SOLUTIONS & ANSWERS FOR JEE MAINS-2013
VERSION – P

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

1. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with ----

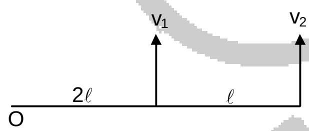
   Ans: $\frac{Mg}{k}\left[1-\frac{LA\sigma}{2M}\right]$

   Sol: $B = \frac{LA\sigma g}{2}$
   $Mg = B + kx_0$
   $\Rightarrow x_0 = \frac{Mg}{k} - \frac{B}{k}$
   $= \frac{Mg}{k}\left[1-\frac{LA\sigma}{2M}\right]$

2. A metallic rod of length l is tied to a string of length 2l and made to rotate with angular speed ω ----

   Ans: $\frac{5B\omega\ell^2}{2}$

   Sol:



   $v_1 = 2\ell\omega;\ v_2 = 3\ell\omega$
   $v_{Av} = \frac{v_1+v_2}{2} = \frac{5\ell\omega}{2}$
   $E = B\ell v_{Av} = \frac{5B\omega\ell^2}{2}$

3. Statement – 1
   A point particle of mass m moving with speed v collides with ----

   Ans: Statement-I is false, statement – II is true.

   Sol: $\Delta KE = \frac{1}{2}\frac{mM}{(M+m)}(v-0)^2(1-e^2)$
   e = 0 for ΔKE_{MAX}
   $\Rightarrow \Delta KE = \frac{1}{2}mv^2\left(\frac{M}{M+m}\right)$
   Not $\frac{1}{2}mv^2\left(\frac{m}{M+m}\right)$
   Statement 1 is false.

4. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum----

   Ans: $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

   Sol: $\varepsilon_0 = C^2\ N^{-1}\ m^{-2}$
   $= (AT)^2\ [MLT^{-2}]^{-1}\ L^{-2}$
   $= M^{-1}L^{-3}\ T^4\ A^2$

5. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/ s, where $\hat{i}$ is----

   Ans: $y = 2x - 5x^2$

   Sol: $u\cos\theta = 1\ m\ s^{-1}$;
   $\tan\theta = \frac{2}{1} = 2$
   $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$
   $= 2x - \frac{1}{2}\times\frac{10x^2}{1^2}$
   $\Rightarrow y = 2x - 5x^2$

6. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude ----

   Ans: 0.729

   Sol: $A = 0.9A_0 \times 0.9 \times 0.9$
   $= 0.729A_0$

7. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found ----

   Ans: $3C_1 = 5C_2$

   Sol: $Q_{net} = 0 \Rightarrow C_1V_1 + C_2V_2 = 0$
   But C_1 and C_2 are positive.
   $\therefore$ V_1 and V_2 are of opposite signs.
   $\Rightarrow C_1V_1 - C_2V_2 = 0$
   $\Rightarrow C_1V_1 = C_2V_2 \Rightarrow 120C_1 = 200C_2$
   or $3C_1 = 5C_2$

8. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. ----

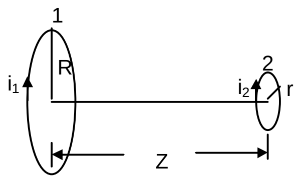
   Ans: 178.2 Hz

   Sol: $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{\varepsilon Y}{\rho}}$
   $= \frac{1}{2\times1.5}\sqrt{\frac{0.01\times2.2\times10^{11}}{7.7\times10^3}}$
   $= 178.2$ Hz

9. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the ----

   Ans: 9.1×10^{-11} weber

Sol:



We have $M_{12} = M_{21}$ (R for large coil 1, r for small coil 2, Z = distance between them)

$$\frac{\mu_0 . i_1 R^2}{2\left[R^2 + Z^2\right]^{3/2}} . \pi r^2 = \phi_2 = M_{21} i_1$$

$$\Rightarrow M_{21} = \frac{\pi \mu_0 R^2 r^2}{2\left(R^2 + Z^2\right)^{3/2}} = M_{12}$$

$\phi_1 = M_{12}\, i_2 = M_{21}\, i_2$ ($\because$ $i_2 = 2$ A)

$$= \frac{\pi \mu_0 R^2 r^2}{2\left(R^2 + Z^2\right)^{3/2}} \times 2$$

$$= \frac{\pi \times 4\pi \times 10^{-7} \times (0.2)^2 \times (0.003)^2}{\left[0.2^2 + 0.15^2\right]^{3/2}}$$

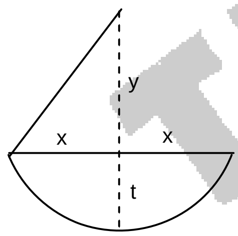
$$= \frac{1.4212 \times 10^{-12}}{0.015625}$$

$\cong 9.1 \times 10^{-11}$ Wb

10. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light ----

Ans: 30 cm

Sol:



x = 3 cm; y = 0.3 cm

$x^2 = yt$

$\Rightarrow y = \frac{x^2}{t} = 30$ cm ; 2R = 30.3 cm

$\Rightarrow$ R = 15.15 cm

$\mu = \frac{c}{v} = 1.5$; $\frac{1}{f} = 0.5\left[\frac{1}{15.15} - \frac{1}{\infty}\right]$

$= \frac{1}{2 \times 15.5} \Rightarrow$ f = 30.3 cm

$\cong$ 30 cm

11. What is the minimum energy required to launch a satellite of mass m from the surface ----

Ans: $\frac{5GmM}{6R}$

Sol: $-\frac{GMm}{R} + E = -\frac{GMm}{2(R + 2R)}$

$\Rightarrow E = \frac{5GmM}{6R}$

12. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of ----

Ans: 10.62 MHz

Sol: $\frac{1}{f} << RC = 2.5 \times 10^{-5}$

$\Rightarrow f >> \frac{1}{RC} = 4 \times 10^4$ Hz

Select the highest value of f from given choices!

13. A beam of unpolarised light of intensity I_0 is passed through a Polaroid A and then through another Polaroid B ----

Ans: $\frac{I_0}{4}$

Sol: $I_E = \frac{I_0}{2} \times \cos^2 45°$

$= \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$

14. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already ----

Ans: 10.04 volt

Sol: r = 6 Ω

$R_1 = \frac{V^2}{P_1} = \frac{120 \times 120}{60} = 240\ \Omega$

$V_1 = \frac{VR_1}{(R_1 + r)} = \frac{120 \times 240}{246} = 117.07$ V

$R_2 = \frac{V^2}{P_2} = \frac{120 \times 120}{240} = 60\ \Omega$

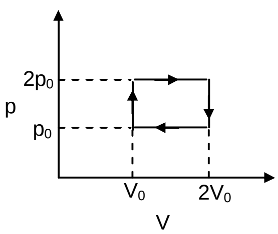
$R_P = \frac{R_1 R_2}{(R_1 + R_2)} = \frac{240 \times 60}{300} = 48\ \Omega$;

$V_2 = \frac{48 \times 120}{(48 + 6)} = 106.67$ V

$\therefore \Delta V = 117.07 - 106.67$

= 10.4 V (Nearest answer is 10.04 volt)

15.



The above pV diagram represents the thermodynamic cycle of an engine -----

Ans: $\left(\frac{13}{2}\right)p_0V_0$

Sol: $H = Q_1 + Q_2$

Q_1 = isochoric = $nC_V(T_2 - T_1)$

$= \frac{C_V(p_2V_2 - p_1V_1)}{R} = \frac{3}{2}R.\frac{(2p_0V_0 - p_0V_0)}{R}$

$= \frac{3}{2}p_0V_0$

Q_2 = isobaric = $nC_p(T_3 - T_2)$

$= \frac{C_p(p_3V_3 - p_2V_2)}{R}$

$= \frac{5}{2}R.\frac{(4p_0V_0 - 2p_0V_0)}{R} = 5p_0V_0$

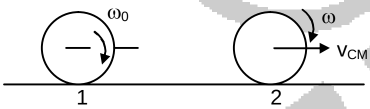
$\therefore H = \frac{3}{2}p_0V_0 + 5p_0V_0$

$= \left(\frac{13}{2}\right)p_0V_0$

16. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface ----

Ans: $\frac{r\omega_0}{2}$

Sol:



$L_1 = L_2$;

$L_1 = L_{CM} + mv_{CM}r = L_{CM} = mr^2\omega_0$

$L_2 = L_{CM} + mv_{CM}r = mr^2\omega + mr^2\omega = 2mr^2\omega$

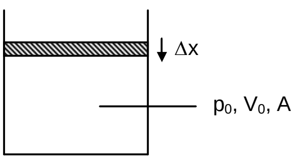
$2mr^2\omega = mr^2\omega_0 \qquad \omega = \frac{\omega_0}{2}$

$\Rightarrow v_{CM} = r\omega = \frac{r\omega_0}{2}$

17. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal ----

Ans: $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma p_0}{MV_0}}$

Sol:



$\Delta V = A\Delta x \ ; \ \frac{\Delta V}{V_0} = \frac{A\Delta x}{V_0}$

$\Delta p = -B\frac{\Delta V}{V_0} = -\frac{BA\Delta x}{V_0}$

$\Delta F = \Delta pA = -\frac{BA^2\Delta x}{V_0} = -\frac{\gamma p_0A^2}{MV_0}\Delta x$

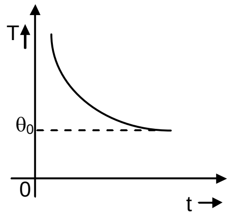
($\because B = \gamma p_0$ for adiabatic process)

$\therefore a = \frac{\Delta F}{M} = -\frac{\gamma p_0A^2}{MV_0}\Delta x \Rightarrow$ SHM

$\Rightarrow \omega^2 = \frac{\gamma p_0A^2}{MV_0} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma p_0}{MV_0}}$

18. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature ----

Ans:



Sol: Temperature – time graph for Newton's law of cooling.

19. Statement – 1
Higher the range, greater is the resistance of ammeter ----

Ans: Statement - I is false, Statement - II is true.

Sol: Higher the range of an ammeter, less is its resistance.
$\Rightarrow$ Statement-I is false.

20. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. ----

Ans: At $t = 2\tau$, $q = CV[1 - e^{-2}]$

Sol: $q = Q_0\left[1 - e^{-\frac{t}{\tau}}\right]$

$= CV\left[1 - e^{-\frac{t}{\tau}}\right]$

when $t = 2\tau$, $q = CV[1 - e^{-2}]$

21. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown ----

Ans: Concentric circles

Sol: Wavefront of each source is spherical. When two spheres interfere, locus is circular
$\Rightarrow$ concentric circles.

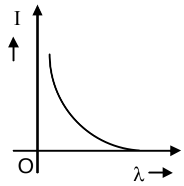
22. The magnetic field in a travelling electromagnetic wave has a peak value of ----

Ans: 6 V / m

Sol: $E_0 = B_0 c$
$= 20 \times 10^{-9} \times 3 \times 10^{8}$
$= 60 \times 10^{-1}$ V m^{-1}
$= 6$ V m^{-1}

23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode----

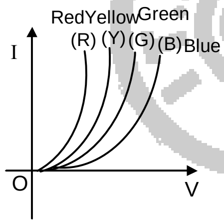
Ans:



Sol: At λ_{max}, i becomes zero.
For smaller λ, $i \neq 0$
Only option (4) fulfills.

24. The I-V characteristic of an LED is ----

Ans:



Sol: LED is PN junction diode in forward bias.
$\Rightarrow$ option (1) is correct.

25. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should ---

Ans: $\frac{2T}{\rho L}$

Sol: $E = 4\pi r^2 T$

$\frac{dE}{dr} = 8\pi r T$ —(i)

$\left(\frac{dm}{dr}\right)L = L\frac{d}{dr}\left(\frac{4}{3}\pi r^3 \rho\right)$

$= 4\pi r^2 \rho L$ —(ii)

(i) = (ii)

$8\pi r T = 4\pi r^2 \rho L \Rightarrow r = \frac{2T}{\rho L}$

26. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with ----

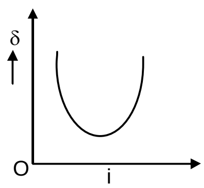
Ans: $\frac{1}{n^3}$

Sol: $\Delta E = 13.6 Z^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = h\upsilon$

$\Rightarrow \upsilon \propto \frac{1}{n^3}$

27. The graph between angle of deviation (δ) and angle of incidence (i) ----

Ans:

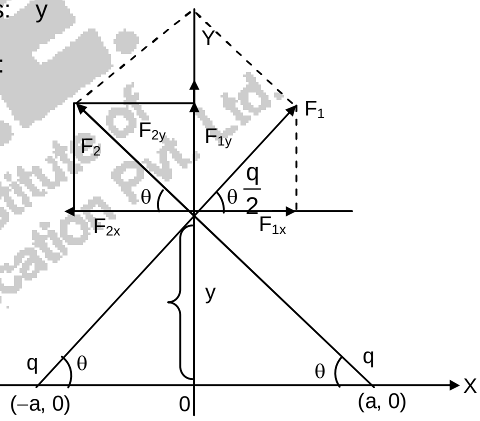


Sol: As i increases, δ decreases, reaches a minimum value and then increases.
$\Rightarrow$ option (3)

28. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge ----

Ans: y

Sol:



The charge $\frac{q}{2}$ experiences a repulsive force due to each charge on the X-axis. i.e. F_1 and F_2

$F_1 = F_2 = \frac{kq^2}{2r^2} = \frac{q^2}{8\pi\varepsilon_0(a^2 + y^2)}$

The x-components F_{1x} and F_{2x} cancel each other. The y-components F_{1y} and F_{2y} add together.

Net force $F\left(\text{on } \frac{q}{2}\right) = 2F_{1y} = 2F_1 \sin\theta$

$= \frac{2q^2}{8\pi\varepsilon_0(a^2 + y^2)} \cdot \frac{y}{(a^2 + y^2)^{1/2}}$

$\left(\because \sin\theta = \frac{y}{(a^2 + y^2)^{1/2}}\right)$

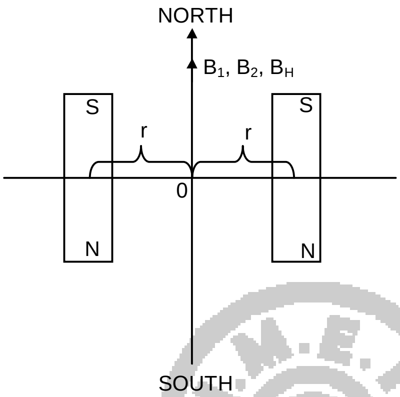
$$= \frac{q^2 y}{4\pi\varepsilon_0 \left(a^2 + y^2\right)^{3/2}}$$

If y << a, $F = \frac{q^2 y}{4\pi\varepsilon_0 a^3} \Rightarrow F \propto y$

29. Two short bar magnets of length 1 cm each have magnetic moments 1.20 A m^2 and 1.00 A m^2 respectively. They are placed on a horizontal ----

Ans: 2.56×10^{-4} Wb/m^2

Sol:



$$B_0 = B_1 + B_2 + B_H = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

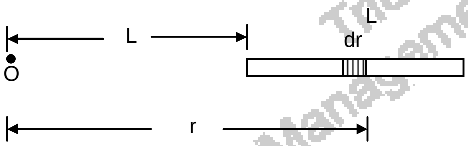
$$= \frac{10^{-7}[1.2 + 1]}{(0.1)^3} + 3.6 \times 10^{-5}$$

$= 2.56 \times 10^{-4}$ Wb m^{-2}

30. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential ----

Ans: $\frac{Q\ell n2}{4\pi\varepsilon_0 L}$

Sol:



$$\lambda = \frac{Q}{L}$$

$$dq = \lambda dr$$

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda dr}{4\pi\varepsilon_0 r}$$

$$V = \int_L^{2L} \frac{\lambda dr}{4\pi\varepsilon_0 r}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \ell nr\Big|_L^{2L} \quad = \frac{\lambda \ell n2}{4\pi\varepsilon_0}$$

$$= \frac{Q\ell n2}{4\pi\varepsilon_0 L}$$

PART B – CHEMISTRY

31. Which of the following complex species is not expected ----

Ans: $[Co(NH_3)_3Cl_3]$

Sol: Complex of the type Ma_3b_3

32. Which one of the following molecules is expected to exhibit ----

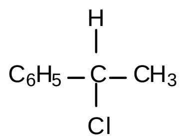
Ans: C_2 and N_2

Sol: C_2 and N_2 are diamagnetic in nature

33. A solution of (–)-1-chloro-1-phenylethane in toluene racemises ----

Ans: Carbocation

Sol:



1-chloro-1-phenylethane

It can form a stable secondary benzylic carbocation

34. Given

$E^\circ_{Cr^{3+}/Cr} = 0.74V; E^\circ_{MnO_4^-/Mn^{2+}} = 1.51$ V ----

Ans: MnO_4^-

Sol: Among the given, highest E° value is possessed by MnO_4^-.
Higher the reduction potential the better the oxidising agent

35. A piston filled with 0.04 mol of an ideal gas expands ----

Ans: q = +208 J, w = –208 J

Sol:

$$w = -nRT \ln \frac{V_2}{V_1}$$

$= -0.04 \times 8.314 \times 310 \times \ln 7.5$

$= -208$ J

36. The molarity of a solution obtained by mixing ----

Ans: 0.875 M

Sol: $M = \frac{750 \times 0.5 + 250 \times 2}{1000} = 0.875$ M

37. Arrange the following compounds in order of decreasing acidity :----

Ans: III > I > II > IV

Sol: Presence of electron withdrawing group increases the acid strength of phenol. Since $-NO_2$ group is more electron withdrawing than Cl, p-nitrophenol is the strongest acid among the given compounds

38. For gaseous state, if most probable speed is denoted by C*, average speed ----

Ans: $C^* : \overline{C} : C = 1 : 1.128 : 1.225$

Sol: $C^* : \overline{C} : C = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$

$= 1 : 1.128 : 1.225$

39. The rate of a reaction doubles when its temperature ----

Ans: 53.6 kJ mol^{-1}

Sol: $\log \frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$

$$E_a = \frac{\log 2 \times 2.303 \times 8.314 \times 300 \times 310}{10}$$

$= 53.59$ kJ mol^{-1}

40. A compound with molecular mass 180 is acylated with CH_3COCl ----

Ans: 5

Sol: Increase in molecular mass due to acylation = 390 − 180 = 210.
This corresponds to the introduction of 5 acetyl groups. So the number of amino groups present is 5.

41. Which of the following arrangements does no represent ----

Ans: $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

Sol: No. of unpaired electrons in Mn^{2+} is 5 and Fe^{2+} is 4

42. The order of stability of the following carbocations :----

Ans: III > I > II

Sol: Benzyl carbocation (III) is more resonance stabilized than allyl carbocation (I). Primary carbocation (II) is the least stable ion

43. Consider the following reaction : ----

Ans: 2, 5 and 16

Sol: The balanced equation is

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + \frac{16}{2}H_2O$$

x = 2, y = 5, z = 16

44. Which of the following is the wrong statement?---

Ans: No answer

Sol: All the given four statements are correct

45. A gaseous hydrocarbon gives upon combustion 0.72 g.----

Ans: C_7H_8

Sol: Ratio of C and H atoms
= No. of moles CO_2 : 2 ×No. of moles of water

$= \frac{3.08}{44} : \frac{2 \times 0.72}{18}$

= 7 : 8

∴ Empirical formula = C_7H_8

46. In which of the following pairs of molecules/ions, both----

Ans: H_2^{2+}, He_2

Sol: $H_2^{2+} - \sigma 1s^0$

$He_2 - \sigma 1s^2\ \sigma^* 1s^2$

47. Which of the following exists as covalent crystals----

Ans: Silicon

Sol: Silicon is a covalent crystal

48. Synthesis of each molecule of glucose in ----

Ans: 18 molecules of ATP

Sol: 18 molecules of ATP

49. The coagulating power of electrolytes having ----

Ans: $Na^+ < Ba^{2+} < Al^{3+}$

Sol: Higher the charge associated with the ion, the coagulating power increases.

50. Which of the following represents the correct order of ----

Ans: Ba < Ca < Se < S < Ar

Sol: The ionisation enthalpy in kJ mol^{-1} of
Ar – 1520
S – 1000
Se – 941
Ca – 590
Ba – 502

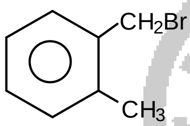
51. Energy of an electron is given by $E = -2.178 \times^{-18}$J ----

Ans: 1.214×10^{-7} m

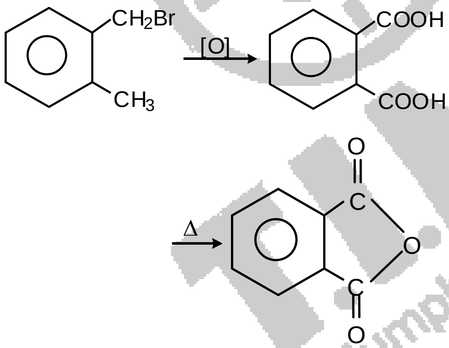
Sol: $\lambda = \frac{hc}{\Delta E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8} \times 4}{2.178 \times 10^{-18} \times 3}$
$= 1.214 \times 10^{-7}$ m

52. Compound (A), C_8H_9Br, gives a white precipitate when warmed with ----

Ans:



Sol:



53. For successive members of the first row transition elements ----

Ans: Co(Z = 27)

Sol: Highest $E^{\circ}_{M^{3+}/M^{2+}}$ is possessed by cobalt

54. How many litres of water must be added to 1 litre ----

Ans: 9.0 L

Sol: $\left.\begin{array}{l} pH = 1 \rightarrow [H^{+}] = 0.1 \\ pH = 2 \rightarrow [H^{+}] = 0.01 \end{array}\right\}$ 10 times dilution

1 L HCl + 9 L water

55. The first ionisation potential of Na is 5.1 eV----

Ans: –5.1 eV

Sol: $Na \rightarrow Na^{+} + e^{-}$ 5.1 eV
$Na^{+} + e^{-} \rightarrow Na$ $\Delta_{eg}H = -5.1$ eV

56. An organic compound A upon reacting with NH_3 gives B.----

Ans: CH_3CH_2COOH

Sol: $\underset{(A)}{CH_3CH_2COOH} \xrightarrow{NH_3} \underset{(B)}{CH_3CH_2COONH_4}$
$\xrightarrow{\Delta} \underset{(C)}{CH_3CH_2CONH_2} \xrightarrow{Br_2/KOH}$
$CH_3CH_2NH_2$

57. Stability of the species Li_2, Li_2^{-} and Li_2^{+} ----

Ans: $Li_2^{-} < Li_2^{+} < Li_2$

Sol: Li_2 : $\sigma 1s^2$, $\sigma^* 1s^2$, $\sigma 2s^2$, $\sigma^* 2s^0$ B.O = 1
Li_2^{+} : B.O = $\frac{1}{2}$
Li_2^{-} : B.O = $\frac{1}{2}$
In Li_2^{-} there is an e^{-} in antibonding M.O and hence it is less stable than Li_2^{+} :

58. An unknown alcohols is treated with the "Lucas reagent"----

Ans: tertiary alcohol by S_N1

Sol: Tertiary alcohols react most rapidly with Lucas reagent

59. The gas leaked from a storage tank of the ----

Ans: Methylisocynate

Sol: Methylisocynate

60. Experimentally it was found that a metal oxide has formula $M_{0.98}O$.----

Ans: 4.08%

Sol: Let x be M^{2+} ions
2x + (98 – x) 3 = 200
∴ x = 94
∴ M^{3+} ions = 4
Ratio = $\frac{4}{98} = 4.08\%$

PART C – MATHEMATICS

61. Distance between two parallel planes 2x + y + 2z = 8 ----

Ans: $\frac{7}{2}$

Sol: Distance = $\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$

$$= \frac{\left|-8-\frac{5}{2}\right|}{\sqrt{4+1+4}} = \frac{21}{2\times 3} = \frac{7}{2}$$

62. At present, a firm is manufacturing 2000 items. It is ----

Ans: 3500

Sol: $x = 0,\ p_0 = 2000$

$dp = 100 - 12\sqrt{x}$

$\Rightarrow p = 100x - 8x^{\frac{3}{2}} + p_0$

$\therefore\ p = 100x - 8x^{\frac{3}{2}} + 2000$

At x = 25, p = 3500

63. Let A and B two sets containing 2 elements and 4 elements respectively----

Ans: 219

Sol: Total number of subsets = 2^8

Subset containing less than three elements

$= {}^8C_0 + {}^8C_1 + {}^8C_2$

$= 1 + 8 + 28 = 37$

$\therefore$ Required no. of subsets = 256 – 37

= 219

64. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ ----

Ans: exactly two values

Sol: $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$1(1+2k) + (1+k^2) - (2-k) = 0$

$1 + 2k + 1 + k^2 - 2 + k = 0$

$k^2 + 3k = 0$

$k(k+3) = 0$

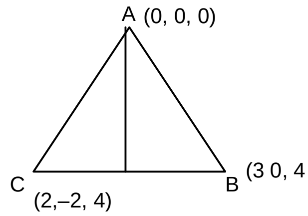
$k = 0, -3$

exactly two values

65. If the vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ ----

Ans: $\sqrt{33}$

Sol:



midpoint of BC = D (4, –1, 4)

$\therefore$ Length of the median

$= \sqrt{16+1+16} = \sqrt{33}$

66. The real number k for which the equation, ----

Ans: does not exist

Sol: $f(x) = 2x^3 + 3x + k = 0$

$f'(x) = 6x^2 + 3 > 0$ for all real x

$\therefore$ f(x) is monotonic increasing

No value of k exists

67. The sum of first 20 terms of the sequence 0.7, 0.77----

Ans: $\frac{7}{81}\left[179 + 10^{-20}\right]$

Sol: $\frac{9}{7}S = .9 + .99 + \ldots\ldots\ldots$

$= 1 - 1 + 1 - .01 + \ldots\ldots\ldots$

$= 20 - .1 + .1^2 + .1^3 + \ldots\ldots + .1^{20})$

$= 20 - \frac{.1(1-.1^{20})}{1-.1} = 20 - \frac{1}{9}(1-.1^{20})$

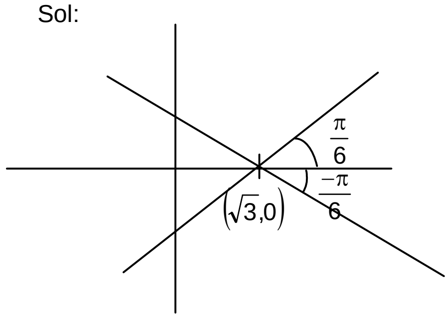
$S = \frac{7}{9}\left[20 - \frac{1}{9}(1-10^{-20})\right]$

$= \frac{7}{81}\left[179 + 10^{-20}\right]$

68. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected----

Ans: $\sqrt{3}y = x - \sqrt{3}$

Sol:



$\sqrt{3}y = \sqrt{3} \Rightarrow \theta = \frac{-\pi}{6}$ at $(\sqrt{3},0)$

$\therefore$ the required ray make angle $\frac{\pi}{6}$ with x axis at $(\sqrt{3},0)$

$\therefore$ The equation is

$y = \frac{1}{\sqrt{3}}(x-\sqrt{3}) \Rightarrow \sqrt{3}y = x-\sqrt{3}$

69. The number of values of k, for which the system of equations----

Ans: 1

Sol: Since system has no solution, the lines are parallel

$\therefore \frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$

$\frac{k+1}{k} = \frac{8}{k+3} \Rightarrow k = 1 \text{ or } 3$

When k = 1

$\frac{8}{k+3} = \frac{4k}{3k-1} \Rightarrow k \in R$

$\therefore$ There is only two value of k = 3 is feasible

70. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$----

Ans: 1: 2: 3

Sol: Since $x^2 + 2x + 3 = 0$ has non – real roots and its is given that one root is common, implies that both roots of equations are common, Hence coefficients are proportional

$\therefore$ a: b: c = 1: 2: 3

71. The circle passing through (1, –2) and touching the axis----

Ans: (5, –2)

Sol: Since circle touches x axis at (3, 0) the center is (3, α)

$\therefore (x-3)^2 + (y-\alpha)^2 = \alpha^2$, it passes through (1,–2) $\Rightarrow \alpha = -2$

$\therefore$ The equation of circle is $(x-3)^2 + (y+2)^2 = 4$ that circle passes through (5, –2)

72. If x, y, z are in A. P and $\tan^{-1}x$, $\tan^{-1}y$----

Ans: x = y = z

Sol: Given that 2xy = x + z and $2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1+xz}\right)$

$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$

$\frac{2y}{1-y^2} = \frac{2y}{1-xz}$

$\Rightarrow xz = y^2 \Rightarrow$ x, y, z are in GP

Hence, x = y = z

73. Consider: statement - I:----

Ans: Statement -I is true; statement II is true; Statement - II is a correct explanation for statement I.

Sol: $P \lambda \sim q \wedge \sim P \wedge q \Leftrightarrow (p \wedge \sim p) \wedge (q \wedge \sim q)$

$\Leftrightarrow F \wedge F \Leftrightarrow F$

$\therefore$ It is a following statement I is true

Since $\sim q \rightarrow \sim p$ is contra positive of $p \rightarrow q$ $\therefore$ $p \rightarrow q \Leftrightarrow (\sim q - \sim p)$ is a tautology, and statement is true. Both statements I and II are true but II does not simply I

74. If $\int f(x)dx = \psi(x)$, then ----

Ans: $\frac{1}{3}x^3\psi(x^3) - \int \chi^2\psi(x^3)dx + c$

Sol: $\int x^5 f(x^3)dx = \int x^3 \bullet x^2 f(x^3)dx \backslash$

$= \frac{1}{3}\int t\ f(t)dt$

$= \frac{1}{3}t\int f(t)dt - \frac{1}{3}\int 1\int f(t)dt$

$= \frac{1}{3}t\psi(t) - \frac{1}{3}\int \psi(t)dt$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

75. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to:---

Ans: 2

Sol: $\frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$

$= \frac{2\sin^2 x}{x \tan 4x}[3+\cos x]$

$= 2\ (3+\cos x)\ \frac{\left(\frac{\sin x}{x}x\right)^2}{\left(\frac{\tan 4x}{4x}\right)} \times 4$

limit $= \frac{2\times 4}{4} = 2$

76. Statement – I: The value of the integral----

Ans: Statement - I is false; statement - II is true.

Sol: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx \int f(x)dx$

$= \int_a^b f(a+b-x)dx$

Adding, $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = \frac{\pi}{3}-\frac{\pi}{6}$

$= \frac{\pi}{6}$

$I = \frac{\pi}{12}$

77. The equation of the circle passing through the foci of the ellipse----

Ans: $x^2 + y^2 - 6y - 7 = 0$

Sol: $a^2 = 16$; $b^2 = 9$
$9 = 16(1 - e^2)$
$16e^2 = 7$
$e = \frac{\sqrt{7}}{4}$
Foci are at $(-\sqrt{7},0)(\sqrt{7},0)$
Centre at (0, 3)
Radius = $\sqrt{(\sqrt{7})^2 + 9} = 4$
Equation of the circle is
$(x-0)^2 + (y-3)^2 = 16$
$x^2 + y^2 - 6y - 7 = 0$

78. A multiple choice examination has 5 questions.----

Ans: $\frac{11}{3^5}$

Sol: 5 questions and 3 choices.
$\therefore$ Probability of correct choice by guessing = $\frac{1}{3}$
Probability of getting exactly four correct answers
$= {}^5C_4 \times \left(\frac{1}{3}\right)^4 \times \frac{2}{3}$
Probability of getting all correct answers
$= \left(\frac{1}{3}\right)^5$

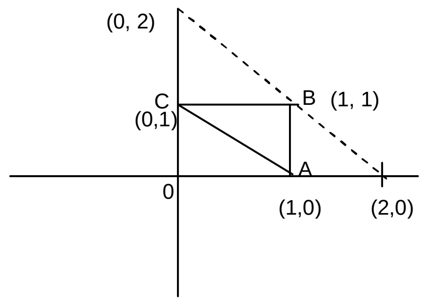
Required answer = $\left(5 \times \frac{2}{3^5}\right) + \frac{1}{3^5}$

$= \frac{10}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

79. The x- coordinate of the incentre of the triangle that has the coordinates of mid points of its----

Ans: $2-\sqrt{2}$

Sol:



Lengths of the sides are 2, $2\sqrt{2}$, 2
x coordinate of the in centre

$= \frac{2\times 0 + 2\sqrt{2}\times 0 + 2\times 2}{4+2\sqrt{2}}$

$= \frac{2}{2+\sqrt{2}}$

$= \frac{2(2-\sqrt{2})}{2}$

$2\sqrt{2}$

80. The term independent of x in expansion of ----

Ans: 210

Sol: Expression

$= \frac{(x+1)\left(x^{\frac{1}{3}}+1\right)}{(x+1)} - \frac{(x-1)\left(x+x^{\frac{1}{2}}\right)}{x^2-x}$

$= \left(x^{\frac{1}{3}} - \frac{1}{x^{\frac{1}{2}}}\right)^{10}$

$T_{r+1} = {}^{10}C_r \left(x^{\frac{1}{3}}\right)^{10-r} \left(\frac{1}{x^{\frac{1}{2}}}\right)^r (-1)^r$

${}^{10}C_r \left(x^{\frac{10}{3}-\frac{r}{3}}\right)\left(\frac{1}{x^{\frac{1}{2}}}\right)^r (-1)^r$

$(-1)^r \frac{{}^{10}C_r}{2^r} \times x^{\frac{10}{3}-\frac{5r}{6}}$

$\frac{10}{3} - \frac{5r}{6} = 0 \Rightarrow r = 4$

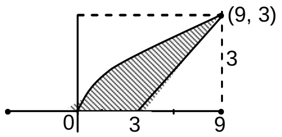
term independent of x

$$(-1)^4 \quad {}^{10}C_4 = \frac{10\times 9\times 8\times 7}{24x}$$
$= 10 \times 3 \times 7$
$= 210$

81. The area (in square units) bounded by the curves $y = \sqrt{x}$, ----

Ans: 9

Sol:



$x - 3 = 2y \quad (x \geq 3)$
Substituting for y $\Rightarrow$
$(x - 3)^2 = 4x$
$x^2 - 10x + 9 = 0$
$\therefore$ x = 9
When x = 9, y = 3
After integrating, the required area
= 18 – 9 = 9.

82. Let T_n be the number of all possible triangles formed by ----
Ans: 5

Sol: $${}^{n+1}C_3 - {}^{n}C_3 = 10$$
$$\frac{(n+1)(n)(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$$
$$\frac{n(n-1)}{6} - \{n+1-(n-2)\} = 10$$
$$\frac{n(n-1)(3)}{6} = 10$$
n (n –1) = 20
n = 5

83. If z is a complex number of units modulus and argument θ, then arg----

Ans: θ

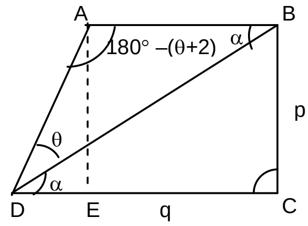
Sol: Let z = cosθ + isinθ

$$\frac{1+z}{1+\bar{z}} = \frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}$$
$$= \frac{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$
$$= \frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}}$$
$$\arg\left(\frac{1+z}{1+\bar{z}}\right) = \frac{\theta}{2} - \left(\frac{-\theta}{2}\right) = \theta$$

84. ABCD is a trapezium such that AB and CD are parallel----

Ans: $\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

Sol:



$$\frac{AB}{\sin\theta} = \frac{BD}{\sin\left[180^\circ - (\theta+\alpha)\right]} \quad BD^2 = p^2 + q^2$$
$$= \frac{BD}{\sin(\theta+\alpha)} =$$
$$= \frac{BD}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$$
$$\frac{BD}{(\sin\theta)\left(\frac{q}{BD}\right) + (\cos\theta)\left(\frac{p}{BD}\right)}$$
$$= \frac{BD^2}{p\cos\theta + q\sin\theta}\sin$$
$$= \frac{(p^2+q^2)}{p\cos\theta + q\sin\theta}$$
$$AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

85. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 $\times$ 3----

Ans: 11

Sol: $|\text{AdjA}| = |A|^2$
$$1\begin{vmatrix} 3 & 3 \\ 4 & 4 \end{vmatrix} - \alpha\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 16$$
$2\alpha - 6 = 16 \Rightarrow \alpha = 11$

86. The intercepts on x- axis made by tangents to the curve, y = ----

Ans: ± 1

Sol: $y = \int_0^x |t|dt$, $x \in R$
$x \in (-\infty, 0)$
$$\int_0^x - tdt$$
$$= \left(\frac{-t^2}{2}\right)_0^x = \frac{-x^2}{2}$$
$x \in (-\infty, 0)$
$$y = \int_0^x tdt = \frac{x^2}{2}$$

Therefore, $f(x) = \begin{cases} -\frac{x^2}{2}, & -\infty < x < 0 \\ \frac{x^2}{2} & 0 \le x < \infty \end{cases}$

$f'(x) = \begin{cases} -x, & -\infty < x < 0 \\ x & 0 \le x < \infty \end{cases}$

$y = \frac{x^2}{2}$

y' = x

Points are (2, 2) and (–2, –2)

Tangents are y – 2 = 2 (x – 2)

y + 2 = 2 (x + 2)

y = 2x – 2, y = 2x + 2

× intercept → x = ±1

87. Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola, ----

Ans: Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I

Sol:

$x^2 + y^2 = \frac{5}{2} \longrightarrow (1)$

$y^2 = 4\sqrt{5}x \longrightarrow (2)$

$y = mx + \frac{\sqrt{5}}{m}$ is a tangent to (2)

If the above line is to be tangent to (1)

$\frac{5}{m^2} = \frac{5}{2}(1 + m^2)$

$\frac{1}{m^2} = \frac{1}{2}(1 + m^2)$

$2 = m^2 + m^4$

$m^4 + m^2 - 2 = 0$

$\therefore m^2 = 1$, which satisfies $m^4 - 3m^2 + 2 = 0$

Statement I is true, Statement II is true but Statement II is not a correct explanation for Statement I.

88. If y = sec ($\tan^{-1}x$), then ----

Ans: $\frac{1}{\sqrt{2}}$

Sol: $y = \sec(\tan^{-1}x)$

$= \sec \sec^{-1} \sqrt{x^2+1}$

$= \sqrt{x^2+1}$

$\therefore \frac{dy}{dx} = \frac{1 \; 2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$

$\left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{2}}$

89. The expression ----

Ans: secA + cosecA+1

Sol:

$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} =$

$\frac{\sin^2 A}{\cos A(\cos A - \sin A)} + \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$

$= \frac{\cos^3 A - \sin^3 A}{\sin A \cos A(\cos A - \sin A)}$

Simplifying ⇒ secA + cosecA+1

90. All the students of a class performed poorly in mathematics.----

Ans: Variance

Sol: Variance is unaffected by linear change of sampling