B)

45. $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$

$y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v + \sec(v)$

$x\ \frac{dy}{dx} = \sec v$

$\cos v\ dv = \frac{dx}{x}$

$\sin v = \ell nCx$

$\sin\frac{y}{x} = \ell n\ (Cx)$

$x = 1,\ y = \frac{\pi}{6} \Rightarrow \frac{1}{2} = \ell n\,C$

$\sin\left(\frac{x}{y}\right) = \frac{1}{2} + \log x$

Choice (A)

46. $f'(x) < 2(f(x)$

$f'(x) - 2f(x) < 0 \quad = -k$ (say)

Where $k > 0$

$f'(x) - 2f(x) = -k$

$\frac{df}{dx} - 2f(x) = -k$

$f(e^{-2x}) = \int -ke^{-2x}dx + c$

$= \frac{k}{2}e^{-2x} + C$

$x = \frac{1}{2}\ 1 \times e^{-1} = \frac{k}{2}e^{-1} + C$

$C = \frac{1}{e}\left(1 - \frac{k}{2}\right)$

Since $f > 0$ in $\left(\frac{1}{2}, 1\right)$

$\frac{k}{2} + \frac{1}{e}\left(1 - \frac{k}{2}\right)e > 0$

$f(x) = \frac{k}{2} + Ce^{2x}$

$= \frac{k}{2} + \frac{1}{e}\left(1 - \frac{k}{2}\right)e^{2x}$

$\int_{\frac{1}{2}}^{1} f(x)dx = \frac{k}{2} \times \frac{1}{2} + \left(\frac{1 - \frac{k}{2}}{e}\right)\left(\frac{e^{2x}}{2}\right)_{\frac{1}{2}}^{1}$

$= \frac{k}{4} + \frac{e}{2}\left(1 - \frac{k}{2}\right) - \frac{1}{2}\left(1 - \frac{k}{2}\right)$

$\frac{k}{2} + \frac{e}{2} - \frac{ek}{4} - \frac{1}{2}$

$= \frac{e-1}{2} + \frac{k}{2}\left(1 - \frac{e}{2}\right)$

$< \frac{e-1}{2}$

Since $f > 0$ in $\left(\frac{1}{2}, 1\right)$ and $f\left(\frac{1}{2}\right) = 1$

Graph of f is above the x– axis

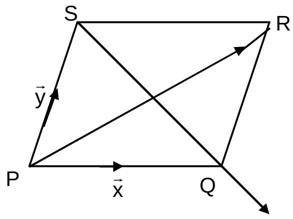
$\Rightarrow \int_{\frac{1}{2}}^{1} f(x)dx > 0$

Hence, $\int_{\frac{1}{2}}^{1} f(x)$ lies in $\left(0, \frac{e-1}{2}\right)$

47. $\overline{PR} = 3\bar{i} + \bar{j} - 2\bar{k}$

$\overline{SQ} = \bar{i} - 3\bar{j} - 4\bar{k}$

$\overline{PT} = \bar{i} + 2\bar{j} + 3\bar{k}$



$\bar{x} + \bar{y} = 3i + \bar{j} - 2\bar{k}$

$y + \overrightarrow{SQ} = \vec{x}$

$\bar{x} - \bar{y} = \bar{i} - 3\bar{j} - 4\bar{k}$

$\bar{x} = 2\bar{i} - \bar{j} - 3\bar{k}$

$\bar{y} = \vec{i} + 2\vec{j} + \vec{k}$

Volume = $[xyp\vec{T}]$

$= \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 10$

48. Point on the line is (–2, –1, 0)

D. Ratio normal to plane <1, 1, 1>

Equation of the line $\perp^r$ plane is

$\frac{x+2}{1} = \frac{y+1}{1} = \frac{z}{1} = \lambda$

Point on the line (λ–2, λ–1, λ)

Which lies on the plane x + y + z –3 = 0

$\therefore \lambda = 2$

$\therefore$ point is (0, 1, 2)

Another point is (0, –2, 3)

$\therefore x = \frac{2}{3}\quad y = \frac{-4}{3},\quad z = \frac{11}{3}$

$\begin{matrix} 0, & 1, & 2 \\ \frac{2}{3} & \frac{-4}{3} & \frac{11}{3} \\ \hline \frac{2}{3} & \frac{-7}{3} & \frac{5}{3} \end{matrix}$

$\therefore$ D . Ratio < (2, –7, 5>

Equation line $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

Choice (D)

49. Required probability

$= 1 - P\left(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}\right)$

$1 - P(\bar{A}).\ P(\bar{B}).\ P(\bar{C}).P(\bar{D})$

$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$

$= 1 - \frac{21}{256} = \frac{235}{256}$

Choice (A)

50. α lies on $|z - z_0| = r$

$\Rightarrow |\alpha - z_0| = r$

$(\alpha - z_0)\left(\bar{\alpha} - \bar{z}_0\right) = r^2$

$|\alpha|^2 - \left(\alpha\bar{z}_0 + \bar{\alpha}z_0\right) + z_0\bar{z}_0 = r^2$

$|\alpha|^2 - \left(\alpha\bar{z}_0 + \bar{\alpha}z_0\right) + \frac{r^2+2}{2} = r^2$

$|\alpha|^2 - \left(\alpha\bar{z}_0 + \bar{\alpha}z_0\right) = \frac{r^2}{2} - 1$ ——— (1)

$\frac{1}{\bar{\alpha}}$ lies on $|z - z_0| = 2r$

$\frac{1}{\bar{\alpha}} = \frac{\alpha}{|\alpha|^2}$ lies on $|z - z_0| = 2r$

$\left|\frac{\alpha}{|\alpha|^2} - z_0\right| = 2r$

$\left(\frac{\alpha}{|\alpha|^2} - z_0\right)\left(\frac{\bar{\alpha}}{|\alpha|^2} - \bar{z}_0\right) = 4r^2$

$\frac{1}{|\alpha|^2} - \frac{1}{|\alpha|^2}\left(\alpha\bar{z}_0 + \bar{\alpha}z_0\right) + z_0\bar{z}_0 = 4r^2$

ie, $\frac{1}{|\alpha|^2} - \frac{1}{|\alpha|^2}\left\{|\alpha|^2 - \frac{r^2}{2} + 1\right\} + \frac{r^2+2}{2} = 4r^2$

using (1)

$-1 + \frac{r^2}{2|\alpha|^2} + \frac{r^2}{2} + 1 = 4r^2$

$\Rightarrow \frac{r^2}{2|\alpha|^2} + \frac{7r^2}{2}$

$|\alpha|^2 = \frac{1}{7}$

$|\alpha| = \frac{1}{\sqrt{7}}$

Option (C)

Section II

51. The line ℓ in along the line of S. D between ℓ_1 & ℓ_2

$\therefore$ the d. r. ' s are x, y, z where

$x + 2y + 2z = 0$

$2x + 2y + z = 0$

$\Rightarrow \frac{x}{2} = \frac{y}{-3} = \frac{z}{2}$

$\therefore$ line (λ) is $\bar{r} = \lambda\,(2i - 3j + 2k)$

where ℓ meets ℓ_1, $\lambda = 1$

So the point is $2i - 3j + 2k$

$(3 + 2s - 2)^2 + (2s + 6)^2 + s^2 = 17$

$9s^2 + 28s + 20 = 0$

$(9s+10)(s+2) = 0$

$\Rightarrow s = \frac{-10}{9}$ or -2

points are $(-1, -1, 0)$ & $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Ans (B) & (D)

52. $f'(x) = \sin\pi x + \pi x\cos\pi x$

$f'(n) = (-1)^n\,\pi n$

$f'\left(n + \frac{1}{2}\right) = (-1)^n$

$f'(n+1) = (-1)^{n+1}\,\pi\,(n+1)$

$\Rightarrow f'(n)\, f'\left(n + \frac{1}{2}\right)$ is +ve

$f'(n)\, f'(n+1)$ is –ve

$f'\left(n + \frac{1}{2}\right) f'(n+1)$ is –ve

$\therefore$ one point exists in $(n, n+1)$

& is in $\left(n + \frac{1}{2}, n+1\right)$

Ans (B, C)

53. $S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + \text{-----} + (4n-1)^2 + (4n)^2$

$= \sum_{r=1}^{n}\left[-(4r-3)^2 - (4r-2)^2 + (4r-1)^2 + (4r)^2\right]$

$= \sum_{r=1}^{n} 4(8r-3)$

$= 4\left[.8.\frac{n(n+1)}{2} - 3n\right]$

$= 4n\,[4n+1]$

$4n(4n+1) = 1056 \Rightarrow n = 8$

$4n(4n+1) = 1332 \Rightarrow n = 9$

[(B), (C) do not give integral n]

Ans (A, (D)

54. (A) $(N^T MN)^T = N^T M^T N$

$= N^T MN$ when M is symmetric

$\Rightarrow N^T MN$ is symmetric

(B) $(MN - NM)^T = NM - MN$, is skew symmetric when M, N are symmetric

(C) $(MN)^T = NM$ where M, N are symmetric

So MN is not symmetric

(D) (adjM) (adjN) = adj (NM) & not adj (MN)

So statement is not correct

Ans (C), & (D)

55. If 46 n is the perimeter

and x is side of square removed

$V = (15n - 2x)(8n - 2x)x$

$= 4x^3 - 46nx^2 + 120n^2x$

$V' = 12x^2 - 92nx + 120n^2$

$= 0$ when $x = 5$

$\Rightarrow 120n^2 - 460n + 300 = 0$

$\Rightarrow 6n^2 - 23n + 15 = 0$

$\Rightarrow 6n^2 - 18n - 5n + 15 = 0$

$\Rightarrow (6n - 5)(n - 3) = 0$

$\therefore n = 3$ or $\frac{5}{6}$

Possible side – lengths are 45, 24, 12.5, $\frac{20}{3}$

. Ans (A), (C)

Section III

56. The eight vectors can be represented as

(1) 1 1 1
(2) 1 1 –1
(3) 1 –1 1
(4) –1 1 1
(5) –1 –1 –1
(6) –1 –1 1
(7) –1 1 –1
(8) 1 –1 –1

From the eight vectors, 3 vectors may be chosen in ${}^8C_3 = 56$ ways. Among 56 sets, we need to find coplanar sets

With pair of vectors 1 and 5, any one of the other six may be chosen to complete coplanar set.

Likewise for each pair of vectors (2 and 6), (3 and 7), (4 and 8), six coplanar sets can be chosen.

$\therefore 6 \times 4 = 24$ coplanar sets $\Rightarrow$

$56 - 24 = 32$ non- coplanar sets $= 2^p$

$\therefore p = 5$

57. Let $P(E_1) = x$

$P(E_2) = y$

$P(E_3) = z$

$x(1-y)(1-z) = \alpha$

$y(1-z)(1-x) = \beta$

$z(1-x)(1-y) = \gamma$

Also $(1-x)(1-y)(1-z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$(\beta - 2r)p = 2\beta r$

$[x(1-y)(1-z) - 2y(1-z)(1-x)]p$
$= xy(1-z)^2(1-x)(1-y)$

$[x(1-y) - 2y(1-x)]p$
$xy(1-z)(1-x)(1-y)$

$x(1-y) - 2y(1-x) = xy$

$x - xy - 2y + 2xy = xy$

$x + xy - 2y = xy$

$x - 2y = 0$ ——— (1)

$[y(1-z)(1-x) - 3z(1-x)(1-y)]p =$
$2yz(1-z)(1-x)^2(1-y)$

$[y(1-z) - 3z(1-y)]p = 2yz(1-x)(1-y)(1-z)$

$y - yz - 3z + 3zy = 2yz$

$y - 3z + 2zy = 2yz$

$y - 3z = 0$ ——— (2)

$$\frac{P(E_1)}{P(E_3)} = \frac{x}{z} = \frac{2y}{\frac{1}{3}y} = \frac{2}{\frac{1}{3}} = 6$$

58. $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = 2; \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{7}{5}$

$\Rightarrow \frac{n-r+6}{r} = 2, \frac{n-r+5}{r+1} = \frac{7}{5}$

$\Rightarrow n - 3r = -6$

$5n - 12r = -18$

$\Rightarrow n = 6$

59. $\frac{n(n+1)}{2} - (2k+1) = 1224$

$\Rightarrow n(n+1) = 2450 + 4k$

$\Rightarrow (n+50)(n-49) = 4k$

$n = 49 \Rightarrow k = 0$

$n = 50 \Rightarrow k = 25$

(The next possible value of k is 103, for n = 53, is not feasible)

$\therefore$ the required integer is $25 - 20 = 5$

60. Let $x = h$ meet the ellipse at $(2\cos\theta, \sqrt{3}\sin\theta)$, so that $\cos\theta = \frac{h}{2}$

tangent at the point is $\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$

where it meets $y = 0$, $x = \frac{2}{\cos\theta} = \frac{4}{h}$

Required area, $\Delta(h)$

$$2 = \frac{1}{2}\sqrt{3}\sin\theta\left(\frac{2}{\cos\theta} - 2\cos\theta\right)$$

$$= 2\sqrt{3}\sin\theta\left(\frac{2}{h} - \frac{h}{2}\right)$$

$$= \frac{2\sqrt{3}(4-h^2)}{2h}\sqrt{1 - \frac{h^2}{4}}$$

$$= \frac{\sqrt{3}(4-h^2)^{\frac{3}{2}}}{2h}$$

$\Delta(1) = \frac{9}{2}$ and

$$\Delta\left(\frac{1}{2}\right) = \sqrt{3}\left(4 - \frac{1}{4}\right)^{\frac{3}{2}} = \frac{\sqrt{3}.\ 15\sqrt{15}}{8}$$

$$= \frac{45\sqrt{5}}{8}$$

$$\therefore \frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = 45 - 36 = 9$$