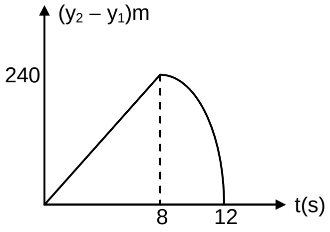
SOLUTIONS & ANSWERS FOR JEE MAINS-2015
VERSION – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

1. Two stones are thrown up simultaneously from the edge of a cliff ----

Ans:



Sol: $S_1 = u_1 t - \frac{1}{2}gt^2$

$S_2 = u_2 t - \frac{1}{2}gt^2$

$(S_2 - S_1) = (u_2 - u_1)t$ upto T = 8 s

From t = 8 s to t = 12 s,

$S = -40t - 5t^2$

T_1 for 1st stone is

$-240 = 10t - 5t^2$

$-5t^2 - 10t - 240 = 0$

$t = \frac{10 \pm \sqrt{100 + 4800}}{10} = 1 \pm 7 = 8$ s

T_2 for 2nd stone is $-240 = 40t - 5t^2$

$\rightarrow T = \frac{40 \pm \sqrt{1600 + 4800}}{12} = 12$ s

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is ----

Ans: 3%

Sol: $\frac{\Delta T}{T} + \frac{1}{2}\frac{\Delta L}{L} = \frac{1}{2}\frac{\Delta g}{g}$

$\frac{1}{90} + \frac{1}{2} \times \frac{1}{200} = \frac{1}{2}\frac{\Delta g}{g}$

$\left(\frac{1}{45} + \frac{1}{200}\right) \times 100 = \frac{\Delta g}{g} = 2.7\%$

$\cong 3\%$

3. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being ----

Ans: 120 N

Sol: $f_B = \omega t$ of A + ωt of B

= 20 + 100

= 120 N

4. A particle of mass m moving in the x-direction with speed 2v is hit by another particle of mass 2m ----

Ans: 56%

Sol: $\bar{p}_x = 2mv\hat{i}$

$\bar{p}_y = 2mv\hat{j}$

$\bar{v}_x = \frac{2mv}{3m} = \frac{2}{3}v\hat{i}$

$\bar{v}_y = \frac{2}{3}v\hat{j}$

$\bar{v} = \frac{2\sqrt{2}}{3}v\hat{n}$

$KE_1 = \frac{1}{2}m(2v)^2 + \frac{1}{2}2mv^2$

$= 3\ mv^2$

$KE_2 = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2$

$= \frac{1}{2}.3m.\frac{8}{9}v^2$

$= \frac{4}{3}mv^2$

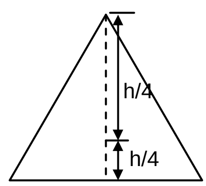
$= \frac{\Delta KE}{KE_1} = \frac{KE_2 - KE_1}{KE_1} = -\frac{5}{9} \times 100$

= 56%

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius ----

Ans: $\frac{3h}{4}$

Sol:



C.G of solid cone for aped = $\frac{3h}{4}$

6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment ----

Ans: $\frac{4}{9\sqrt{3}\pi}MR^2$

Sol: $M = \frac{4}{3}\pi R^3 \rho$

$\lambda = \frac{2R}{\sqrt{3}}$ (Θ 2R = body diagonal)

$M' = \frac{2M}{\sqrt{3}\pi}$

$I = \frac{M'L^2}{6} = \frac{M'}{6}\left(\frac{2R}{\sqrt{3}}\right)^2$

$= \frac{4MR^2}{9\sqrt{3}\pi}$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking ----

Ans: $\frac{-GM}{R}$

Sol: $M' = -\frac{M}{8}$

$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$

$= -\frac{GM}{2R}\left[\frac{11}{4}\right] = -\frac{11GM}{8R}$

$V_2 = -\frac{3GM'}{2R'} = -\frac{3}{2}\frac{G(mM)}{8.\frac{R}{2}}$

$= +\frac{3GM}{8R}$

$V = V_1 + V_2 = -\frac{11GM}{8R} + \frac{3GM}{8R} = -\frac{GM}{R}$

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added ----

Ans: $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

Sol: $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = 4\pi^2\frac{L}{g}$

$T_M = 2\pi\sqrt{\frac{L_1}{g}} \rightarrow T_M{}^2 = 4\pi^2\frac{L_1}{g}$

$\frac{T_M^2}{T^2} = \frac{L_1}{L} \rightarrow \left[\frac{T_M}{T}\right]^2 = \frac{L_1}{L}$

$\left(\frac{T_M}{T}\right)^2 - 1 = \frac{L_1 - L}{L} = \frac{\Delta L}{L} = \frac{Mg}{AY}$

$\Rightarrow \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

9. Consider a spherical shell of radius R at temperature T. The black body radiation ----

Ans: $T \propto \frac{1}{R}$

Sol: $\frac{U}{V} \propto T^4$

$p \propto \frac{U}{V} \Rightarrow p \propto T^4$

$\frac{nRT}{V} \propto T^4$

$\Rightarrow T^3 \propto \frac{1}{V} \propto \frac{1}{\frac{4}{3}\pi R^3}$

$T \propto \frac{1}{R}$

10. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in ----

Ans: λn2, λn2

Sol: dQ = msdT

Entropy $\frac{dQ}{T} = ms\frac{dT}{T}$

$\int \frac{dQ}{T} \int_{100}^{100} ms\frac{dT}{T}$ = msλn2 (nearest answer)

ms = 1

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an ----

Ans: $\frac{\gamma + 1}{2}$

Sol: $\tau = \frac{1}{\pi d^2 V_{avg} n}$

$n \propto \rho$ density

$V_{avg} \propto \sqrt{T}$

$\tau \propto \frac{1}{\sqrt{T\rho}}$ $\qquad p = \frac{1}{3}Qv_{rms}^2$

$p \propto \rho \times T$

$\rho \propto \frac{p}{T} \propto \frac{1}{V}$ ---- (1)

pV^γ = constant

$\frac{nRT}{V}.V^\gamma$ = constant

$T \propto \frac{1}{V^{\gamma-1}}$ --- (2)

From (1) & (2)

$\tau \propto \frac{1}{\sqrt{T}\frac{1}{V}}$

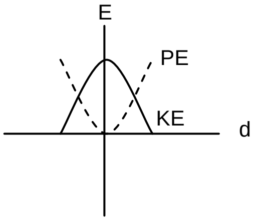
$$\tau \propto \frac{V}{\sqrt{T}} \propto V^{\frac{\gamma-1}{2}+1}$$

$$\tau \propto V^{\frac{\gamma+1}{2}}$$

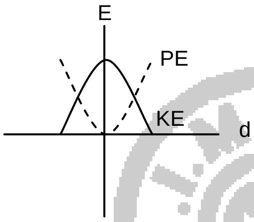
$$q = \frac{\gamma+1}{2}$$

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy ----

Ans:



Sol:



13. A train is moving on a straight track with speed 20 m s^{-1}. It is blowing its whistle at the frequency ----

Ans: 12%

Sol: $f_1 = f_0 \frac{c}{(c+v)}$

$f_2 = f_0 \frac{c}{(c+v)}$

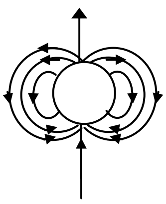
$\frac{f_1}{f_2} = \frac{c+v}{c-v} = \frac{340}{300} = \frac{17}{15} \rightarrow \frac{f_2}{f_1} = \frac{15}{17}$

$\frac{f_2 - f_1}{f_1} = \frac{-2}{17} \times 100$

= 11.7% $\cong$ 12%

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge ----

Ans:



Sol: Electric field in the equatorial line is opposite to dipole moment direction.

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) ----

Ans: $R_1 = 0$ and $R_2 < (R_4 - R_3)$

Sol: $V_i = \frac{kQ}{2R^3}\left[3R^2 - r^2\right]$; $V_0 = \frac{kQ}{R}$

$\frac{+V_0}{2R^2}\left[3R^2 - r^2\right]$

$\Rightarrow$ where $V_1 = \frac{3V_0}{2}$, r = 0 $\Rightarrow R_1 = 0$

$\frac{5V_0}{4} = \frac{3V_0}{2R^2}\left[3R^2 - r\right]$

$\Rightarrow \frac{5}{2} = \frac{3}{R^2}\left(3R^2 - r^2\right)$

$\Rightarrow 5R^2 = 18R^2 - 6r^2$

(**Note:** $2R < R_1$ is also answer.

$R_1 = 0$

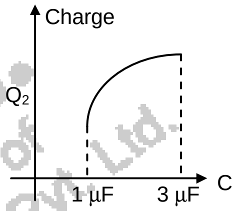
$R_2 = \frac{R}{\sqrt{2}}$

$R_3 = \frac{4R}{3}$

$R_4 = 4R$)

16. In the given circuit, charge Q_2 on the 2 μFcapacitor changes as C is varied from ----

Ans:



Sol: $Q = \frac{2CE}{(3+C)}$ is the $\frac{dQ}{dC}$ is with decreasing the

17. When 5 V potential difference is applied across a wire of length 0.1 m ----

Ans: 1.6×10^{-5} Ω m

Sol: $\rho = \frac{E}{j} = \frac{V}{\lambda nev}$

$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$

$\cong 1.6 \times 10^{-5}$ Ω m

18. In the circuit shown, the current in the 1 Ω resistor ----

Ans: 0.13 A, from Q to P

Sol: $\frac{5}{6} + 3 = \frac{23}{6}$ Ω

$I_0 = \frac{6 \times 6}{23} = \frac{36}{23}$ A

$\rightarrow I_1 = I_0 \times \frac{5}{6} = \frac{36}{23} \times \frac{5}{6}$

$= \frac{30}{23}$ A (↑ Q to P)

$\frac{3}{4} + 5 = \frac{23}{4}\ \Omega$

$I_0' = \frac{9\mu}{23} = \frac{36}{23}$ A

$\Rightarrow I_1' = I_0' \times \frac{3}{4} = \frac{36}{23} \times \frac{3}{4} = \frac{27}{23}$ A

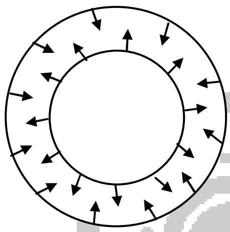
$I = I_1 - I_1' = \frac{30}{23} - \frac{27}{23} = \frac{3}{23}$ A

(from Q to P)

19. Two coaxial solenoids of different radii carry current I in the same ----

Ans: $\overline{F}_1 = \overline{F}_2 = 0$

Sol:



$\overline{F}_1 = \overline{F}_2 = 0$

20. Two long current carrying thin wires, both with current I, are held by insulating ----

Ans: $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$

Sol: $\left(\frac{F}{L}\right) = \frac{\mu_0}{4\pi}.\frac{2I^2}{2L\sin\theta} = \frac{\mu_0 I^2}{4\pi L\sin\theta}$

$\Rightarrow F = \frac{\mu_0 I^2}{4\pi\sin\theta} = T\sin\theta$

$Mg2\lambda Lg = T\cos\theta$

$\rightarrow \frac{F}{Mg} = \frac{\mu_0 I^2}{4\pi\sin\theta\lambda Lg} = \tan\theta$

$\Rightarrow I = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A ----

Ans: (b) and (d), respectively

Sol: Stable → PE minimum

→ $-\overline{M}.\overline{B}$ is minimum (i.e. $\overline{M}$ & $\overline{B}$ in same direction)

→ Fig. (b)

Unstable → PE maximum

→ $-\overline{M}.\overline{B}$ is maximum i.e. $\overline{M}$ & $\overline{B}$ in opposite direction)

→ Fig. (d)

22. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery ----

Ans: 0.67 mA

Sol: $i = I_0 e^{-\frac{t}{\tau}}$

$= \frac{15}{0.15 \times 10^3} - \frac{1 \times 10^{-3}}{e\left(\frac{0.03}{150}\right)}$

$= \frac{1}{10}e^{-5} = \frac{1}{1500}$ A

= 0.67 mA

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the ----

Ans: 2.45 V / m

Sol: $4\pi r^2 I = P$

$\frac{1}{2}\varepsilon_0 E^2 c = I$

$\Rightarrow \frac{1}{2}\varepsilon_0 E^2 c \times 4\pi r^2 = P$

$\Rightarrow E = \sqrt{\frac{2P}{\varepsilon_0 c 4\pi r^2}}$

= 2.45 V/m

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material ----

Ans:

Sol: $r_2 = C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$r_1 = A - r_2 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$

$\theta > \sin^{-1}\left[\mu\sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right]$

25. On a hot summer night, the refractive index of air is smallest near the ground and ----

Ans: $\theta < \cos^{-1}\left[\mu\sin\left[A + \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right]$

Sol: Directed horizontally, is assumed as light horizontal on surface of Earth → coming with grazing incidence → enters at critical angle → light bents upwards.

Note: If light is coming from upper layer, it will not deviate.

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance ----

Ans:

Sol: $\frac{1.22\lambda}{D} = \frac{d}{25 \times 10^{-2}}$; $D = 0.25 \times 2 \times 10^{-2}$

$$d = \frac{1.22\lambda \times 25 \times 10^{-2}}{0.25 \times 2 \times 10^{-2}} = 30\ \mu m$$

27. As an electron makes a translation from an excited state to the ground state ----

Ans:

Sol: KE ↑, PE ↓, TE ↓

28. Match List – I (Fundamental Experiment) with List – II (its conclusion) and select ----

Ans:

Sol: (A) → (ii)
(B) → (i)
(C) → (iii)

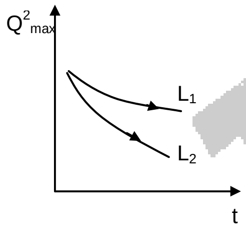
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency ----

Ans:

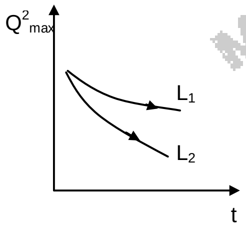
Sol: 2005 kHz
2000 kHz and 1995 kHz

30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged ----

Ans:



Sol: $L_2 < L_1 \rightarrow \tau_2 < \tau_1$
Discharge faster for case 2



PART – B – CHEMISTRY

31. The molecular formula of a commercial resin used for exchanging ions in water----

Ans: $\frac{1}{412}$

Sol: $2C_8H_7SO_3Na + Ca^{2+} \rightarrow (C_8H_7SO_3)_2Ca + 2Na^+$

2 moles resin ⇒ 1 mol Ca^{2+}
2×206 g ⇒ 1 mol Ca^{2+}

32. Sodium metal crystallizes in a body centred cubic lattice with ----

Ans: 1.86 Å

Sol: $4r = \sqrt{3}\,a$

$$\therefore r = \frac{\sqrt{3} \times 4.29}{4}$$

$$= 1.86\ \text{Å}$$

33. Which of the following is the energy of a possible excited state of hydrogen?----

Ans: –3.4 eV

Sol: $E_n = \frac{-13.6\ eV}{n^2}$

When, n = 2, $E_n = -3.4$ eV

34. The intermolecular interaction that is dependant on the inverse ----

Ans: ion-dipole interaction

Sol: ion-dipole interaction

35. The following reaction is performed at 298 K----

Ans: $0.5[2 \times 86,600 - R(298)\ \lambda n(1.6 \times 10^{12})]$

Sol: $\Delta G° = -R(298)\ \lambda n\ 1.6 \times 10^{12}$

$\Delta_f G^o_{(NO)} = 0.5[2 \times 86,600 - R(298)\ \lambda n(1.6 \times 10^{12})]$

36. The vapour pressure of acetone at 20°C is 185 torr.----

Ans: 64

Sol: $\frac{P^o - P_S}{P^o} = \frac{W_2.M_1}{W_1.M_2}$

$$\frac{185 - 183}{185} = \frac{1.2 \times 58}{100 \times M_B}$$

$\therefore M_B = 64$

37. The standard Gibbs energy change at 300 K for the given reaction $2A \rightleftharpoons B + C$ ----

Ans: reverse direction because $Q > K_c$

Sol: $Q = \frac{[B][C]}{[A]^2} = 4$

$$K = e^{\frac{-\Delta G°}{RT}} = e^{-\frac{2494.2}{8.314 \times 300}}$$

$$= 2.718^{-1} = \frac{1}{2.718}$$

Thus, $K_c < Q$

38. Two Faraday of electricity is passed through a solution of $CuSO_4$.----

Ans: 63.5 g

Sol: $Cu^{2+} + 2e^{-} \rightarrow Cu$
2F deposits 1 mole of Cu

39. Higher order (> 3) reactions are rare due to:----

Ans: low probability of simultaneous collision of all the reacting species

Sol: low probability of simultaneous collision of all the reacting species

40. 3 g of activated charcoal was added to 50 mL of aceti acid solution (0.06 N) in a flask.----

Ans: 18 mg

Sol: Amount of acetic acid adsorbed

$$= 50[0.06 - 0.042] \times 60 \text{ mg}$$

Amount adsorbed per gram charcoal

$$= \frac{50 \times 0.018 \times 60}{3} \text{mg}$$

$$= 18 \text{ mg}$$

41. The ionic radii (in $\overset{o}{A}$) of N^{3-}, O^{2-} and F^{-} are ----

Ans: 1.71, 1.40 and 1.36

Sol: For iso-electronic species, ionic radii decreases with increase in atomic number.

42. In the context of the Hall-Heroult process for the extraction of Al----

Ans: Na_3AlF_6 serves as the electrolyte

Sol: Molten Al_2O_3 serves as the electrolyte.

43. From the following statements regarding H_2O_2, choose the incorrect statement:----

Ans: It can act only as an oxidising agent.

Sol: It can act both as an oxidising and reducing agent.

44. Which one of the following alkaline earth metal sulphates has its hydration enthalpy----

Ans: $BeSO_4$

Sol: Be^{2+}, due to its small size, has high hydration enthalpy.

45. Which among the following is the most reactive? ----

Ans: ICl

Sol: Interhalogen compounds are more reactive than respective halogens.

46. Match the catalysts to the correct processes:----

Ans: (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)

Sol: $TiCl_3$ – Zielglar-Natta polymerisation
$PdCl_2$ – Wacker process
$CuCl_2$ – Deacon's process
V_2O_5 – Contact process

47. Which one has the highest boiling point?----

Ans: Xe

Sol: The boiling points of
Xe – 165 K
Kr – 119.7
He – 4.2 K
Ne – 27.1 K

48. The number of geometric isomers that can exist for square planar ----

Ans: 3

Sol: Square planar complex of the type [MABCD] have 3 geometrical isomers.

49. The color on $KMnO_4$ is due to:----

Ans: L $\rightarrow$ M charge transition

Sol: L $\rightarrow$ M charge transition

50. Assertion: Nitrogen and Oxygen are the main components ----

Ans: Both assertion and reason are correct and reason is the correct explanation for the assertion

Sol: Both assertion and reason are correct and reason is the correct explanation for the assertion

51. In Carius method of estimation of halogens, 250 mg of an organic compound----

Ans: 24

Sol: $$\% \text{ of } B = \frac{80 \times 0.141 \times 100}{188 \times 0.250}$$

$$= 24$$

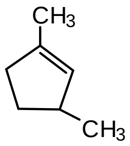
52. Which of the following compounds will exhibit geometrical isomerism? ----

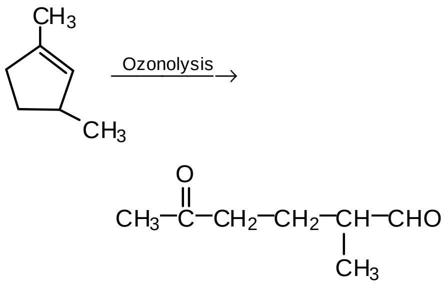
Ans: 1-phenyl-2-butene

Sol: $$\underset{(1\text{–phenyl–2–butene})}{C_6H_5 - CH_2 - CH = CH - CH_3}$$

It shows geometrical isomerism.

53. Which compound would give 5-keto-2-methylhexanal upon ozonolysis? ----

Ans: 

Sol: 

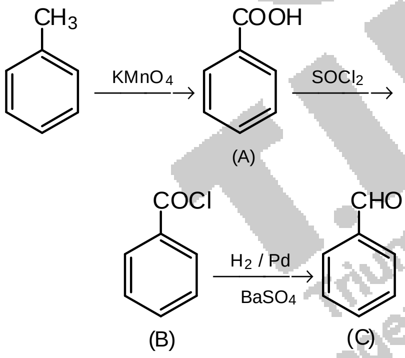
54. The synthesis of alkyl fluorides is best accomplished by----

Ans: Swartz reaction

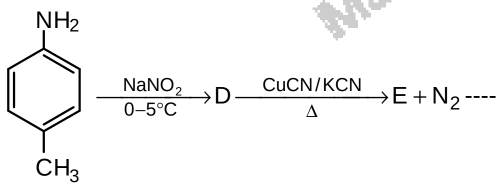
Sol: Swartz reaction is used to synthesise alkyl fluoride.

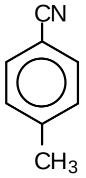
55. In the following sequence of reactions:----

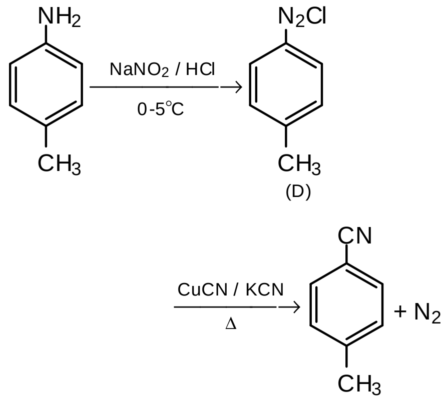
Ans: C_6H_5CHO

Sol: 

56. In the reaction



Ans: 

Sol: 

57. Which polymers is used in the manufacture of paints and lacquers?----

Ans: Glyptal

Sol: Glyptal is used in paints and lacquers.

58. Which of the vitamins given below is water soluble?----

Ans: Vitamin C

Sol: Vitamin C is water soluble.

59. Which of the following compounds is not an antacid? ----

Ans: Phenelzine

Sol: Phenelzine is an antidepressant drug.

60. Which of the following compounds is not colored yellow?----

Ans: $Zn_2[Fe(CN)_6]$

Sol: $Zn_2[Fe(CN)_6]$ is a blush white compound.

PART – C – MATHEMATICS

61. Let A and B be two sets containing four and two elements respectively. ----

Ans: 219

Sol: Number of subsets of $A \times B = 2^8$
Number of subsets of $A \times B$
having no element = 1
having one element = 8
having two elements = ${}^8C_2 = 28$
Therefore, number of subsets of $A \times B$ having at least 3 elements
$= 2^8 - (1 + 8 + 28)$
$= 219$
Choice (a)

62. A complex number z is said to be unimodular if |z| =1----

Ans: circles of radius 2

Sol: $\left|\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$

$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2)$

$= (2 - z_1\bar{z}_2)(2 - \bar{z},\ z_2)$

$\Rightarrow |z_1|^2 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2$

$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + |z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - |z_1|^2\,|z_2|^2 - 4 = 0$

$\Rightarrow (|z_1|^2 - 4) - |z_2|^2\,(|z_1|^2 - 4)\} = 0$

$(|z|^2 - 4)\,(1 - |z_2|^2) = 0$

$1 - |z_2|^2 \neq 0 \ \Rightarrow |z_1|^2 - 4 = 0$

$\Rightarrow$ Choice (c)

63. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$ ----

Ans: 3

Sol: $x^2 - 6x - 2 = 0$ Given $a_n = \alpha^n - \beta^n$

$\therefore \dfrac{a_{10} - 2a_8}{2a_9} = \dfrac{\alpha^{10} - \beta^{10} - 2\alpha^8 + 2\beta^8}{2(\alpha^9 - \beta^9)}$

$= \dfrac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$

But α and β are roots of the quadratic

$\Rightarrow \alpha^2 = 6\alpha + 2$ and $\beta^2 = 6\beta + 2$

$\therefore$ Required expression

$= \dfrac{\alpha^8(6\alpha^2 + 2 - 2) - \beta^8(6\beta + 2 - 2)}{2(\alpha^9 - \beta^9)}$

$= \dfrac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)} = 3$

Choice (c)

64. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$,----

Ans: (–2, –1)

Sol: $AA^T = 9I \Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} =$

$\begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2(a-b+1) \\ a+2b+4 & 2(a-b+1) & a^2+4+b^2 \end{bmatrix}$

$= 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (given)

$\therefore a + 2b + 4 = 0$

$a - b + 1 = 0$

$a^2 + 4 + b^2 = 9$

solving we get a = –2 and b = –1

choice (d)

65. The set of all values of λ for which the system of linear equations----

Ans: contains two elements

Sol: $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$(2 - \lambda)\{\lambda(3 + \lambda) - 4\} + 2\{-2\lambda + 2\}$

$+ 1\{4 - (3 + \lambda)\} = 0$

$\lambda(3 + \lambda)(2 - \lambda) - 4(2 - \lambda)$

$-4\lambda + 4 + 4 - 3 - \lambda = 0$

$\lambda(6 - \lambda - \lambda)^2 - 8 + 4\lambda - 4\lambda + 8 - 3 - \lambda = 0$

$-\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$

$\lambda^3 + \lambda^2 - 5\lambda + 3 = 0$

$\lambda = 1 \rightarrow$ is a root

Division gives $\lambda^2 + 2\lambda - 3 = 0$

$(\lambda + 3)(\lambda - 1) = 0$

$\lambda = 1, 1, -3$

Choice (c)

66. The number of integers greater than 6000 that can be formed, using----

Ans: 192

Sol: 4 digit numbers greater than 6000 may start with 6, 7 or 8

3 ways

Number of 4 digit numbers

$= 3 \times 4 \times 3 \times 2$

$= 72$

All possible 5 digit numbers = 5! = 120

$\therefore$ Total required numbers = 120 + 72

= 192

Choice (b)

67. The sum of coefficients of integral powers of x in the binomial----

Ans: $\frac{1}{2}(3^{50} + 1)$

Sol: Put $y = 2\sqrt{x}$

$(1 - y)^{50} = {}^{50}C_0 - {}^{50}C_1 y + {}^{50}C_2 y^2 + \ldots$ (1)

sum of coefficient lowest integral power is

${}^{50}C_0 - {}^{50}C_2\, 2^2 + {}^{50}C_4 2^4 + \ldots$

put y = 2 and –2 and adding

$1 + 3^{50} = 2[{}^{50}C_0 + 50C_2 \,.\, 2^2 + \ldots]$

$\therefore$ required coefficient $= \dfrac{3^{50} + 1}{2}$

68. If m is the A. M. of two distinct real numbers λ and n----

Ans: $4\lambda m^2 n$

Sol: $m = \dfrac{\lambda + n}{2}$

$\lambda, G_1, G_2, G_3, n$ are in G. P

$n = \lambda r^4$

$$r = \left(\frac{n}{\lambda}\right)^{\frac{1}{4}}$$

$$G_1 = \lambda\left(\frac{n}{\lambda}\right)^{\frac{1}{4}}$$

$$G_2 = \lambda\left(\frac{n}{\lambda}\right)^{\frac{2}{4}}$$

$$G_3 = \lambda\left(\frac{n}{\lambda}\right)^{\frac{3}{4}}$$

$G_1^4 + 2G_2^4 + G_3^4$

$$= \lambda^4 \times \frac{n}{\lambda} + 2 \times \lambda^4 \times \frac{n^2}{\lambda^2} + \lambda^4 \times \frac{n^3}{\lambda^3}$$

$= n\lambda^3 + 2n^2\lambda^2 + \lambda n^3$

$= n\lambda\{\lambda^2 + 2n\lambda + n^2\}$

$= n\lambda(\lambda + n)^2$

$= n\lambda \times 4m^2 = 4\lambda m^2 n$

Choice (b)

69. The sum of first 9 terms of the series----

Ans: 96

Sol: $\dfrac{1^3}{1} + \dfrac{1^3 + 2^3}{1+3} + \ldots\ldots$ n terms

$$n^{th} \text{ term} = \frac{n^2(n+1)^2}{4n^2}$$

$$= \frac{(n+1)^2}{4}$$

We want $\displaystyle\sum_{n=1}^{9} \frac{(n+1)^2}{4}$

$$= \frac{1}{4}\left\{2^2 + 3^2 + 4^2 + \ldots + 10^2\right\}$$

$$= \frac{1}{4}\left\{\frac{10 \times 11 \times 21}{6} - 1\right\}$$

$$= \frac{1}{24}\{10 \times 11 \times 21 - 6\}$$

$$= \frac{6}{24}\{5 \times 11 \times 7 - 1\}$$

$$= \frac{384}{4} = 96$$

Choice (b)

70. $\displaystyle\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ ----

Ans: 2

Sol: $$\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$$

$$= 4 \times \lim_{x \to 0} \frac{(2\sin^2 x)}{x \tan 4x}$$

$$= 4 \times \lim_{x \to 0} \frac{2\left(\frac{\sin x}{x}\right)^2 \times x^2}{x \times \left(\frac{\tan 4x}{4x}\right) \times 4x}$$

$$= 4 \times 2 \times \frac{1}{4} = 2$$

Choice (c)

71. If the function g(x) **=** ----

Ans: 2

Sol: $$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx + 2, & 3 < x \le 5 \end{cases}$$

g(x) should be continuous at x = 3

2k = 3m + 2

2k – 3m = 2 ——— (1)

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}}, & 0 \le x \le 3 \\ m, & 3 < x \le 5 \end{cases}$$

g(x) is differentiable at x = 3

$\dfrac{k}{4} = m$

k = 4m ——— (2)

From (1) and (2)

$5m = 2 \Rightarrow m = \dfrac{2}{5}$

$\Rightarrow k = \dfrac{8}{5}$

k + m = 2

choice (a)

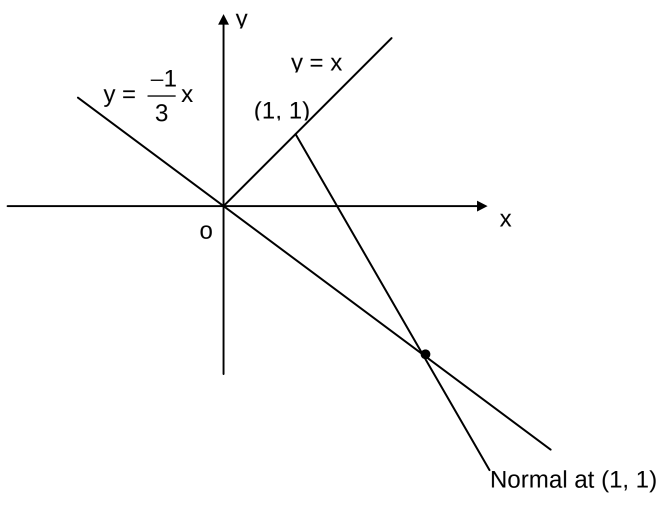
72. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$----

Ans: meets the curve again in the fourth quadrant

Sol: $x^2 + 2xy - 3y^2 = 0$

represent the two lines

x = y and x = –3y



Normal at (1, 1) to the curve meets the curve again at a point in the 4th quadrant
Choice (d)

73. Let f(x) be a polynomial of degree four----

Ans: 0

Sol: Since $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$

f(x) must be of the form

$\equiv ax^4 + bx^3 + cx^2$

$f'(x) = 4ax^3 + 3bx^2 + 2cx$

$4a + 3b + 2c = 0$ ——— (1)

$\underline{32a + 12b + 4c = 0}$

$\Rightarrow 8a + 3b + c = 0$ ——— (2)

Also, since

$\lim_{x\to 0}\left\{1+\frac{f(x)}{x^2}\right\}=3$

$\lim_{x\to 0}\left\{1+\frac{ax^4+bx^3+cx^2}{x^2}\right\}=3$

$1 + c = 3$

$c = 2$

Substituting in (1) and (2)

$4a + 3b = -4$

$\underline{8a + 3b = -2}$

$4a = 2 \Rightarrow a = \frac{1}{2}$

$3b = -4 - 4a$

$= -4 - 2 = -6$

$b = -2$

$f(2) = 16a + 8b + 4c$

$= 16 \times \frac{1}{2} + 8 \times (-2) + 4 \times 2$

$= 8 - 16 + 8 = 0.$

Choice (c)

74. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$ ----

Ans: $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+C$

Sol: $\int \frac{dx}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$

$= \int \frac{dx}{x^2 \times x^3\left\{1+\frac{1}{x^4}\right\}^{\frac{3}{4}}}$

$1+\frac{1}{x^4}=t$

$\frac{-4}{x^5}dx = dt$

$\frac{dx}{x^5}=\frac{-1}{4}dt$

Substituting

$\int = \frac{-1}{4}\int \frac{dt}{\frac{t^3}{4}}$

$= \frac{-1}{4} \times \frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}$

$= -t^{\frac{1}{4}}$

$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}$

$= -\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x^4}+C$

Choice (d)

75. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$ ----

Ans: 1

Sol: Let $I = \int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$

$= \int_2^4 \frac{\log x^2}{\log x^2+\log(6-x)^2}dx$ —— (1)

We know $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\therefore I = \int_2^4 \frac{\log(6-x)^2}{\log x^2+\log(6-x)^2}dx$ —— (2)

(1) + (2)

$\Rightarrow 2I = \int_2^4 1dx$

$I = 1 \Rightarrow$

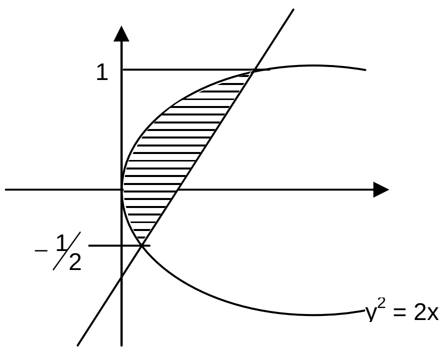
$\int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx = 1$

Choice (c)

76. The area (in sq. units) of the region described by----

Ans: $\frac{9}{32}$

Sol:



Required An = $\int_{-\frac{1}{2}}^{1} (x_1 - x_2)\ dy$

$$= \int_{-\frac{1}{2}}^{1} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy$$

$$= \frac{1}{4}\left(\frac{y^2}{2} + y - \frac{2y^3}{3}\right)_{-\frac{1}{2}}^{1}$$

$$= \frac{1}{4}\left[\frac{3}{4} - \frac{3}{8} - \frac{3}{2}\right] = \frac{9}{32}$$

Choice (d)

77. Let y(x) be the solution of the differential equation----

Ans: 2

Sol: $(x\log x)\dfrac{dy}{dx} + y = 2x\log(x)$

$$\frac{dy}{dx} + \frac{y}{x\log x} = 2$$

$$I.F = e^{\int \frac{1}{x\log x}} = e^{\log(\log x)} = \log x$$

solution is

$y \log x = \int 2\log x\ dx$

$y \log x = 2\left[\log x\ .x - \int \frac{1}{x}.x\ dx\right] + C$

ylogx = 2 [xlogx – x] + C

when x = 1 $\Rightarrow$ c = 2

when x = e

y . 1 = 2[e loge – e] + C

y = C

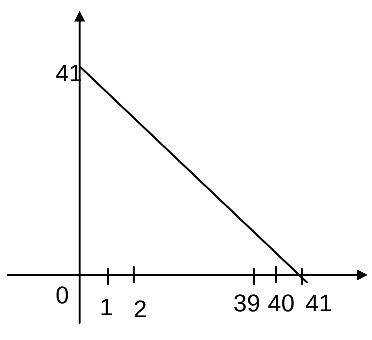
$\therefore$ y (e) = 2

Choice (c)

78. The number of points, having both co-ordinates as integers----

Ans: 780

Sol: Consider the triangle – it is right angled and the hypotenuse is part of the line x+ y = 41.



We require only integer points lying an the interior of the triangle.

$\therefore$ when x = 1 $\Rightarrow$ 41 points of which two are deleted as (1, 0) and (1, 40) lie on the boundary $\Rightarrow$ 39 points.

$\therefore$ when x = 2 $\Rightarrow$ 38 points etc

when x = 39 $\Rightarrow$ 1 points and

when x = 40 $\Rightarrow$ No points

= 1+ 2 + 3 + ….+ 39

$$= \frac{39 \times 40}{2} = 780$$

Choice (d)

79. Locus of the image of the point (2, 3) in the line----

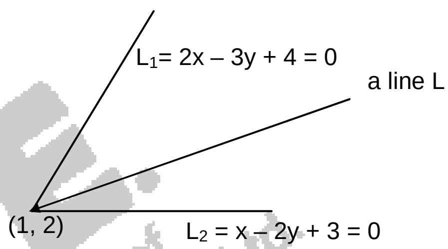
Ans: circle of radius $\sqrt{2}$

Sol: Observe that the two lines

2x – 3y + 4 = 0

and x – 2y + 3 = 0

intersect at the point (1, 2)



The family of lines

L = 2x – 3y + 4 + k(x – 2y + 3) = 0

pass through the point (1, 2)

Distance of the image of (2, 3) in L is at a distance

$$= \sqrt{(2-1)^2 + (3-2)^2}$$

$= \sqrt{2}$ from (1, 2)

$\Rightarrow$ Locus of the image is the circle centered at (1, 2) and whose radius equals $\sqrt{2}$

Choice (c)

80. The number of common tangent to the circles----

Ans: 3

Sol: $S_1: x^2 + y^2 - 4x - 6y - 12 = 0$

$S_2: x^2 + y^2 + 6x + 18y + 26 = 0$

C_1 (2, 3) $r_1 = \sqrt{25} = 5$

C_2 (–3, –9) $r_2 = \sqrt{64} = 8$

$C_1C_2 = \sqrt{25 + 144} = 13$

$C_1C_2 = r_1 + r_2$

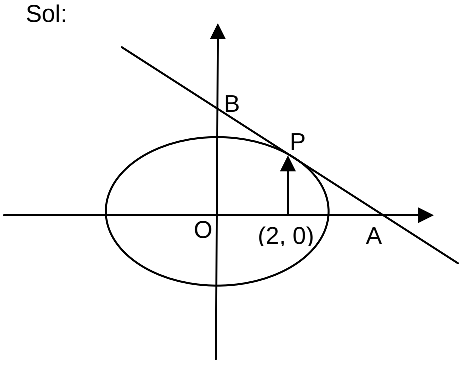
$\therefore$ No: common tangent = 3

Choice (c)

81. -The area (in sq. units) of the quadrilateral formed by the tangents at the end points---

Ans: 27

Sol:



$\frac{x^2}{a}+\frac{y^2}{5}=1$ focus (2, 0)

Co-ordinate of P is $\left(2, \frac{5}{3}\right)$

Equation of tangent at P is $\frac{2x}{9}+\frac{y}{3}=1$

$x = 0 \Rightarrow y = 3$: B (0, 3)

$y = 0 \Rightarrow x = \frac{9}{2}$: A $\left(\frac{9}{2}, 0\right)$

$\therefore$ Ans Δ OAB = $\frac{1}{2}.\frac{9}{2}.3$

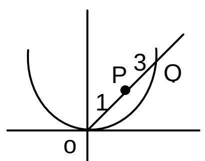
Required Ans = 4. $\frac{1}{4}$.9.3 = 27

Choice (d)

82. Let O be the vertex and Q be any point on the parabola----

Ans: $x^2 = 2y$

Sol:



Let the point Q be $\left(x, \frac{x^2}{8}\right)$

Then by section formula,

x – coordinate of P is $\frac{x}{4}$

y – coordinate of P is $\frac{x^2}{32}$

Clearly, $\left(\frac{x}{4}\right)^2 = 2\left(\frac{x^2}{32}\right)$

$\therefore$ Locus of P is $x^2 = 2y$

Choice (d)

83. The distance of the point (1, 0, 2) from the point of intersection----

Ans: 13

Sol: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=\lambda$

$(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$ lies on plane

$\Rightarrow 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 16 = 0$

$\Rightarrow 11\lambda - 11 = 0 \Rightarrow \lambda = 1$

$\therefore$ point of intersection (5, 3, 14)

$\therefore$ Distance from (1, 0, 2) to (5, 3, 14)

$\therefore \sqrt{16+9+144}$

$= \sqrt{169} = 13$

Choice (d)

84. The equation of the plane containing, the line 2x – 5y + z = 3; ----

Ans: x + 3y + 6z = 7

Sol: The plane can be taken as

$(2x - 5y + z - 3) + \lambda (x + y + 4z - 5) = 0$

i.e., $(2 + \lambda) x + (-5 + \lambda)y + (1 + 4\lambda)z = 5\lambda + 3$

Since this plane is parallel to the plane $x + 3y + 6z = 2$,

$\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\therefore \lambda = \frac{-11}{2}$

$\therefore$ Required plane is

$-7x - 21y - 42z + 49 = 0$

i.e. $x + 3y + 6z = 7$

Choice (c)

85. Let $\bar{a}$, $\bar{b}$ and $\bar{c}$ be three non- zero vectors such that no two of them are collinear----

Ans: $\frac{2\sqrt{2}}{3}$

Sol: $(a \times b) \times c = \frac{1}{3}|b|\ |c|\ \bar{a}$

$(a . c)\ b - (b . c)\ \bar{a} = \frac{1}{3}|\bar{b}|\ |\bar{c}|\ \bar{a}$

$a . c = 0$ and $-\left(\bar{b} . \bar{c}\right) = \frac{1}{3}|b|\ |\bar{c}|$

$\Rightarrow -|b|\ |\bar{c}|\ \cos\theta = \frac{1}{3}|\bar{b}|\ |\bar{c}|$

$\cos\theta = -\frac{1}{3}$

$\therefore \sin\theta = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

86. If 12 identical balls are to be places in 3 identical boxes, then the probability that----

Ans: $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Sol: Since the 3 boxes are identical probability of selecting a box (in which we are to put 3 balls) is $\frac{1}{3}$

$\Rightarrow p = \frac{1}{3}$, $q = 1 - p = \frac{2}{3}$

The given problem is:

Probability that the selected box should contain exactly 3 balls which is the same as " Probability of exactly 3 success in 12 trials (putting the 12 balls in 3 boxes)

$= {}^{12}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^9$

$= \frac{12\times 11\times 10}{6}\frac{1}{3^{12}}\times 2^9$

$= \frac{220\times 2^9}{3^{12}}$

$= 55 \times \frac{2^{11}}{3^{12}}$

$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Choice (a)

87. The mean of the data set comprising of 16 observations is 16. If one----

Ans: 14.0

Sol: $\frac{\sum x}{16} = 16$

$\sum x = 256$

Correct $\sum x.\ 256 - 16 + 3 + 4 + 5$

$= 252$

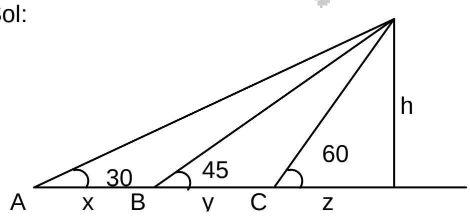
Correct mean = $\frac{252}{18} = 14$

Choice (d)

88. If the angles of elevation of the top of a tower from three collinear points A, B and C----

Ans: $\sqrt{3} : 1$

Sol:



$\tan 60 = \frac{h}{z} \Rightarrow h = \sqrt{3}\ z$

$y + z = h = \sqrt{3}\ z$

$y = \left(\sqrt{3} - 1\right)z$

$\tan 30 = \frac{h}{x+y+z} = \frac{h}{x+\sqrt{3}z} = \frac{1}{\sqrt{3}}$

$h = \frac{x+\sqrt{3}\ z}{\sqrt{3}} = \frac{x}{\sqrt{3}} + z$

$\sqrt{3}\ z = \frac{x}{\sqrt{3}} + z$

$3z = x + \sqrt{3}\ z$

$x = \sqrt{3}\left(\sqrt{3} - 1\right)z$

$y = \left(\sqrt{3} - 1\right)z$

$\therefore \frac{x}{z} = \sqrt{3} : 1$

(Choice a)

89. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ ----

Ans: $\frac{3x - x^3}{1-3x^2}$

Sol: $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

$= \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - \frac{2x^2}{1-x^2}}\right)$

$= \tan^{-1}\left(\frac{x\left(1-x^2\right)+2x}{1-x^2-2x^2}\right)$

$= \tan^{-1}\frac{3x-x^3}{1-3x^2}$

Choice (a)

90. The negation of ~s ∨ (~r ∧ s) is equivalent ----

Ans: s ∧ r

Sol: Negation of ~s ∨ (~r ∧ s)

⇔ s ∧ (r ∨ ~ s)

⇔ (s ∧ r) ∨ (s ∨ ~ s)

⇔ s ∧ r

Choice (d)