T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2013

Paper II – Code 0

PART I

1	2	3	4	5	6	7	8
B, C, D	**A, C**	**D**	**C, D**	**A, D**	**A, B**	**B, D**	**A, D**

9	10	11	12	13	14	15	16
B	**A**	**B**	**A**	**B**	**B**	**C**	**A**

17	**18**	**19**	**20**
A	**C**	**D**	**C**

Section I

1. $\frac{\Delta Q}{\Delta t} = mC\frac{\Delta \theta}{\Delta t}$

m_1, $\frac{\Delta\theta}{\Delta t}$ is given constants

$\frac{\Delta Q}{\Delta t} \propto C$

In the temperature range 0 – 100 K, the curve is non-linear. Hence $\frac{\Delta Q}{\Delta t}$ does not vary linearly. Hence option A is not correct.

2. $r \propto \frac{n^2}{Z} = \frac{9}{Z}$

r = 4.5a₀ ⇒ Z = 2, n = 3 to n = 2
n = 3 to n = 1

$\frac{1}{\lambda} = R_2{}^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$ or $R_2{}^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$

$= 4R\left[\frac{1}{4} - \frac{1}{9}\right]$ or $4R\left[1 - \frac{1}{9}\right]$

$= 4R\frac{5}{36}$ or $4R\frac{8}{9}$, $\lambda = \frac{9}{5R}$ or $\frac{9}{32R}$

3. $d\cos\theta d\theta + d(d)\sin\theta = 0$

$d(d) = -d \cot\theta\, d\theta = \frac{-\lambda}{2\sin\theta}\cot\theta d\theta$

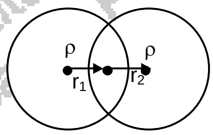
$\Rightarrow d(d) = \frac{-\lambda}{2}\frac{\cos\theta}{\sin^2\theta}d\theta$

$|d(d)|$ decreases with θ.

$\frac{d(d)}{d} = -\cot\theta\, d\theta$

i.e., fractional error decreases with θ.

4.



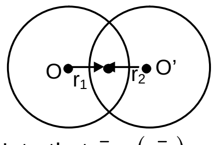
$\bar{E} = \bar{E}_1 + \bar{E}_2$

$= \frac{\rho r_1}{3\varepsilon_0}\hat{i} + \frac{\rho r_2}{3\varepsilon_0}\hat{i} \neq 0$

Since E is not zero, additional W involved
∴ V is not constant.

$E = \bar{E}_1 + \bar{E}_2 = \frac{\rho \bar{r}_1}{3\varepsilon_0} + \frac{\rho}{3\varepsilon_0}(-\bar{r}_2)$

$= \frac{\rho}{3\varepsilon_0}(\bar{r}_1 + (-\bar{r}_2))$

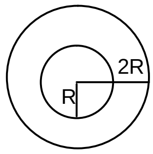


Note that $\bar{r}_1 + (-\bar{r}_2) = \overline{OO'}$ = constant

$= \frac{8}{3\varepsilon_0}(\bar{r}_1 - \bar{r}_2)$ not constant in magnitude.

Direction $+\hat{i}$

5.

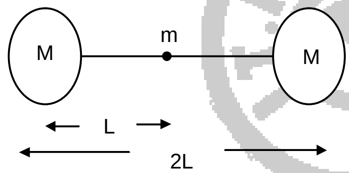


Enclosed current not zero; Option A correct. Due to cylinder, tangential due to solenoid, axial B is wrong
due to solenoid, field exists
C is wrong
due to only cylinder, D correct.

6. $f_2 = \frac{f_1(v \pm w + u)}{v \pm w - u}$

A: means –w, $\therefore f_2 > f_1$, A correct
B: means +w, $f_2 > f_1$, B correct [w << v]
C: means –w, same as A, C wrong
D: same as B, D is wrong

7.



$PE = -\frac{GMm}{L} \times 2 + \text{cons tan t}\left(-\frac{GM^2}{2L}\right)$

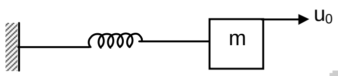
(Initial)

$\therefore$ K.E = $\frac{2GMm}{L}$, (for escape)

$\frac{1}{2}mv_e^2 = \frac{2GMm}{L} \Rightarrow v_e = 2\sqrt{\frac{GM}{L}}$, B is correct

D is correct (Conservative field)

8.



A is correct (reverses velocity]

At collision, $\frac{u_0}{2} = u_0 \cos \omega t_1 \Rightarrow \omega t_1 = \frac{\pi}{3}$

After collision, $-\frac{u_0}{2} = u_0 \cos \omega t_2 \Rightarrow \omega t_2 = -\frac{\pi}{3}$

B) required time = $2t_1 = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$ B wrong

C) required time = $2t_1 + \frac{T}{4} = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{1}{4}2\pi\sqrt{\frac{m}{k}}$

$= \pi\sqrt{\frac{m}{k}}\frac{7}{6}$ C wrong

D) required time = $\pi\sqrt{\frac{m}{k}}\frac{7}{6} + \frac{T}{4} = \pi\sqrt{\frac{m}{k}}\left[\frac{7}{6} + \frac{1}{2}\right]$

$= \pi\sqrt{\frac{m}{k}}\left(\frac{20}{12}\right)$

$= \pi\sqrt{\frac{m}{k}}\frac{5}{3}$ D is correct

Section II

9. At Q: $mgh - mgh \sin 30 - 150 = \frac{1}{2}mv^2$

$\Rightarrow \frac{mgR}{2} - 150 - \frac{1}{2}mv^2$

$200 - 150 = \frac{1}{2}v^2$

v = 10

10. $N - mg \cos 60 = \frac{mv^2}{R}$

$\Rightarrow N = 1 \times 10 \times \frac{1}{2} + 1 \times \frac{100}{40}$

= 7.5 N

11. $i - \frac{P}{V} = \frac{6 \times 10^5}{4 \times 10^3} = 1.5 \times 10^2$ A

Loss = $i^2R = 2.25 \times 10^4 \times 0.4 \times 20$
$= 18 \times 10^4$

$\therefore$ Fractional loss = $\frac{18 \times 10^4}{6 \times 10^5} = 0.3$

$\Rightarrow$ 30%

12. Step up 4000 V $\to$ 40000 V
(1 : 10)
Step down 40000 V $\to$ 200 V
(200 : 1)
Option A is correct.

13. $P = \frac{QW}{2\pi}$, $A = \pi R^2$

$E_{ind} = \frac{d\phi}{dt} = A.\frac{dB}{dt} = \pi R^2 B$

$E_{ind} = \frac{E_{ind}}{2\pi R} = \frac{RB}{2}$

14. E_{ind}. Q = Foce

Acceleration = $\frac{\text{Force}}{m}$

$\therefore \Delta v = a \times t$ (t = 1 s)

$= \frac{EQ}{m}$

$\therefore \Delta L = m\Delta vR = EQR = \left(\frac{RB}{2}\right)QR$

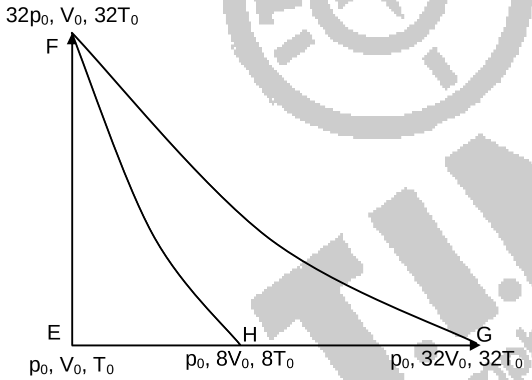
$\therefore \Delta M = \gamma \Delta L$

$= \frac{\gamma B R^2 Q}{2}$ (negative)
(negative : Lenz law)
Option B

15. (A) ${}_3^6Li \to {}_2^4He + {}_1^2H$
015 002 014 – Not possible
(B)
${}_{84}^{210}Po \to {}_1^1H + {}_{83}^{209}Bi$
983 008 980 Not possible
(C) Reverse of A possible
(D) Not possible

16. $\frac{p^2}{2}\left(\frac{1}{M}+\frac{1}{m}\right) = 5422$
M : m
206 : 4
103 : 2
$5422 \times \frac{103}{105}$ = 5319 keV

Section III

17.


$32p_0V_0 = p_0V^{5/3} \Rightarrow V' = 8V_0$
(P) G – E: $W = nR\Delta T = nR.31T_0$
$= 31p_0V_0$: P → 4
(Q) G – H: $W = nR\Delta T = nR.24T_0 = 24p_0V_0$
Q → 3
(R) F – H: $nC_V\Delta T = n.\frac{3}{2}R.24T_0 = 36p_0V_0$
R → 2
(S) F → G: $nR\, 32T_0\, \ell n32$
$= p_0V_0\, 32 \times 5 \times 0.7$
$= 160p_0V_0 \ell n2$
S → 1

18. List II
1. ${}_8^{15}O \to {}_7^{15}N + {}_1^0\beta^+$ [Q]
2. ${}_{92}^{238}U \to {}_{90}^{234}Th + {}_2^4He, (\alpha)$ [P]
3. ${}_{83}^{185}Bi \to {}_{82}^{184}Pb + {}_1^1H(p)$ [S]
4. ${}_{94}^{239}Pu \to {}_{57}^{140}La + {}_{37}^{99}X$ Fission [R]
∴ P → 2, Q → 1, R → 4, S → 3

19. P: e – f: $\mu_2 > \mu_1$ $\mu_3 < \mu_2$ (2)
Q: e → g: $\mu_1 = \mu_2$ (3)
R: e – h: $\mu_1 > \mu_2$, $\mu_2 > \mu_3$ (4)
Also
C > 45°
$\sin C > \frac{1}{\sqrt{2}}$
$\mu_1 \sin C = \mu_2 \Rightarrow \sin C = \frac{\mu_2}{\mu_1} > \frac{1}{\sqrt{2}}$
$\Rightarrow \mu_1 < \sqrt{2}\,\mu_2$
∴ Option (4)
S: e – i: C < 45° $\Rightarrow \mu_1 > \sqrt{2}\mu_2$
(Similar derivation as above)
∴ Option (1)
P → 2 Q → 3, R → 4, S → 1
Option D

20. P: $K = \frac{R}{N_A} = \frac{\frac{pV}{nT}}{N_A} \frac{\frac{Energy}{mole \times temperature}}{\frac{N_0 s}{mole}}$
$= \frac{Energy}{Temperature}$
$= ML^2T^{-2}K^{-1}$: (4)
Q: η: $ML^{-1}T^{-1}$: (2)
R: h = Js : E × Time = ML^2T^{-1} (1)
S: K: $W m^{-1} K^{-1} = \frac{E}{T}.L^{-1}K^{-1} = ML^1T^{-3}K^{-1}$ (3)
P → 4, Q → 2, R → 1, S → 3
∴ Option (C)

PART II

21	22	23	24	25	26	27	28
C, D	**A, B, D**	**A, C, D**	**A, B**	**B**	**C**	**B**	**B, D**

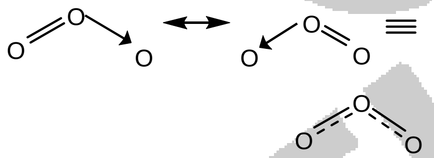
29	30	31	32	33	34	35	36
A	**D**	**B**	**A**	**C**	**B**	**A**	**A**

37	38	39	40
A	**D**	**D**	**A**

Section I

21. Al from Al_2O_3 and Mg from dolomite are obtained by electrolysis.

22. Enthalpies of compounds vary with temperature
K does not depend on the initial amount of the reactant
At a given temperature, K is independent of pressure of CO_2.
ΔH is the same for a reaction at constant temperature in the presence or absence of catalyst

23. 

O_3 is diamagnetic since no unpaired electrons
O_3 has bent structure.

24. $^{9}_{4}Be + \gamma \rightarrow ^{8}_{4}Be + ^{1}_{0}n$
$^{9}_{4}Be + ^{1}_{1}H \rightarrow ^{8}_{4}Be + ^{2}_{1}D$

25. Bromine replaces SO_3H group which is ortho or para to a hydroxyl or amino group. So the final product is 2,4,6-tribromophenol

26. Reaction I
In aqueous NaOH, rate of bromination of acetonr is independent of the concentration of bromine.

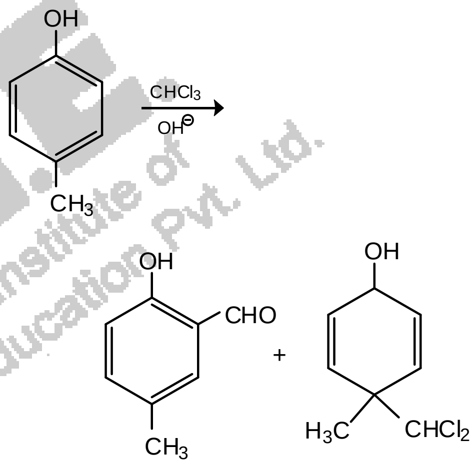
$$CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow{Br_2} CH_3-\overset{\overset{O}{\|}}{C}-Br_3 \xrightarrow{NaOH} CH_3COONa + CHBr_3$$

Excess acetone will remain in the reaction mixture

Reaction II

$$CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow[CH_3COOH]{Br_2} CH_3-\overset{\overset{O}{\|}}{C}-CH_2Br + HBr$$

27. $K_{sp}(Ag_2CrO_4) = [Ag^+]^2[CrO_4^{2-}]$
$1.1 \times 10^{-12} = (0.1)^2 \times [CrO_4^{2-}]$
$= [CrO_4^{2-}] = 1.1 \times 10^{-10}$

28. 

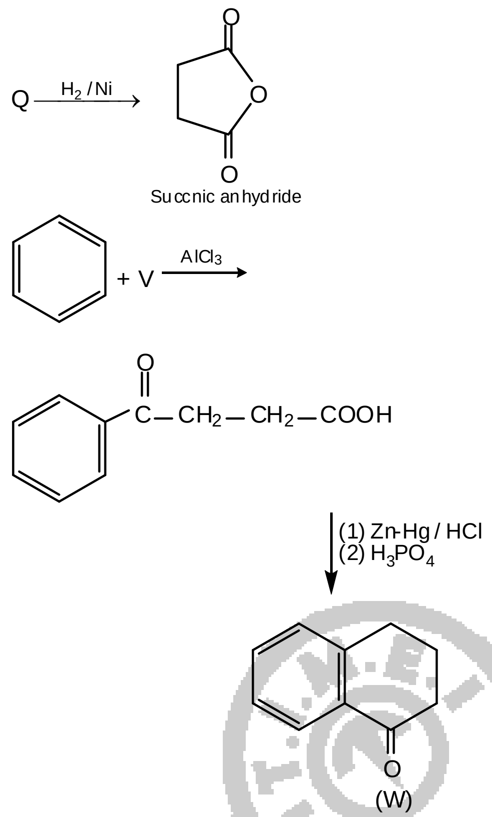
Section II

29. $Pb^{2+} + HCl \rightarrow PbCl_2\downarrow + 2H^+$
$PbCl_2$ is a ppt soluble in hot water

30. Na_2CrO_4 (yellow solution) is the only possibility

31. P is maleic acid. Reaction of P with dilute alkaline $KMnO_4$ gives S (meso form). R is fumaric acid which reacts with dilute alkaline $KMnO_4$ to give racemic mixture containing equivalent amount of T & U

32. Q is fumaric acid



33. K to L and M to N – P constant
$\therefore V \alpha T$
L to M and N to K – V constant
$\therefore P \alpha T$

34. Isochoric process – volume remains as constant

35. $\underset{\text{cold dil}}{2NaOH} + Cl_2 \rightarrow NaCl + NaOCl + H_2O$

$\underset{\text{hot conc}}{6NaOH} + 3Cl_2 \rightarrow 5NaCl + NaClO_3 + 3H_2O$

36. $SO_2 + Cl_2 \rightarrow \underset{[R]}{SO_2Cl_2}$

$P \xrightarrow{SOCl_2} \underset{[S]}{PCl_5}$

$PCl_5 \xrightarrow{H_2O} \underset{(T)}{H_3PO_4}$

Section III

37. P – Addition of $(C_2H_5)_3N$ to CH_3COOH produces $(C_2H_5)_3\overset{+}{N}H$ and CH_3COO^- and hence conductivity increases. $(C_2H_5)_3N$ exists in the molecular form and hence conductivity remains as constant (P–3)
Q – Addition of KI to $AgNO_3$ replaces Ag^+ with K^+, conductivity remains as a constant. Finally KI increases the conductivity (Q–4)
R – Initially OH^- is replaced with CH_3COO^- and hence conductivity decreases.
CH_3COOH does not change conductivity (R–2)
S – Initially H^+ is replaced with Na^+ and hence conductivity decreases. Addition of NaOH increases the conductivity (S–1)

38. P = 3

	E°	nE°
$Fe^{3+} + 1e^- \rightarrow Fe^{2+}$	0.77	0.77V
$Fe^{2+} + 2e^- \rightarrow Fe$	–0.44	–0.88 V
$Fe^{3+} + 3e^- \rightarrow Fe$	–0.037	–0.11 V

Q = 4
$2H_2O \rightarrow O_2 + 4H^+ + 4e^-$ $E° = -1.23$ V
$2H_2O + O_2 + 4e^- \rightarrow 4OH^-$ $E° = 0.40$ V
$4H_2O \rightarrow 4H^+ + 4OH^-$ $E° = -0.83$ V
R = 1
$Cu / Cu^{2+} // Cu^+ / Cu$
$2Cu^+ \rightarrow Cu^{2+} + Cu$ $E^\circ_{cell} = 0.52 - 0.34$
$= 0.18$ V
S = 2

	E°	nE°
$Cr^{3+} + 3e^- \rightarrow Cr$	–0.74	–2.22 V
$Cr^{2+} + 2e^- \rightarrow Cr$	–0.91	–1.82 V
$Cr^{3+} + 1e^- \rightarrow Cr^{2+}$	–0.4	– 0.4 V

39. P – Warm
Q – Cl_2
R – I_2
S – NO

40. (P) is dehydrohalogenation reaction. (Q) is Williamson's synthesis. (R) involves addition of water according to Markovnikov's rule for which oxymercuration-reduction is used. (S) involves addition of water against Markovnikov's rule for which hydroboration-oxidation is used

PART III

41	42	43	44	45	46	47	48
B, C, D	**A, B**	**C, D**	**A, B, C**	**A, D**	**B, D**	**B**	**A, C**

49	50	51	52	53	54	55	56
D	**C**	**B**	**D**	**B**	**C**	**A**	**D**

57	58	59	60
C	**A**	**B**	**A**

Section I

41. $1, \omega, \omega^2$

$$P = \begin{bmatrix} \omega^2 & \omega^3 \text{ ------------ } & \omega^{n+1} \\ \omega^3 & \omega^4 \text{ ------------ } & \omega^{n+2} \\ -- & --- & --- \\ -- & --- & --- \\ \omega^n & \omega^{n+1} \text{ ------ } & \omega^{2n} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^4 & \omega^6 & \omega^{2n+2} \\ \omega^6 & \omega^8 \text{ -------- } & \omega^{2n+4} \\ --- & --- & --- \\ \omega^{2n} & \omega^{2n+2} \text{ ----- } & \omega^{4n} \end{bmatrix} \neq 0$$

$P^2 \neq 0$

$\Rightarrow \omega^4 + \omega^6 + \text{------} + \omega^{2n+2} \neq 0$

$\Rightarrow \omega^4 (1 + \omega^2 + \omega^4 + \text{-------} \omega^{2n-2}) \neq 0$

$\Rightarrow 2n - 2 \neq 4$

$\Rightarrow n \neq 3$

$\therefore$ 55, 56, 58 are not multiple of

$\therefore$ B, C, D

42. Let is analyze $| \, |x+2| - 2|x| \, | = |g(x)|$

$x < -2 \quad g(x) = -(x+2) - (-2x) = x - 2 < 0$

$\therefore |g(x)| = 2 - x$

$-2 \le x < 0$

$g(x) = x + 2 + 2x = 3x + 2$

$3x + 2 = 0 \Rightarrow x = \dfrac{-2}{3}$

$-2 \le x < \dfrac{-2}{3} \Rightarrow |g(x)| = -3x - 2$

$\dfrac{-2}{3} < x < 0 \Rightarrow |g(x)| = 3x + 2$

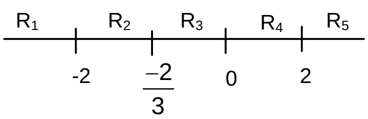
$x \ge 0$

$g(x) = x + 2 - 2x = 2 - x$

$2 - x = 0 \Rightarrow x = 2$

$0 \le x < 2 \Rightarrow |g(x)| = 2 - x$

$x \ge 2 \Rightarrow |g(x)| = x - 2$



$R_1 : f(x) = -2x - x - 2 - (2 - x) = -2x - 4$

$R_2 : f(x) = -2x + x + 2 + 3x + 2 = 2x + 4$

$R_3 : f(x) = 2x + x + 2 - 3x - 2 = -4x$

$R_4 : f(x) = 2x + x + 1 - (2 - x) = 4x$

$R_5 : f(x) = 2x + x + 2 - (x - 2) = 2x + 4$

f(x) continuous at $x = -2, \dfrac{-2}{3}, 0, 2$

f'(x) changes sign at $-2, \dfrac{-2}{3}$, 0 only

Option : A and B

43. $\omega = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2}$

$= \cos\dfrac{\pi}{6} + i \sin\dfrac{\pi}{6}$

$= e^{\frac{in\pi}{6}}$

$P = \left\{ e^{\frac{in\pi}{6}}, n = 1, 2, 3, ---- \right\}$

Cos $n\theta$

$0 \le \theta < \dfrac{\pi}{3}$

$H_1 : z = e^{1\theta}, 0 \le \theta \; \dfrac{\pi}{3}$

$H_2 : z = e^{i\theta}, \; \dfrac{2\pi}{3} < 0 \le \pi$

$P \cap H_1 = z_1 = e^{\frac{i\pi}{6}}$

$P \cap H_2 = z_2 = e^{\frac{i.5\pi}{6}}, e^{i\pi}$

$\text{Arg}\left(\dfrac{z_1}{z_2}\right) = \dfrac{2\pi}{3}$ or $\dfrac{5\pi}{6}$

C and D

44. $3^x = 4^{x-1}$

taking log

$x\log 3 = (x - 1) \log 4$

x (log4 –log3) = log4

$\therefore x = \dfrac{\log 4}{\log 4-\log 3} = \dfrac{2\log 2}{2\log 2-\log 3}$

A, B, C are correct

45. $L_1 : \dfrac{x-5}{0} = \dfrac{y}{3-\alpha}\,\dfrac{3}{-2}$

L_2: $\dfrac{x-\alpha}{0} = \dfrac{y}{-1} = \dfrac{ze}{2-\alpha}$ coplanar

$$\begin{vmatrix} \alpha-5 & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (\alpha-5)\,((3-\alpha)(2-\alpha)-2) = 0$

$\Rightarrow (\alpha-5)\,(4-5\alpha+\alpha^2) = 0$

$\Rightarrow \alpha = 5,\ \alpha^2-5\alpha+4 = 0$

$\Rightarrow \alpha = 5,\ \alpha = 4, 1$

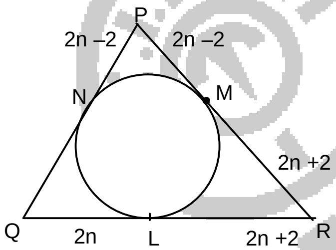
A and D are correct

46. $\cos p = \dfrac{1}{3} = \dfrac{q^2+r^2-p^2}{2qr}$

$\therefore$ p = 4n +2

q = 4n

r = 4n –2



$\Rightarrow \dfrac{1}{3} = \dfrac{16n^2+(4n-2)^2-(4n+2)^2}{8n(4n-2)}$

$\Rightarrow \dfrac{1}{3} = \dfrac{16n^2-32n}{32n^2-16n} = \dfrac{2n^2-4n}{4n^2-2n}$

$\dfrac{1}{3} = \dfrac{n-2}{2n-1}$

2n –1 = 3n –6

$\Rightarrow$ n = 5

$\therefore$ sides 22, 20, 18

$\therefore$ Choice (B), (D)

47. $$\dfrac{n^a\left(\left(\dfrac{1}{n}\right)^a+\left(\dfrac{2}{n}\right)^a+----\left(\dfrac{n}{n}\right)^a\right)}{n^{a-1}\left(1+\dfrac{1}{n}\right)^a}$$

$$= \dfrac{\dfrac{1}{n}\left\{\left(\dfrac{1}{n}\right)^a+\left(\dfrac{2}{n}\right)^a+----+\left(\dfrac{n}{n}\right)^a\right\}}{\left(1+\dfrac{1}{n}\right)^{a-1}\left\{a+\dfrac{1}{2}\left(1+\dfrac{1}{n}\right)\right\}}$$

$$\lim_{n\to\infty} = \dfrac{\int_0^1 x^a dx}{\left(a+\dfrac{1}{2}\right)} = \dfrac{2}{(2a+1)}\times\dfrac{1}{(a+1)}$$

$$\dfrac{2}{(a+1)(2a+1)} = \dfrac{1}{60}$$

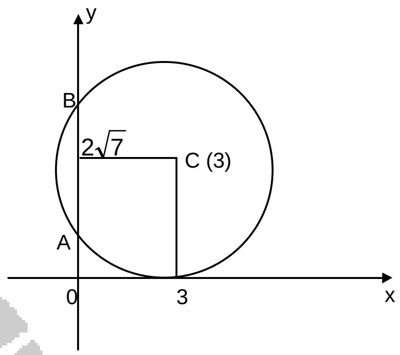
(a +1) (2a +1) = 120

$a = 7,\ a = \dfrac{-17}{2}$

But when $a = \dfrac{-17}{2}$, the integral $\int_0^1 x^a dx$ does not exist.

and hence a = 7.

48.



Choice (A)

$(x-3)^2 + (y-4)^2 = -9 + 25$

$= 16$

x = 0

$(y+4)^2 = 16 - 9 = 7$

$y = 4 = \pm\sqrt{7}$

$y = -4 \pm\sqrt{7}$

$\left(0,-4+\sqrt{7}\right)\quad\left(0,-4-\sqrt{7}\right)$

Distance = $2\sqrt{7}$

Choice B

$(x-3)^2 + \left(y+\dfrac{7}{2}\right)^2 = -9+9+\dfrac{49}{4}$

x = 0 $\qquad = \dfrac{49}{4}$

$\left(y+\dfrac{7}{2}\right)^2 = \dfrac{49}{4} = 9$

$= \dfrac{13}{4}$

Choice (C)

$(y-4)^2 = \pm\sqrt{7}$

$y = 4\pm\sqrt{7}$

Choice (D)

$(x-3)^2 + (y-4)^2 = -9 + 25$

$= 16$

x = 0

$(y+4)^2 = 16 - 9 = 7$

$y = 4 = \pm\sqrt{7}$

$y = -4 \pm\sqrt{7}$

$\left(0,-4+\sqrt{7}\right) \quad \left(0,-4-\sqrt{7}\right)$

Distance = $2\sqrt{7}$

(A) and (c) true

$(y-4)^2 = \pm\sqrt{7}$

$y = 4 \pm\sqrt{7}$

Section II

49. $e^{-x}(f''(x) - 2f'(x) + f(x)) > 1$

Let $g(x) = e^{-x} f(x) \Rightarrow g'(x) = e^{-x}[f'(x) - f(x)]$

$g''(x) = e^{-x}[f''(x) - 2f'(x) + f(x)]$

$\Rightarrow g''(x) > 1$

Integrating twice we get

$g(x) \geq \left(\frac{x^2}{2} + ax + b\right)$

$\therefore f(x) \geq \left(\frac{x^2}{2} + ax + b\right)e^x$

given $f(0) = f(1) = 0 \Rightarrow b = 0$ $a = \frac{-1}{2}$

$\therefore f(x) \geq \left(\frac{x^2 - x}{2}\right)e^x$ _____ (2)

Let $h(x) = (x^2 - x)\, e^x$

$h'(x) = (x^2 - x + 2x - 1)\, e^x$

$= (x^2 + x - 1)\, e^x$

$h''(x) = (x^2 + x - 1 + 2x + 1)\, e^x$

$= (x^2 + 3x)\, e^x$

$h'(x) = 0 \Rightarrow x^2 + x - 1 = 0$

$\Rightarrow x = \frac{-\sqrt{5}-1}{2} \notin (0,1)$; $\frac{\sqrt{5}-1}{2} \in (0, 1)$

at $x = \frac{\sqrt{5}-1}{2}$, $h''(x) > 0$ so it is minimum of h(x)

at $x = \frac{\sqrt{5}-1}{2}$, $\frac{x^2-x}{2}e^x$ is negative

and $f(0) = f(1) = 0$

$\Rightarrow f(x) < 0\ \forall x \in (0, 1)$

50. $e^{-x} f(x)$ has its minimum at $\frac{1}{4}$ in [0, 1]

$\therefore \left(0, \frac{1}{4}\right)$, $e^{-x}\ [f'(x) - f(x)] < 0$

& in $\left(\frac{1}{4}, \frac{3}{4}\right)$, $e^{-x}\ [f'(x) - f(x)] > 0$

Since $e^{-x} > 0$

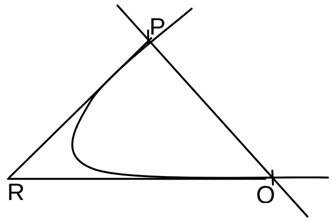
$f(x) - f(x) < 0$ in $\left(0, \frac{1}{4}\right)$ (C)

51. Since tangents at the extremities of focal chord intersect at the direction of $8^2 = 4ax$, the x – coordinate of point of intersection is a, this point lies on y = 2x + a

$\Rightarrow y = -a$

$\therefore$ Equation of PQ is $y(-a) = 2a(x - a)$

$\Rightarrow 2x + y = 2a$ _______ (1)



Area f $\Delta PQR = \frac{\left(y_1^2 - 4ax_1\right)^{\frac{3}{2}}}{2a}$

$= \frac{5\sqrt{5}a^2}{2}$

Perpendicular distance (–a, a) to 2x + y = 2a is $\sqrt{5}\,a$

$\therefore \frac{1}{2}\ PQ \times \sqrt{5}\,a = \frac{5\sqrt{5}a^2}{2} \Rightarrow PQ = 5a$

52. The line PQ is 2x + y = 2a

$\therefore$ equation of the pair of lines joining vertex to P & Q is

$2ay^2 = 4ax\,(2x + y)$

i.e., $4x^2 + 2xy - y^2 = 0$

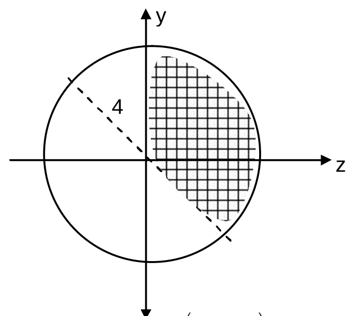
If θ is the angle, $\tan\theta = \pm\frac{2\sqrt{1+4}}{4-1}$

$= \pm\frac{2\sqrt{5}}{3}$

But θ is obtuse

$\therefore \tan\theta = -\frac{2\sqrt{5}}{3}$

53.



$\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{x-1+i\left(y+\sqrt{3}\right)}{1-i\sqrt{3}}$

$= \frac{\left[x-1+i\left(y+\sqrt{3}\right)\right]\left[1+i\sqrt{3}\right]}{1+3}$

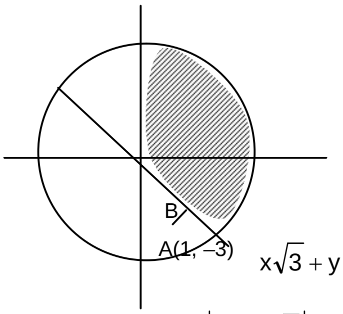
$= \frac{1+i\left[y+\sqrt{3}+x\sqrt{3}-\sqrt{3}\right]}{4}$

$\therefore\ y + x\sqrt{3} > 0$

Required area $= 8\pi - \frac{1}{2}\times 16\times\frac{\pi}{6}$

$= 8\pi\left(1-\frac{1}{6}\right) = \frac{40\pi}{6} = \frac{20\pi}{3}$

54.



$x\sqrt{3} + y = 0$

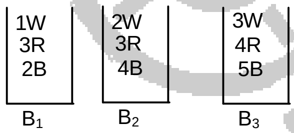
Min |1–3i–3| = AB $= \left|\frac{-3+\sqrt{3}}{2}\right| = \frac{3-\sqrt{3}}{2}$

55. Required probability

$= \frac{1.2.3 + 3.3.4 + 2.4.5}{6.9.12}$

$= \frac{82}{648}$

56.



Required probability

$$= \frac{\frac{1}{3}\left(\frac{2c_1.3c_1}{pc_2}\right)}{\frac{1}{3}\left(\frac{1c_1.3c_1}{6c_2}\right)+\frac{1}{3}\left(\frac{2c_1.3c_1}{9c_2}\right)+\frac{1}{3}\left(\frac{3c_1.4c_1}{12c_2}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{55}{181}$$

Section III

57. P: [abc] = 2

$[a \times b \quad b \times c \quad c \times a] = [abc]^2$

$\therefore$ [2(a × b) 3 (b × c) (c × a)]

$= 6[abc]^2$

= 6 × 4 = 24

$\therefore$ choice (3)

Q: [a + b b + c c + a] = 2 [abc]

$\therefore$ [3(a + b) b + c (c + a)] = 12[abc]

But [abc] = 5

$\therefore$ 12 × 5 = 60

$\therefore$ choice (4)

R: $\frac{1}{2}$ (a × b) = 20

$\therefore\ \frac{1}{2}$ ((2a + 3b) × (a – b)) = $\frac{1}{2}$ (5 (a × b))

= 5 × 20

= 100

choice (1)

S: a × b = 30

$\therefore$ Area = (a + b) × a = a × b = 30

$\therefore$ choice (2)

$\therefore$ code (c)

58. Gen point L_1 : $(2\lambda + 1, -\lambda, \lambda - 3)$

h_2: $(\mu + 4, \mu - 3, 2\mu - 3)$

for sime λ and μ

$\Rightarrow 2\lambda + 1 = \mu + 4$ and $-\lambda = \mu - 3 \quad \lambda - 3 = 2\mu - 3$

$$\Rightarrow \begin{array}{l} 2\lambda - \mu \ = 3 \\ \lambda + \mu \ = 3 \\ \hline 3\lambda \qquad = 6 \end{array} \Rightarrow \lambda = 2$$

$\therefore\ \mu = 1$

$\therefore$ Point of intersection (5, –2, –1)

$$\begin{array}{l} 7a + b + 2c \ = 0 \\ 3a + 5b - 6c \ = 0 \\ \hline \end{array}$$

$\frac{a}{-6-10} = \frac{b}{6+42} = \frac{c}{35-3}$

$\frac{a}{-16} = \frac{b}{48} = \frac{c}{32}$

$\frac{a}{-1} = \frac{b}{3} = \frac{c}{2}$

$\therefore$ <a, b, c> = <1, –3, –2>

$\therefore$ a = 1, b = –3, c = –2

$\therefore$ d = 13

59. P:

$$\left(\frac{1}{y^2}\left(\frac{\cos\cos^{-1}\left(\frac{1}{\sqrt{1+y^2}}\right)+y\sin\sin^{-1}\frac{y}{\sqrt{1+y^2}}}{\cot\cot\left(\frac{\sqrt{1-y^2}}{y}\right)+\tan\tan^{-1}\left(\frac{y}{\sqrt{1-y^2}}\right)}\right)^2_{\frac{1}{2}}+y^4\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{y^2}\left(\frac{\frac{1+y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right)^2+y^4\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}.y\sqrt{1-y^2}}{1-y^2+y^2}\right)^2+y^2\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{y^2}\left(y^2\left(1-y^4\right)+y^4\right)\right)^{\frac{1}{2}}$$

$= \left(1-y^4+y^4\right)^{\frac{1}{2}} = 1^{\frac{1}{2}} = 1$

Choice (4)

S: $\cot\cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sin\sin^{-1}\left(\frac{x\sqrt{6}}{\sqrt{6x^2+1}}\right)$

$\frac{x}{\sqrt{1-x^2}} = \frac{\sqrt{6}x}{\sqrt{6x^2+1}}$

$\frac{x^2}{1-x^2} = \frac{6x^2}{6x^2+1}$

$\Rightarrow 12x^4 = 5x^2$

$\Rightarrow x^2 = \frac{5}{12} \Rightarrow \therefore x = \frac{\sqrt{5}}{2\sqrt{3}}$

Q: cosx + cosy = – cosz _____ (1)

Sinx + siny = – sinz _____ (2)

Squaring and adding (1) and (2)

We get 1 + 2 cos (x – y) = 0

$\Rightarrow 4\cos^2\left(\frac{x-y}{2}\right) = 1$

$\Rightarrow \cos\left(\frac{x-y}{2}\right) = \frac{1}{2}$

R: $\cos\left(\frac{\pi}{4}-x\right)$. cos2x + 2sin²x

$= \sin 2x + \cos 2x \cos\frac{\pi}{4} x$

$\cos 2x\left[2\sin x.\frac{1}{\sqrt{2}}\right] = 2\sin^2 x + \sin^2 x$

$\sqrt{2}\ \sin x . \cos 2x = \sin 2x + 2\sin^2 x$

$\sqrt{2}\ \cos 2x = 2\cos x + 2\sin x$

$\cos 2x = \cos\left(x+\frac{\pi}{4}\right)$

$\Rightarrow x = \frac{\pi}{4} \Rightarrow \sec x = \sqrt{2}$

60. E is (0, 3)

F is (x_0, y_0),

G is $(0, y_1)$

where $y_0^2 = 16x_0$, $y_0 = mx_0 + 3$

and $(0, y_1)$ satisfies $yy_0 = 8(x + x_0)$

so that $y_0 y_1 = 8x0$

Area, A = area of Δ with vertices

$(0, 0), (x_0, y_0 - 3), (0, y - 3)$

$= \frac{1}{2}\left[x_0(y_1-2)\right]$

$= \frac{1}{2}\left[x_0\left(\frac{8x_0}{4\sqrt{x_0}} - 3\right)\right]$

$= \frac{1}{2}\left[2x_0^{\frac{3}{2}} - 3x_0\right]$

$\frac{dA}{dx_0} = 0 \Rightarrow \frac{3}{2}\sqrt{x_0} - \frac{3}{2} = 0 \Rightarrow x_0 = 1$

$\Rightarrow y_0 = 4\ (y_0 > 0)$

$\Rightarrow m = 1$

$\Rightarrow y = 2$

and them area $= \frac{1}{2}$

So Ans (A)