T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2015

Paper I – Code 0

PART I

1	2	3	4	5	6	7	8
7	**3**	**7**	**2**	**2**	**3**	**6**	**2**

9	10	11	12	13
C	**B**	**A, B, C**	**A, B, D**	**B**

14	15	16	17	18
A, C	**A, C**	**A, C, D**	**B, D**	**D**

19	20
A – R, T	**A – P, Q, R, T**
B – P, S	**B – Q, S**
C – Q, R, T	**C – P, Q, R, S**
D – Q, R, T	**D – P, R, T**

Section I

1. **Case – 1**

Mirror

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow -\frac{1}{15}+\frac{1}{v}=\frac{1}{10}$

$\Rightarrow v = -30$

$m_1 = -\frac{v}{u} = -2$

Lens

u = 50 – 30 = 20. f = 10

$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow v = 20$

$m_2 = \frac{v}{u} = \frac{20}{-20} = -1$

$M_1 = m_1m_2 = -2 \times -1 = 2$

Case – 2

$\frac{1}{10} = 0.5\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f_\ell} = \left(\frac{1.5\times 6}{7}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Dividing and simplify $f_\ell = 17.5$

$\frac{1}{v'} = \frac{1}{17.5}-\frac{1}{20} \Rightarrow v' = 140$

$m_2' = \frac{140}{-20} = -7$

$M_2 = m_1m_2' = 1 \times -7 = -7$

$\frac{M_2}{M_1} = \frac{m_1m_2'}{m_1m_2} = \frac{m_2'}{m_2} = \frac{7}{1} = 7$

2. $S_1 = \sqrt{x^2+d^2}$

$S_2 = \mu\sqrt{x^2+d^2}$

$\Delta S = S_1 - S_2 = (\mu - 1)(x^2 + d^2)^{1/2} = m\lambda$

$x^2 + d^2 = \frac{m^2\lambda^2}{(\mu-1)^2}$

$x^2 = \frac{m^2\lambda^2}{(\mu-1)^2} - d^2$

$p^2 = \frac{1}{(\mu-1)^2} \Rightarrow p = \frac{1}{\mu-1} = \frac{1}{\frac{4}{3}-1} = 3$

3. $\frac{1}{2}mv_1^2\left(1+\frac{k^2}{R^2}\right)+m\times10\times30$

$= \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

$\frac{1}{2}mv_2^2\left(1+\frac{k^2}{R^2}\right)+m\times10\times27$

$= \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

$\frac{1}{2}v_1^2\left(1+\frac{k^2}{R^2}\right)+300 = \frac{1}{2}v_2^2\left(1+\frac{k^2}{R^2}\right)\times270$

$\frac{3}{4}v_1^2+300 = \frac{3}{4}v_2^2+270$

$30 = \frac{3}{4}\left(v_2^2-v_1^2\right)$

$40 - v_2^2 - v_1^2 = v_2^2 - 3^2$

$v_2 = \sqrt{49} = 7$

4. $-\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{R+h}$

$u = \frac{v_e}{n}$; $h = \frac{R}{n^2-1}$

$g = \frac{GM}{R^2}$

$g' = \frac{GM}{(R+h)^2} = \frac{g}{4} \Rightarrow R + h = 2R \Rightarrow h = R$

$\frac{R}{n^2-1} = R \Rightarrow n^2 - 1 = 1 \quad n^2 = 2 , \; n = \sqrt{2}$

n = 2

5. $P_A = E_A A_A = \sigma T_A^4 . r_A^2$

$P_B = E_B A_B = \sigma T_B^4 . r_B^2$

$\frac{P_A}{P_B} = 10^4 = \left(\frac{T_A}{T_B}\right)^4 (400)^2$

$1 = \left(\frac{T_A}{T_B}\right)^4 \times 2^4$

$1 = \frac{T_A}{T_B}\times2 \Rightarrow \frac{T_A}{T_B} = \frac{1}{2}$

$\frac{\lambda_B}{\lambda_A} = \frac{1}{2} \Rightarrow \frac{\lambda_A}{\lambda_B} = 2$

6. $\frac{P_A}{P_R} = \frac{100}{12.5} = 8 = 2^{t/T} = 2^n \Rightarrow n = 3$

7. $\bar{E} = \frac{\lambda}{2\pi\varepsilon_0 r}.\hat{r} = \frac{\lambda}{2\pi\varepsilon_0 r^2}.\bar{r}$

At any x, $r^2 = x^2 + z^2 = x^2 + \frac{3}{4}a^2$

and $\bar{r} = x\hat{i} - \frac{\sqrt{3}}{2}a\hat{k}$

Take elemental area at x, width dx, breadth L.

$d\phi_E = \frac{\lambda}{2\pi\varepsilon_0\left(x^2+\frac{3}{4}a^2\right)}.\frac{\sqrt{3}a}{2}(Ldx)$

(only $\hat{k}$ component gives flux)

Integrating between $x = -\frac{a}{2}$ to $x = +\frac{a}{2}$

$\phi_E = \frac{\lambda}{2\pi\varepsilon_0}.\frac{\sqrt{3}}{2}aL.\frac{2}{\sqrt{3}a}.\frac{\tan^{-1}2x}{\sqrt{3}a}\Bigg|_{-a/2}^{+a/2}$

$= \frac{\lambda}{2\pi\varepsilon_0}.L.\frac{\pi}{3} = \frac{\lambda L}{6\varepsilon_0}$

$\Rightarrow n = 6$

8. $E_P = \frac{1240}{90} = 13.8$ eV

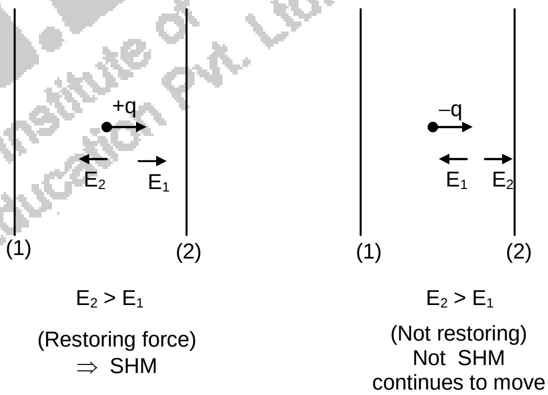
$KE_e = 10.4$ eV

Ionisation energy = 13.8 – 10.4 = 3.4 eV = $\frac{13.6}{n^2}$

$n^2 = 4 \Rightarrow n = 2$

Section II

9.



10. For S_1 $\quad \frac{1}{v} - \frac{1.5}{-50} = \frac{(1-1.5)}{-10}$

$\frac{1}{v} = -\frac{1.5}{50} + \frac{0.5}{10} = \frac{-1.5+2.5}{50} = \frac{1}{50}$

v = 50 cm

For S_2 $\quad \frac{1.5}{\infty} - \frac{1}{-u} = \frac{1.5-1}{+10}$

$\frac{1}{u} = \frac{0.5}{10}$ $\qquad$ u = 20 cm

Assuming d > 50 cm

d = 50 + 20 = 70 cm

No answer of d is less than 50 cm.

Hence object for S_2 is not virtual.

11. dF = $i\overline{d\ell} = i\overline{d\ell}\times\overline{B}$

F = $\left[\int i\overline{d\ell}\right]\times\overline{B}$

(A) F = i[L + R + R + L] × B × sin90°

⇒ F ∝ (L + R)

(A) correct, (D) wrong.

(B) $i\overline{d\ell}\times\overline{B} = 0$ ⇒ (B) correct

(C) F = i[L + R + R + L] × B × sin90°

⇒ F ∝ (L + R) (C) correct

12. $\dfrac{\frac{5}{2}RT + \frac{3}{2}RT}{2} = 2RT$ → A is correct

$C_{mix} = \sqrt{\dfrac{1.5RT}{3}} = \sqrt{\dfrac{RT}{2}}$ $\quad C_V = 2R$

$C_{He} = \sqrt{\dfrac{5}{3}\dfrac{RT}{4}} = \sqrt{\dfrac{5RT}{12}}$ $\quad C_p = 3R$

$\gamma = 1.5$

$\dfrac{C_{mix}}{C_{He}} = \sqrt{\dfrac{RT12}{2\times 5RT}} = \sqrt{\dfrac{6}{5}}$ → B is correct

$(v_{rms})_{He} = \sqrt{\dfrac{3RT}{4}}$

$(v_{rms})_H = \sqrt{\dfrac{3RT}{2}}$

∴ $\dfrac{V_{He}}{v_H} = \sqrt{\dfrac{3RT}{5}\times\dfrac{2}{3RT}} = \dfrac{1}{\sqrt{2}}$ → D is correct

13. 1 → $A\ell$ $\quad$ 2 → Fe

$R_1 = \dfrac{\rho\ell}{A} = \dfrac{2.7\times10^{-8}\times50\times10^{-3}}{(49-4)\times10^{-6}}$

$= \dfrac{2.7\times50}{45}\times10^{-5} = \dfrac{27\times50}{45}\times10^{-6}\ \Omega$

$= 30\ \mu\Omega$

$R_2 = \dfrac{1\times10^{-7}\times50\times10^{-3}}{4\times10^{-6}} = \dfrac{50}{4}\times10^{-4}\times\dfrac{10^{-2}}{10^{-2}}$

$= \dfrac{5000}{4}\ \mu\Omega$

$R_{eq} = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{30\times\frac{5000}{4}}{30+\frac{5000}{4}}$

$= \dfrac{30\times5000}{5120} = \dfrac{15000}{512}$

Answer upto 1 significant figure = 30

14. $eV_0 = \dfrac{hc}{\lambda} - \phi$

$V_0 = \left(\dfrac{hc}{e}\right)\times\dfrac{1}{\lambda} - \dfrac{\phi}{e}$

V_0 v/s $\dfrac{1}{\lambda}$ ⇒ +ve slope

–ve intercept

(C) Correct

$\left(V_0 + \dfrac{\phi}{e}\right)\lambda = \dfrac{hc}{e}$ → Rectangular hyperbola

At $\lambda = 0$, $\quad V_0 = \infty$

At $V_0 = 0$, λ is +ve

(A) correct.

15. Vernier

1 div of MS = $\dfrac{1}{8}$ cm

5 VSD = 4 MSD = $4\times\dfrac{1}{8} = \dfrac{1}{2}$ cm

1 VSD = $\dfrac{1}{2}\times\dfrac{1}{5} = \dfrac{1}{10}$ cm

LC = 1 MSD – 1 VSD = $\dfrac{1}{8} - \dfrac{1}{10} = \dfrac{1}{4}$ mm

= $\dfrac{1}{40}$ cm

Screw Gauge

CSD = 100

100 CSD = 2 PSD

LC = 1 CSD = $\dfrac{1}{50}$ PSD

LC = $\dfrac{1}{50}\times2\times\dfrac{1}{40} = \dfrac{1}{1000}$ cm = 0.01 mm

A is correct

C is also correct

16. $[L] = [h]^x\,[C]^y\,[G]^z$

$= [ML^2T^{-1}]^x\,[LT^{-1}]^y$

$[M^{-1}L^3T^{-2}]^z$

x – z = 0 $\quad$ x = z

2x + y + 3z = 1 $\quad$ 2x – 3x + 3x = 1

–x – y – 2z = 0 $\quad$ y = –3x

x = $\dfrac{1}{2}$, y = $-\dfrac{3}{2}$, z = $\dfrac{1}{2}$

$[L] = h^{1/2}\,C^{-3/2}\,G^{1/2}$

(C) & (D) correct

For [M] $\quad$ x – z = 1

2x + y + 3z = 0

–x – y – 2z = 0

----

x + z = 0

x – z = 1

2x = 1

x = $\dfrac{1}{2}$

z = $-\dfrac{1}{2}$

y = –x – 2z

$= -\frac{1}{2} + 2\times\frac{1}{2} = \frac{1}{2}$

$[M] = h^{1/2} C^{1/2} G^{-1/2}$

(A) correct (B) wrong

17. $\frac{a}{b} = n^2$ $\quad \frac{a}{R} = n$ $\quad$ Let $m_1 = m_2$

$b = \frac{a}{n^2}$ $\quad R = \frac{a}{n}$ $\quad = m = 1$ kg

$A_1^2 = a^2 \Rightarrow A_1 = a$ $\quad A_2^2 = R^2 \Rightarrow A_2 = R$

$A_1^2\omega_1^2 = b^2 \Rightarrow A_1\omega_1 = b$ $\quad A_2^2\omega_2^2 = R^2$

$\omega_1 = \frac{b}{a} = \frac{1}{n^2}$ $\quad \Rightarrow A_2\omega_2 = R$

$\omega_2 = 1$

$A_2 = \frac{a}{n}$

$E_1 = \frac{1}{2}\times\omega_1^2 A_1^2 = \frac{1}{2}\times\frac{1}{n^4}\times a^2$

$E_2 = \frac{1}{2}\times\omega_2^2\times A_2^2 = \frac{1}{2}\times 1\times\frac{a^2}{n^2}$

$E_1\omega_1 = \frac{a^2}{n^4}\times\frac{1}{n^2} = \frac{a^2}{n^6}$

$E_2\omega_2 = \frac{a^2}{n^2}\times\frac{a^2}{n^2} = \frac{a^4}{n^4}$

$\Rightarrow$ (A) wrong

$\frac{\omega_2}{\omega_1} = \frac{1}{\frac{1}{n^2}} = n^2 \Rightarrow$ (B) correct

$\omega_1\omega_2 = \frac{1}{n^2}\times 1 = \frac{1}{n^2}$ $\quad$ (C) wrong

$\frac{E_1}{\omega_1} = \frac{a^2}{n^4\times\frac{1}{n^2}} = \frac{a^2}{n^2}$

$\frac{E_2}{\omega_2} = \frac{a^2}{n^2\times 1} = \frac{a^2}{n^2}$

$\Rightarrow$ (D) correct

18. Conservation of angular momentum

$MR^2\times\omega = \left[MR^2 + \frac{M}{8}\times\frac{9}{25}R^2 + \frac{M}{8}\times d^2\right]\times\frac{8}{9}\omega$

$R^2 = \left[R^2 + \frac{9R^2}{200} + \frac{d^2}{8}\right]\times\frac{8}{9}$

$\frac{9}{8}R^2 - R^2 - \frac{9R^2}{200} = \frac{d^2}{8}$

$\frac{1800R^2 - 1600R^2 - 72R^2}{8\times 200} = \frac{d^2}{8}$

$\frac{128}{200}R^2 = d^2 = \frac{64}{100}R^2$

$d = \frac{8}{10}R$

$= \frac{4}{5}R$

Section III

19. A : R, T (In some fusion reaction positrons are emitted)

B: P, S

C: Q, R, T (P not correct as only ${}^{238}_{92}U$ absorbs neutron and produces β decay.

D: Q, R, T

20. A: P, Q, R, T

B: Q, S

C: P, Q, R, S

D: P, R, T

Using $F = -\frac{dU}{dx}$

(A) $F = -\frac{U_0}{2}.2.\left[1-\frac{x^2}{a^2}\right]\left[-\frac{2x}{a^2}\right]$

$= 0$ at $x = \pm a$ and at $x = 0$

$\therefore$ P, Q, R, T

$F \propto x$

$\therefore$ S, T are not true

$\therefore$ A $\rightarrow$ P, Q, R

(B) $F = -\frac{U_0}{2}.\frac{2x}{a^2} = -\frac{U_0 x}{a^2} = 0$ at $x = 0$; Hence Q

$F \propto -x$ $\therefore$ S true but T not true

$\therefore$ B $\rightarrow$ Q and S

(C) $F = -\frac{U_0}{2}\left[\frac{x^2}{a^2}.e^{\frac{-x^2}{a^2}}(-2x) + \frac{2x}{a^2}.e^{-x^2/a^2}\right]$

$= -\frac{U_0}{2}.e^{-x^2/a^2}.\frac{2x}{a^2}\left[1-\frac{x^2}{a^2}\right]$

$= -\frac{U_0}{a^2}.e^{-x^2/a^2}x.\left(1-\frac{x^2}{a^2}\right)$

$= 0$ at $x = \pm a$ and at $x = 0$

$\therefore$ P, Q, R

Since F is negatively proportional to x for $|x| < a$, S is true.

(D) $F = -\frac{U_0}{2}\left[\frac{1}{a} - \frac{x^2}{a^3}\right] = -\frac{U_0}{2a}\left(\frac{1-x^2}{a^2}\right)$

$= 0$ at $x = \pm a$

Hence P, R

Also, F is negative for $|x| < a$, it is attractive only for $0 < x < a$.

Hence S not true.

Also T is true

PART II

21	22	23	24	25	26	27	28
1	**4**	**4**	**8**	**4**	**3**	**2**	**9**

29	30	31	32	33
B	**A, B, C**	**B, C, D**	**C, D**	**B**

34	35	36	37	38
A	**B, D**	**A**	**D**	**A**

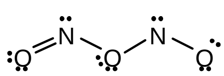
39	40
A – P, Q, S **B – T** **C – Q, R** **D – R**	**A – R, T** **B – P, Q, S** **C – P, Q, S** **D – P, Q, S, T**

Section I

21. $\Delta Tf = i \times K_f \times m$
    $i = \frac{0.0558}{1.86 \times 0.01} = 3$
    $[Co(NH_3)_5Cl]Cl_2$

22. $M^+ \rightarrow M^{3+} + 2e^-$, $E^\circ = -0.25$ v
    $\Delta G^\circ = -nFE^\circ$
    $= -2 \times 96500 \times -0.25$ J
    $= 48.25$ kJ
    No. of moles of M^+ oxidised by
    193 kJ = $\frac{193}{48.25} = 4$

23. $[Fe(SCN)_6]^{-3}$ the no. of unpaired electron is 5
    $\therefore \mu_{BM} = 5.9$
    $[Fe(CN)_6]^{-3}$. The no. of unpaired electron = 1
    $\mu = 1.79$
    Difference in BM = 4

24. 

    No. of lone pairs = 8

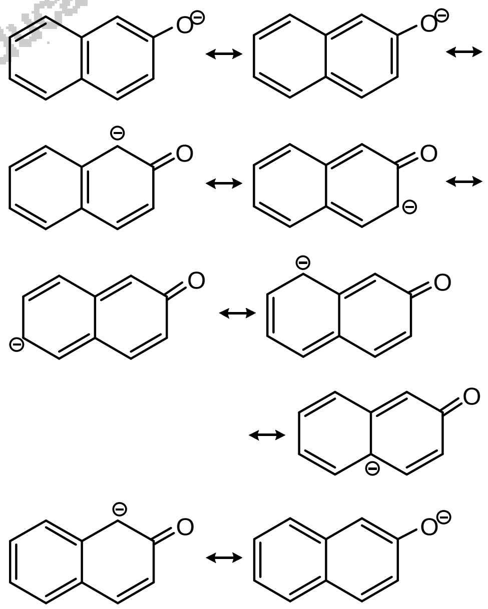
25.

$BeCl_2$	sp	linear
N_3^-	sp	linear
N_2O	sp	linear
NO_2^+	sp	linear
O_3	sp	bent
SCl_2	sp^3	bent
ICl_2^-	sp^3d	linear
I_3^-	sp^3d	linear
XeF_2	sp^3d	linear

26. Energy of an orbital of a multielectronic system is decided by (n+ℓ) value. Ground state configuration of H^- is $1s^2$. Second excited state is 2p orbital. Degeneracy of a p orbital is 3

27. The given structure is that of camphor. There are only two optical isomers for camphor although it has two chiral carbon atoms. The trans pair of enantiomers is impossible in this case because the bridge must be cis

28. The different structures are



Section II

29. Inversion of configuration occurs

30. (a) Cr^{+2} is a reducing agent tends to become Cr^{+3}
    (b) Mn^{+3} is oxidising, tends to become Mn^{+2}
    (c) Both Cr^{+2} and Mn^{+3} are d^4
    (d) Wrong

31. (b) Acidified $CuSO_4$
    (c) Pure Cu at cathode
    (d) Impurities settle as anode mud

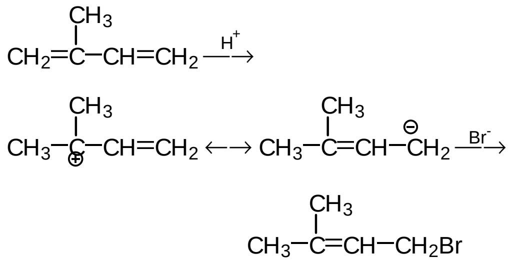
32. In alkaline medium Fe^{2+} (as $Fe(OH)_2$) is oxidised to Fe^{3+} (as $Fe(OH)_3$ precipitate)

33. $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$, $\Delta H < O$
    It is an exothermic reaction. The initial rate increases with increase of temperature but the final yield will be less at high temperature

34. O^{2-} forms ccp
    No. of O^{2-} per unit cell = 4
    $\therefore$ Formula is MgO Al_2O_3
    No. of octahedral voids = 4
    $4 \times \boxed{\frac{1}{2}}$ = No. of Al^{3+} ions
    No. of tetrahedral voids = 8
    $8 \times \boxed{\frac{1}{8}}$ = No. of Mg^{2+} ions

35. The produces hydrogenation of B and D do not contain chiral carbon atoms

36. Intramolecular aldol condensation occurs

37. 

38. L(–) glucose is the mirror image of D(+) glucose

Section III

39. (a) $\rightarrow$ p, q, s
    (b) $\rightarrow$ t
    (c) $\rightarrow$ q, r
    (d) $\rightarrow$ r

    Siderite – $FeCO_3$
    Malachite – $Cu(OH)_2.CuCO_3$
    Bauxite – $AlO_x(OH)_{3-2x}$
    Calamine – $ZnCO_3$
    Argentite – Ag_2S

40. (a) $\rightarrow$ r, t, s
    (b) $\rightarrow$ p, q, s
    (c) $\rightarrow$ p, q, s
    (d) $\rightarrow$ p, q, s, t
    (a) Freezing is accompanied with decrease of entropy. At the freezing point the process is at equilibrium
    (b) The system is isolated, q = 0
    expansion is against vacuum, w = 0
    $\therefore \Delta U = 0$
    (c) Mixing of equal volume of two ideal gases, q = 0 and w = 0
    (d) Cyclic reversible process, q = 0, w = 0, $\Delta U = 0$ and $\Delta G = 0$

PART III

41	42	43	44	45	46	47	48
3	**4**	**6**	**8**	**2**	**0**	**8**	**4**

49	50	51	52	53
A, D	**A, C**	**B, C**	**A, D**	**A, B, C**

54	55	56	57	58
A, C, D	**C, D**	**B, C**	**B, D**	**A, B**

59	60
A – P, Q	**A – P, R, S**
B – P, Q	**B – P**
C – P, Q, S, T	**C – P, Q**
D – Q, T	**D – S, T**

Section I

41. Given

$$F(x) = \int_{x}^{x^2+\frac{\pi^2}{6}} 2\cos^2 t\ dt$$

$$F'(\alpha) + 2 = \int_{x}^{\alpha} f(x)\ dx$$

$F''(\alpha) = f(\alpha)$

$F''(0) = f(0)$ ———(1)

Now,

$$F'(x) = \left[2\cos^2\left(x^2 + \frac{\pi}{6}\right)\right]2x - 2\cos^2 x$$

$$F''(x) = 4x \times 2\cos x$$

$$\left(x^2 + \frac{\pi^2}{6}\right) \times \left[-\sin\left(x^2 + \frac{\pi^2}{6}\right)\right] \times 2x$$

$$+\left[2\cos^2\left(x^2 + \frac{\pi^2}{6}\right)\right] \times 2$$

$$-4\cos x \sin x$$

$$F''(0) = 4\cos^2\left(\frac{\pi^2}{6}\right)$$

$$= 4 \times \frac{3}{4} = 3$$

42. $V = \pi r^2 h$

V = volume of outside

$= \pi(v + 2)^2\ (h + 2)$

$(V^* - v) \rightarrow$ Volume of the container

$= \pi(r + 2)^2\ (h + 2) - \pi r^2 h$

$= \pi(r^2 + 4 + 4r)\ (h + 2) - \pi r^2 h$

$= \pi\{2r^2 + 4h + 8 + 4vh + 8r\}$

$$\pi\left\{2r^2 + \frac{4v}{\pi r^2} + 8 + \frac{4rV}{\pi r^2} + 8r\right\}$$

$$\frac{d(V^* - v)}{dv} = 0 \quad \text{when } r = 10 \text{ (Given)}$$

$$\Rightarrow \left[\pi\left\{4v - \frac{8v}{\pi r^3} - \frac{4v}{\pi r^2} + 8\right\}\right] = 0$$

$$\Rightarrow 40 - \frac{8v}{1000\pi} - \frac{4v}{100\pi} + 8 = 0$$

$$\frac{8v}{1000\pi} + \frac{4v}{100\pi} = 48$$

$$\Rightarrow \frac{48}{1000\pi} = 48$$

$$\Rightarrow \frac{v}{250\pi} = 4$$

43. 5 boys & 5 girls

n ways $\rightarrow$ all girls consecutively. 5 girls may be arranged in 5! ways and they can stand consecutively in a queue of 10 people in 6 ways. 5 boys may be arranged in 5! ways.

$\therefore$ n = 5! $\times$ 5! $\times$ 6

m ways $\rightarrow$ exactly 4 girls consecutively. 4 girls out of 5 may be chosen in 5 ways and arranged in 4! ways. They can stand consecutively in 7 ways. 5 boys + 1 girls $\rightarrow 6!$.
But this includes the case "all 5 girls consecutive" in it.

$\therefore$ m = $(5 \times 4! \times 7 \times 6!) - (5! \times 5! \times 6)$

$= 5! \times 5! \times 36$

$\therefore \dfrac{m}{n} = \dfrac{5! \times 5! \times 36}{5! \times 5! \times 6} = 6$

44. P(Getting atleast two heads)
= 1 – P(getting no heads + getting exactly 1 head)
Let E be the required event and let the number of tosses be 2, then

$P(E) = 1 - \left(\left(\frac{1}{2}\right)^2 + {}^2C_1\left(\frac{1}{2}\right)^2\right) = 1 - \left(\frac{1}{2}\right)^3 (3)$

Let the number of tosses be 3, then

$P(E) = 1 - \left(\left(\frac{1}{2}\right)^3 + {}^3C_2\left(\frac{1}{2}\right)^3\right) = 1 - \left(\frac{1}{2}\right)^3 (3+1)$

Let the number of tosses be 4, then

$P(E) = 1 - \left(\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4\right) = 1 - \left(\frac{1}{2}\right)^4 (4+1)$

$\therefore$ If the number of tosses in n, then

$P(E) = 1 - \left(\frac{1}{2}\right)^n (n+1)$

$\therefore 1 - \left(\frac{1}{2}\right)^n (1+n) > 0.96$

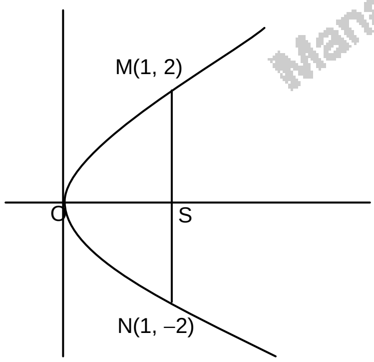
$0.04 > \left(\frac{1}{2}\right)^n (1+n)$

By trial, for n = 7, $\left(\frac{1}{2}\right)^7 (8) = 0.06$

for n = 8, $\left(\frac{1}{2}\right)^8 (9) = 0.035$

$\therefore$ n = 8 is the least value

45.



$y^2 = 4x$

$x = 1$

$2yy' = 4$

$y' = \dfrac{4}{2y}$

Slope of normal at M

$= -\dfrac{2y}{4}$ where y = 2

$= -1$

Equation of the normal at (1, 2) is

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

$x + y = 3$

x + y = 3 is a tangent to the circle

$(x - 3)^2 + (y + 2)^2 = r^2$

Put y = 3 – x in $(x - 3)^2 + (y + 2)^2 = r^2$

$(x - 3)^2 + (5 - x)^2 = r^2$

$2x^2 - 16x + (34 - r^2) = 0$

Discr = 0

$256 = 8(34 - r^2)$

$34 - r^2 = \dfrac{256}{8} = 32$

$r^2 = 2$

46. $f(x) = \begin{cases} -2, & -2 < x < -1 \\ -1, & -1 < x < 0 \\ 0, & 0 < x < 1 \\ 1, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$

$f(x^2) = \begin{cases} 0, & -1 < x < 0 \\ 0, & 0 < x < 1 \\ 1, & 1 < x < \sqrt{2} \\ 2, & \sqrt{2} < x < \sqrt{3} \\ 3, & \sqrt{3} < x < 2 \end{cases}$

$f(x+1) = [x+1] = 1 + [x_1] \begin{cases} -1 & -2 < x < -1 \\ 0 & -1 < x < 0 \\ 1 & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$

$I = \int_{-1}^{0} + \int_{0}^{1} + \int_{1}^{\sqrt{2}} + \int_{\sqrt{2}}^{\sqrt{3}} + \int_{\sqrt{3}}^{2}$

since f(x) = 0, x > 2,
3^{rd} and 4^{th} integrals are zero
Therefore,

$I = \int_{-1}^{0} \frac{x \times 0}{2+0} dx + \int_{0}^{1} \frac{x \times 0}{2+1} dx + \int_{1}^{\sqrt{2}} \frac{x \times 1}{2+0} dx$

$= \left(\frac{x^2}{4}\right)_1^{\sqrt{2}} = \frac{1}{4}$

$4I - 1 = 0$

47. $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x\cos^2 x + 1 - 3\sin^2 x\cos^2 x = 2$

$\frac{5}{4}\cos^2 2x - \frac{1}{2}\sin^2 2x - \frac{3}{4}\sin^2 2x + 2 = 2$

$\frac{5}{4}\cos^2 2x - \frac{5}{4}\sin^2 2x = 0$

$\frac{5}{4}\left(\cos^2 2x - \sin^2 2x\right) = 0$

$\frac{5}{4}\cos 4x = 0$

$\Rightarrow 4x = (2n + 1)\ \frac{\pi}{2}$

$\Rightarrow x = (2n + 1)\ \frac{\pi}{8}$

$\therefore$ Number solution = 8

48. $y^2 = 4x$
$x_1 = t^2$, $y_1 = 2t$
Let (x_2, y_2) denote the image of (x_1, y_1)
in $x + y + 4 = 0$
Then,

$\frac{x_2 - t^2}{1} = \frac{y_2 - 2t}{1} = \frac{-2\left(t^2 + 2t + 4\right)}{2}$

$x_2 = t^2 - (t^2 + 2t + 4)$
$= -2t - 4$
$y_2 = 2t - (t^2 - 2t + 4)$
$= -t^2 - 4$

$t = \frac{x_2 + 4}{-2}$

$y_2 = -4 - \frac{(x_2 + 4)^2}{4}$

$4y_2 = -16 - (x_2 + 4)^2$
$4y_2 + 16 = -(x_2 + 4)^2$
Image curve is
$(x + 4)^2 = -4(y + 4)$
Put y = –5 in the above curve
$x + 4 = \pm 2$
$(x + 4)^2 = 4$
x = –6, –2
Difference = 4

Section II

49. $\overline{OP} \perp \overline{OQ}$

If P is $(2t^2, 2t)$, then Q is $\left(\frac{2}{t^2}, \frac{-2}{t}\right)$

$= \frac{1}{2}\sqrt{4t^2\left(1 + t^2\right)}\ \sqrt{\frac{4}{t^2}\left(1 + \frac{1}{t^2}\right)} = 3\sqrt{2}$

i.e. $2\ \frac{1 + t^2}{t} = 3\sqrt{2}$

$t^2 - \frac{3}{\sqrt{2}}t + 1 = 0$

$t = \frac{3}{2\sqrt{2}} \pm \frac{1}{2\sqrt{2}} = \sqrt{2}$ or $\frac{1}{\sqrt{2}}$

$\therefore$ P is $\left(4,\ 2\sqrt{2}\right)$ or $\left(1,\ \sqrt{2}\right)$

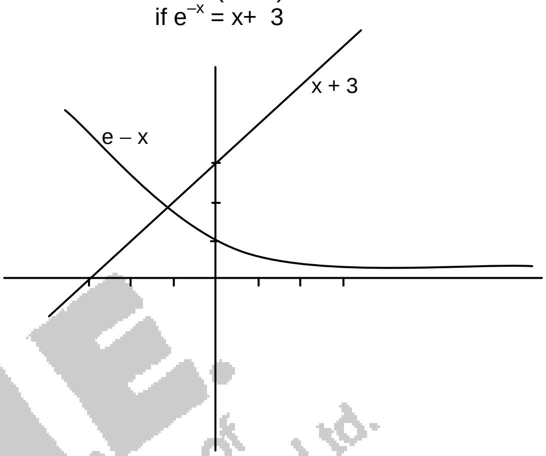
50. $(1 + e^x)\ y' + ye^x = 1$

$\frac{d}{dx}\ y\left(e^x + 1\right) = 1$

$\therefore y(e^x + 1) = x + C$
$y(0) = 0$
C = 4
$y(1 + e^x) = x + y$

$y = \frac{x + y}{1 + e^x}$

$y(-4) = 0,\ y(-2) \neq 0$
$y'(1 + e^x)^2 = 1 + e^x - (x + 4)\ e^x$
$= 1 - (x + 3)e^x = 0$
if $e^{-x} = x + 3$



When x = –1, e^{-x} e = 2.7
x + 3 = 2
x = 0 $e^{-x} = 1$
x + 3 = 3
x + 3 = e^{-2}
in (–1, 0)

51. If (λ, λ) is centre & R, the radius
equation is $(x - \lambda)^2\ (y - \lambda)^2 = R^2$
$\therefore x - \lambda + (y - \lambda)y' = 0$
$1 + (y - \lambda)y'' + y'^2 = 0$

$= \frac{x + yy'}{1 + y'}$

$1 + \left(y - \frac{x + yy'}{1 + y'}\right)y'' + y'^2 = 0$

$1 + y' + (y + yy' - x - yy')\ y'' + y'^2\ (1 + y') = 0$
$(y - x)y'' + 1 + y'(1 + y' + y'^2) = 0$
$\therefore P = y - x \quad Q = 1 + y' + y'^2$
$P + Q = 1 - x + y + y' + y'^2$

52. $g : R \to R$ defined with g(0) = 0, g'(0) = 0,
$g'(1) \neq 0$

$f(x) = \frac{x}{|x|}\ g(x),\ x \neq 0 \qquad h(x) = e^{|x|}$

f(0) = 0

$$\frac{f(x)-f(0)}{x} = \frac{\frac{x}{|x|}g(x)}{x} = \frac{g(x)}{|x|}$$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x} = \lim_{x\to 0}\frac{g(x)}{-x} = \lim_{x\to 0^-}\frac{g'(x)}{-1} = 0$$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x} = \lim_{x\to 0^-}\frac{g(x)}{x} = \lim_{x\to 0}\frac{g'(x)}{-1} = 0$$

∴ f'(0) exists (f'(0) = 0) (A) is true

$$\frac{h(x)-h(0)}{x} = \frac{e^{|x|}-1}{x}$$

$$\lim_{x\to 0^-}\frac{e^{|x|}-1}{x} = \lim_{x\to 0}\frac{e^{-x}-1}{x} = -1$$

$$\lim_{x\to 0^+}\frac{e^{|x|}-1}{x} = \lim_{x\to 0}\frac{e^{x}-1}{x} = 1$$

∴ h'(0) does not exist

$$(h\circ f)x = e^{|g(x)|}$$

$$\frac{(h\circ f)x(h\circ f)(0)}{x} = \frac{e^{|g(x)|}-1}{x}$$

$$= \frac{e^{|g(x)|}-1}{g(x)}\ \frac{g(x)}{x}$$

as $x \to 0$, R H S $\to$ 1 . 0 = 0

∴ $(h\circ f)$ is differentiable at 0

$$\frac{(f\circ h)(x)-(f\circ h)0}{x} = \frac{g(e^{|x|})-g(1)}{e^{|x|}-1}\cdot\frac{e^{|x|}-1}{x}$$

does not have lim at x = 0

∴ $f\circ h$ is not differentiable at 0, D is true

53. $(f\circ g)x = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$

$$-\frac{\pi}{2} \le \frac{\pi}{2}\sin x \le \frac{\pi}{2}$$

$$\Rightarrow\ -\frac{\pi}{2} \le \frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right) \le \frac{\pi}{2}$$

$$\Rightarrow\ -\frac{\pi}{6} \le \frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\sin x\right)\right) \le \frac{\pi}{6}$$

$$(f\circ g)x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$f(x) = \sin\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$$

$$\frac{-\pi}{6} \le \frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right) \le \frac{\pi}{6}$$

∴ range of f = $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$\lim_{x\to 0}\frac{f(x)}{g(x)} = \lim_{x\to 6}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{2}\sin x}$$

$$\lim_{x\to 6}\frac{\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)}{\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)}\ \frac{\pi}{6}\ \frac{\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x}$$

$$= 1 - \frac{\pi}{2}\ 1\ = \frac{\pi}{6}$$

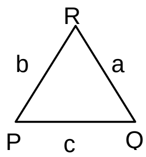
$$(g\circ f)x = \frac{\pi}{2}\sin\left(\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\right)$$

range is $\left[\frac{\pi}{2}\sin\left(\frac{-1}{2}\right), \frac{\pi}{2}\sin\left(\frac{1}{2}\right)\right]$

i.e. $\left[-\frac{\pi}{2}\sin\frac{-1}{2}, \frac{\pi}{2}\sin\frac{1}{2}\right]$

and 1 is not in this ways so D is not true

54. Since P, QR is a triangle



$$\bar{a}+\bar{b}+\bar{c} = 0$$

$$\bar{b}\bullet\bar{c} = 24$$

$$\bar{b}+\bar{c} = -\bar{a}$$

$$|\bar{b}|^2+|\bar{c}|^2+2\bar{b}.\ \bar{c} = |\bar{a}|^2$$

$$48 + |\bar{c}|^2 + 2\times 24 = 144$$

$$|c|^2\ = 48$$

(a) $\Rightarrow \frac{|c|^2}{2} - |a| = \frac{48}{2} - 12 = 12$

(b) $\Rightarrow \frac{|c|^2}{2} + |a| = \frac{48}{2} + 12 = 36 \ne 30$

(d) $\bar{a}+\bar{b} = -\bar{c}$

$|a|^2 + |b|^2 + 2\bar{a}.\bar{b} = |\bar{c}|^2$

144+ 48 + 2 a. b = 48

2 a. b = –144

a. b = –72

(c) $|b - c|^2 + |b + c|^2 = 2[|b|^2 + |c|^2]$

$\Rightarrow |b - c|^2 + |a|^2 = 2(|b|^2 + |c|^2)$

$\Rightarrow |b - c|^2 = 2(48 + 48) - 144$

$\because$ b + c = –a

$(a + b + c)^2 = |a|^2 + |b|^2 + |c|^2$

+ 2(a . b + b . c + c. a)

0 = 144 + 48 + 48 + 2(–72 + 24 + a. c)

$\Rightarrow$ a. c = 72

a. (b – c) = a. b – a. c = –72 +72 = 0

∴ angle between $\bar{a}$ and $(\bar{b}-\bar{c})$ = 90°

$|a\times b+\bar{c}\times\bar{a}| = |\bar{a}\times(b-\bar{c})|$ = |a| |b – c|sin90°

$= 12 \times 4\sqrt{3}$ .1

$= 48\sqrt{3}$

55. X, Y – Skew symmetric matrices
z – symmetric matrix
Option (A) $\Rightarrow (Y^3Z^4 - Z^4Y^3)^T$
$= (Y^3Z^4)^T - (Z^4Y^3)^T = (Z^4)^T(Y^3)^T$
$\quad -(Y^3)^T (Z^4)^T$
$= Y^3Z^4 - Z^4Y^3$
This is symmetric
Option (B) $\Rightarrow (X^{44} + Y^{44})^T$
$= (X^{44})^T + (Y^{44})^T$
Even power of a skew symmetric matrix will not be skew symmetric
Option (C) $\Rightarrow (X^4Z^3 - Z^3X^4)^T$
$= (X^4Z^3)^T - (Z^3 X^4)^T$
$= (Z^3)^T (X^4)^T - (X^4)^T (Z^3)^T$
$= Z^3 X^4 - X^4Z^3$
$= -(X^4Z^3 - Z^3 X^4)$ is skew symmetric
Option (D) $\Rightarrow (X^{23} + Y^{23})^T$ is skew symmetric

56. $\begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha+4\alpha^2 & 1+6\alpha+9\alpha^2 \\ 4+4\alpha+\alpha^2 & 4+8\alpha+4\alpha^2 & 4+12\alpha+9\alpha^2 \\ 9+6\alpha+\alpha^2 & 9+12\alpha+4\alpha^2 & 9+18\alpha+9\alpha^2 \end{vmatrix}$

$R_2 \to R_2 - R_1 \;\; R_3 \to R_3 - R_2$

$$\begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha+4\alpha^2 & 1+6\alpha+9\alpha^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix}$$

$R_2 \to R_3 - R_2$

$$\begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha+4\alpha^2 & 1+6\alpha+9\alpha^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix}$$

$$2\begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha+4\alpha^2 & 1+6\alpha+9\alpha^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 1 & 1 & 1 \end{vmatrix}$$

$$2\begin{vmatrix} 1+2\alpha+\alpha^2 & 2\alpha+3\alpha^2 & 4\alpha+8\alpha^2 \\ 3+2\alpha & 2\alpha & 4\alpha \\ 1 & 0 & 0 \end{vmatrix}$$

$2(8\alpha^2 + 12\alpha^3 - 8\alpha^2 - 16\alpha^3)$
$= -8\alpha^3 = -648\alpha$
$\alpha^2 = \frac{648}{8} = 81$
$\alpha = \pm 9$

57. P_3 is $x + \lambda y + z - 1 = 0$
$\therefore \frac{\lambda - 1}{\sqrt{2+\lambda^2}} = \pm 1$
$(\lambda - 1)^2 = 2 + \lambda^2$
$\Rightarrow \lambda = \frac{-1}{2}$
$\therefore P_3$ is $2x - y + 2z - 2 = 0$
$\frac{2\alpha - \beta + 2\gamma - 2}{3} = \pm 2$
$2\alpha - \beta + 2\gamma = 2 \pm 6$
i.e. $2\alpha - \beta + 2\gamma - 8 = 0$
or $2\alpha - \beta + 2\gamma + 4 = 0$

58. The line L is parallel to the intersection of P_1 & P_2 and passes through the origin
$\therefore$ L is $\frac{x}{-1} = \frac{y}{3} = \frac{z}{5}$
Any point on the line is $(-\lambda, 3\lambda, 5\lambda)$
M is given by
$$\frac{x+\lambda}{1} = \frac{y-3\lambda}{2} = \frac{z-5\lambda}{-1} = -\frac{(-\lambda+6\lambda-5\lambda+1)}{6} = \frac{-1}{6}$$
$x = -\lambda - \frac{1}{6}, \quad y = 3\lambda - \frac{1}{3}, \quad z = 5\lambda + \frac{1}{6}$
$\lambda = -\frac{1}{6}$ given (A)
$\lambda = 0$ given (B)
(c), (d) do not satisfy

Section III

59. (a) Project of $\alpha i + \beta j$ are $\sqrt{3}i + j = \pm\sqrt{3}$.
Also $\alpha = 2 + \sqrt{3}\beta \Rightarrow \beta = \frac{\alpha - 2}{\sqrt{3}}$
$\Rightarrow \frac{(\alpha i + \beta j).\left(\sqrt{3}i + j\right)}{\left|\sqrt{3}i + j\right|} = \sqrt{3}$
$\Rightarrow \frac{\sqrt{3}\alpha + \frac{\alpha - 2}{\sqrt{3}}}{2} = \pm\sqrt{3}$
$\Rightarrow \frac{4\alpha - 2}{2\sqrt{3}} = \pm\sqrt{3}$
$\Rightarrow 4\alpha - 2 = \pm 6$
$\Rightarrow 4\alpha = 8 \quad 4\alpha = -4$
$\Rightarrow \alpha = 2 \quad \alpha = -1$
$|\alpha| = 2, 1$
Option (p, q)

(b) $f(x) = -3ax^2 - 2 \quad x < 1$
$bx + a^2 = x \geq 1$
since x is differentiable at $x \in R$
since n is continuous
$-3a - 2 = b + a^2$ $\qquad -6ax = b$
$\Rightarrow a^2 - 3a + 2 = 0$ $\qquad x = \frac{-b}{6a}$
$\Rightarrow a = 1, 2$ $\qquad$ when x = 1
$6a = -b$
Option (p, q)

(c) Let $n = (3\omega^2 - 3\omega + 2)^{4x+3} \neq 0$ at x = 3
The given expression is
$\text{fn}\left[\frac{1}{\omega^{4n+3}} + 1 + \omega^{4n+3}\right] = 0$

$\therefore \text{fn}\left[\frac{1}{\omega^{4n}} + 1 + \omega^{4n}\right] = 0$

$\Rightarrow \text{fn}\left[\frac{1}{\omega^{n}} + 1 + \omega^{n}\right] = 0$

$\frac{1}{\omega^n} + 1 + \omega^n = 0$ for $n \neq 3$

$\therefore$ The given expression is zero when
n = 1, 2, 4, 5, since $f_n \neq 0$ at $n \neq 3$
Options p, q, s, t

(d) Let d be the common difference a, 5, q , b
$\therefore$ q – a = 2d, a + d = 5 and b = a + 3d

a, n, b are in H P $\Rightarrow 2 = \frac{ab}{a+b}$

$2(a + b) = ab \Rightarrow 2d^2 - 3d - 5 = 0$

$\therefore d = -1$ or $\frac{5}{2} \Rightarrow |2d| = 2$ or 5

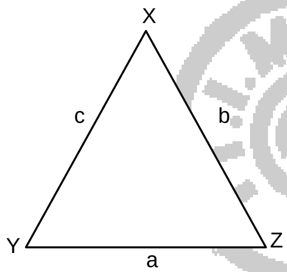
60. (a) $2(a^2 - b^2) = c^2 \Rightarrow 2(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow 2\sin(x + y) \sin(x - y) = \sin^2 z$

$\Rightarrow \frac{\sin(x-y)}{\sin z} = \frac{1}{2} \Rightarrow \cos\left(\frac{n\pi}{2}\right) = 0$

$\therefore$ n = 1, 3, 5

(b)



$1 + \cos 2x - 2\cos 2y = 2\sin x \sin y$
$1 + 1 - 2\sin^2 x - 2[1 - 2\sin^2 y] = 2\sin x \sin y$
$-2\sin^2 x + 4\sin^2 y = 2\sin x \sin y$
$-2a^2 + 4b^2 = +2ab$

$-a^2 + ab - 2b^2 = 0 \left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) - 2 = 0$

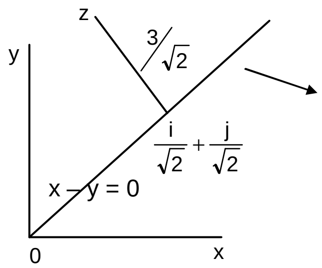
$\frac{a}{b} = -2$ or $+1 \Rightarrow \frac{a}{b} = 1$

(c) $\overline{X} = \sqrt{3}\,i + j$

$\overline{Y} = i + \sqrt{3}\,i$

$\overline{Z} = \beta i + (1-\beta)\bar{j}$

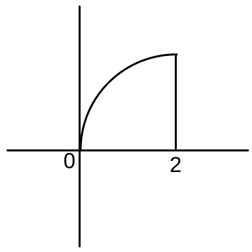
$\overline{Z} = (\beta,\ (1-\beta))$



$\frac{\beta - (1-\beta)}{\sqrt{2}} = \pm\frac{3}{\sqrt{2}}$

$2\beta - 1 = \pm 3$
$2\beta = 4$ or -2
$\beta = 2$ or -1
$|\beta| = 2$ or 1

(d)



For $y^2 = 4x$, $\int_0^2 y\,dx = \frac{8}{3}\sqrt{2}$

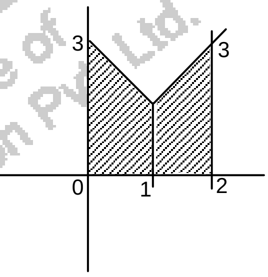
$\therefore F(\alpha) + \int_0^2 \frac{8}{3}\sqrt{2} = F(\alpha) + \int_0^2 y\,dx$

when $\alpha = 0$, y = 3,

$\therefore F(0) + \frac{8}{3}$ = Area bounded by y = 3

and x = axis between x = 0 and x = 2 is 6.
when $\alpha = 1$, $y = |x - 1| + |x - 2| + \alpha$

$y = \begin{cases} 3 - x & x < 1 \\ x + 1 & 1 < x < 2 \\ 3x - 3 & x > 2 \end{cases}$



$\therefore F(1) + \frac{8}{3}\sqrt{2} = 2 \times \frac{1}{2}[3 + 2] = 5$

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