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Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE 2009

Paper I

PART I

1	2	3	4	5	6	7	8
B	**B**	**C**	**A**	**B**	**A**	**D**	**B**

9	10	11	12
B, C	**C, D**	**A, C, D**	**A, D**

13	14	15	16	17	18
D	**C**	**B**	**B**	**A**	**B**

19	20
A – p, q, r, t	**A – p, q, s, t**
B – q, r, s, t	**B – s, t**
C – p, q, r	**C – p**
D – p, q, r, s	**D – r**

Section I

1. Atomic mass of Fe

$$= \frac{(54\times5)+(56\times90)+(57\times5)}{100}$$

$$= 55.95$$

2. $\frac{an^2}{v^2}$ is the term that corrects for the attractive forces present in a real gas in the van der Waals equation.

3. Sb_2S_3 sol is negatively charged.
$\therefore$The most effective coagulating agent among the given is $Al_2(SO_4)_3$ due to the highest charge on the cation (Al^{3+}).

4. $P_2 = Kx_2$
5×0.8 atm $= 1 \times 10^5$ atm $\times x_2$
$x_2 = 4 \times 10^{-5}$
Mole fraction of N_2 dissolved in 10 moles of water $= 4 \times 10^{-5} \times 10$
$= 4 \times 10^{-4}$

5. P_4O_6 is formed when P_4 is burnt in a limited supply of air. O_2 diluted with N_2 produces that condition.

6. Carboxylic acids are more acidic than phenols. Presence of electron donating groups such as $-CH_3$ group decreases the acid strength of carboxylic acids. Presence of electron withdrawing group such as $-Cl$ increases the acid strength of phenol.

7. Natural rubber is an elastomer. The intermolecular force of attraction is the weakest for elastomers.

8. $-CN$ group has higher priority over $-OH$ and $-Br$ which are given in alphabetical order.

Section II

9. Frenkel defect is favoured by a large difference in sizes of cation and anion. It is a dislocation

effect. Trapping of electrons in lattice sites leads to the formation of F-centres. Schottky defects have effect on the physical properties of solids.

10. $[Pt(en)_2Cl_2]Cl_2$ and $Pt(NH_3)_2Cl_2$ exhibit geometrical isomerism.

11. In excess of air Na_2O is not formed only Na_2O_2 is formed. Small amounts of NaO_2 is also formed which is responsible for the yellow colour of commercial Na_2O_2. Pure Na_2O_2 is colourless. Air always contains varying amounts of moisture which produces small amounts of NaOH.

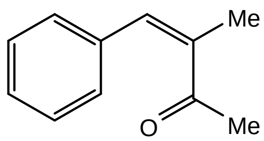
12. (A) Total number of stereo isomers is 6
cis d, l and cis l, d (enantiomers),
trans d, l and trans l, d (enantiomers),
cis d, d (same as cis l, l) meso (plane of symmetry),trans d, d (same as trans l l) meso (centre of symmetry)
(D) Two enantiomers are possible
cis d, l and its mirror image cis l, d

Section III

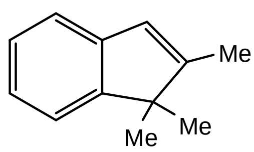
13. Na_2S
Na_2S forms a sulphur bridge in two p-amino-N,N-dimethyl aniline.

14. $FeCl_3$
$FeCl_3$ oxidises the above compound to methylene blue

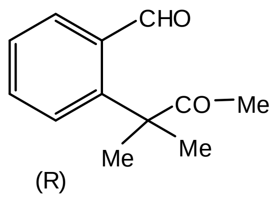
15. $Fe^{3+} + [Fe(CN)_6]^{3-} \rightarrow Fe[Fe(CN)_6]$

16. 

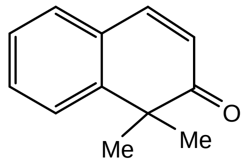
(P)

17.  and

(Q)

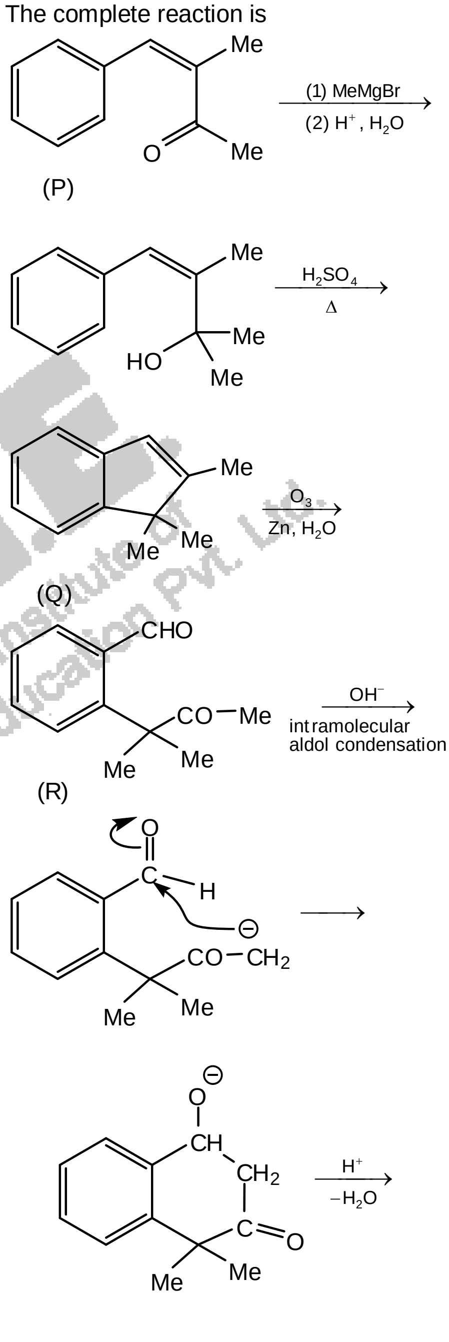


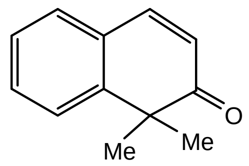
(R)

18. 

(S)

The complete reaction is





(S)

Section IV

19. (A) (p) By MOT B_2 is paramagnetic
(q) Boron can be burnt to B_2O_3
(r) Boron can be reduced with metals to form metal borides.
(t) In B_2 molecule by MOT 2s and 2p orbitals mix to bring the energy of $\sigma 2p_z$ above that of $\pi 2p_x$ and $\pi 2p_y$ (It is equivalent to say that $\sigma 2p_z$ and σ^*2s interact to bring $\sigma 2p_z$ above the $\pi 2p_x$ and $\pi 2p_y$).

(B) (q) N_2 can be oxidised to NO by air.
(r) N_2 undergoes reduction to NH_3.
(s) Bond order in N_2 is 3.
(t) In N_2 molecule also there is mixing of 2s and 2p as in the above case of B_2.

(C) (p) O_2^- is paramagnetic by MOT.
(q)
(r) } In hydrolysis of NaO_2 with water it is oxidized to O_2 and reduced to H_2O_2 simultaneously.

(D) (p) By MOT O_2 is paramagnetic.
(q) O_2 can be oxidized to OF_2 by F_2 and $O_2^+ PtF_6^-$ by PtF_6
(r) O_2 can be reduced to CaO by Ca and CO_2 by C
(s) Bond order in O_2 is 2.

20. (A) $\rightarrow$ p, q, s, t
(B) $\rightarrow$ s, t
(C) $\rightarrow$ p
(D) $\rightarrow$ r

Alkyl cyanides can be reduced to amines by H_2 / Pd / C. Reduction of cyanides with $SnCl_2$ / HCl or DIBAL–H followed by hydrolysis gives corresponding aldehydes. Cyanides can undergo alkaline hydrolysis to form sodium salt of carboxylic acid and NH_3. DIBAL –H reduces esters to aldehydes.

Esters can be catalytically reduced to alcohols and they undergo alkaline hydrolysis.

Double bonds undergo catalytic reduction . Primary amines undergo Hofmann's carbylamine reaction with $CHCl_3$ and alcoholic KOH.

PART II

21	22	23	24	25	26	27	28
A	**B**	**C**	**A**	**D**	**C**	**D**	**C**

29	30	31	32
B, C, D	**A, C**	**B, C**	**B, A**

33	34	35	36	37	38
A	**B**	**B**	**A**	**B**	**D**

39	40
A – p, q, s	**A – p**
B – p, t	**B – s, t**
C – p, q, r, t	**C – r**
D – s	**D – q, s**

Section I

21. $\frac{x-1}{-3} = \frac{y+1}{1} = \frac{z-2}{5} = \mu$

Q($-3\mu + 1$, $\mu - 1$, $5\mu + 2$)

P(3, 2, 6)

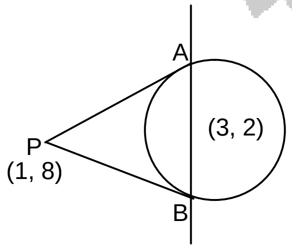
$\overrightarrow{PQ} = [-3\mu - 2,\ \mu - 3,\ 5\mu - 4]$

$[1,\ -4,\ 3]$

$-3\mu - 2 - 4\mu + 12 + 15\mu - 12 = 0$

$8\mu - 2 = 0 \Rightarrow \mu = \frac{1}{4}$

22.



$r = \sqrt{3^2 + 2^2 + 11} = \sqrt{24}$

Equation of AB is

$x \times 1 + y \times 8 - 3(x + 1) - 2(y + 8) - 11 = 0$

$x + 8y - 3x - 3 - 2y - 16 - 11 = 0$

$-2x + 6y - 30 = 0$

$x - 3y + 15 = 0$

Let the circle be

$x^2 + y^2 - 6x - 4y - 11 + \lambda(x - 3y + 15) = 0$

It passes through (1, 8)

$1 + 64 - 6 - 32 - 11$

$+ \lambda(1 - 24 + 15) = 0$

$16 - 8\lambda = 0$

$\lambda = 2$

$x^2 + y^2 - 6x - 4y - 11 + 2(x - 3y + 15) = 0$

$x^2 + y^2 - 4x - 10y + 19 = 0$

23. $\int_0^x \sqrt{1 - (f'(t))^2}\, dt = \int_0^x f(t)dt$

Differentiating w.r.t. x :

$\sqrt{1 - \left(\frac{dy}{dx}\right)^2} = f(x)$

$y^2 = 1 - \left(\frac{dy}{dx}\right)^2$

$\left(\frac{dy}{dx}\right)^2 = 1 - y^2$

$\frac{dy}{dx} = \pm\sqrt{1 - y^2}$

$\pm \frac{dy}{\sqrt{1 - y^2}} = dx$

Integrating ,

(+) $\sin^{-1} y = x + C$

$0 = 0 + C \Rightarrow C = 0$

$y = \sin x$

(–) $\cos^{-1} y = x + C$

But $\frac{\pi}{2} = 0 + C$

$\therefore \cos^{-1} y = x + \frac{\pi}{2}$

$y = \cos(x + \frac{\pi}{2})$

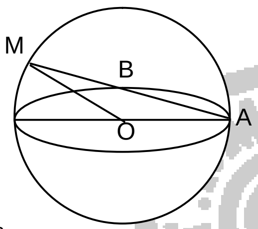
$= -\sin x$

But f(x) is non negative in [0, 1]
$\therefore$ f(x) = sin x

$$\left.\begin{array}{l}\sin\frac{1}{2} < \frac{1}{2} \\ \sin\frac{1}{3} < \frac{1}{3}\end{array}\right\}$$

24. $(z\bar{z})(\bar{z})^2 + (\bar{z}\,z)z^2 = 350$
$|z|^2\left(z^2 + \bar{z}^2\right) = 350$
$\left(x^2+y^2\right)\left\{2\left(x^2-y^2\right)\right\} = 350$
$x^4 - y^4 = 175$
$(x^2 + y^2)(x^2 - y^2) = 175$
$x^2 = 16 \quad \Rightarrow x = \pm 4$
$y^2 = 9 \quad \Rightarrow y = \pm 3$
$\therefore$ Area of the rectangle = 8 $\times$ 6 = 48

25.



$$\frac{x^2}{9} + \frac{y^2}{1} = 0$$
Auxiliary O is $x^2 + y^2 = 9$
A(3, 0)
B(0, 1)

Slope of AB = $-\frac{1}{3}$

$y = -\frac{1}{3}(x - 3)$
$3y = -x + 3$
$y = \frac{-x}{3} + 1$

$$x^2 + \left(\frac{-x}{3} + 1\right)^2 = 9$$

$$x^2 + \frac{x^2}{9} + 1 - \frac{2x}{3} = 9$$
$9x^2 + x^2 + 9 - 6x = 81$
$10x^2 - 6x - 72 = 0$
$5x^2 - 3x - 36 = 0$
$$x = \frac{3 \pm \sqrt{9+720}}{10} = \frac{3 \pm 27}{10}$$
$$= 3, -\frac{12}{5}$$
$$y = \frac{-12}{5x-3} + 1$$
$$= \frac{4}{5} + 1 = \frac{9}{5}$$
Area OAM = $\frac{27}{5} \times \frac{1}{2} = \frac{27}{10}$

26. Given $\left(\bar{a}\times\bar{b}\right).\left(\bar{c}\times\bar{d}\right) = 1$
$\left|\bar{a}\times\bar{b}\right|\left|\bar{c}\times\bar{d}\right|\cos\gamma = 1$ where γ is the angle between $\left(\bar{a}\times\bar{b}\right)$ and $\left(\bar{c}\times\bar{d}\right)$
$\Rightarrow \sin\alpha\ \sin\beta\ \cos\gamma = 1$ (since $|\bar{a}| = |\bar{b}| = |\bar{c}| = |\bar{d}| = 1$ and we assume that angle between $\bar{a}$ and $\bar{b}$ is α and that, the angle between $\bar{c}$ and $\bar{d}$ is β)
$\Rightarrow \sin\alpha = 1, \sin\beta = 1, \cos\gamma = 1$
$\Rightarrow \alpha = \beta = \frac{\pi}{2},\ \gamma = 0$
$\Rightarrow$ $\bar{a}$ and $\bar{b}$ are orthogonal; $\bar{c}$ and $\bar{b}$ are orthogonal; $\bar{a}\times\bar{b}$ is parallel to $\bar{c}\times\bar{d}$.
$\Rightarrow$ $\bar{a}, \bar{b}, \bar{a}\times\bar{b}$ form a mutually orthogonal triad
$\bar{c}, \bar{d}, \bar{c}\times\bar{d}$ form a mutually orthogonal triad
Suppose $\bar{a} \parallel \bar{d}$ and $\bar{b} \parallel \bar{c}$
Let $\bar{b} = k\bar{c}$
$\bar{a} \perp \bar{b} \Rightarrow \bar{a}.\bar{b} = 0$
$\Rightarrow \bar{a}.k\bar{c} = 0,$
a contradiction
$\therefore$ D is false.
As $\bar{a}$ not parallel to $\bar{c}$ we should have that $\bar{b}$ parallel to $\bar{c}$
$\therefore$ (C) is the choice

27. $\displaystyle\sum_{m=1}^{15} \operatorname{Im} z^{2m-1} = \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\theta$
We have $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots + \sin(\alpha + \overline{n-1}\beta)$
$$= \frac{\sin\left(\frac{\alpha + \alpha + \overline{n-1}\,\beta}{2}\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}}$$
Here $\beta = 2\theta$
$\therefore \sin\theta + \sin 3\theta + \ldots + \sin 29\theta$
$$= \frac{\sin\left(\frac{\theta + \theta + 14\times 2\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\frac{2\theta}{2}}$$
$$= \frac{\sin^2 15\theta}{\sin\theta}$$
$$= \frac{\sin^2 30^\circ}{\sin 2^\circ} = \frac{1}{4\sin 2^\circ}$$

28. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 10$
$(x + x^2 + x^3)^7 = x^7(1 + x + x^2)^7$
Coefficient of x^3 in $(1 + x + x^2)^7$
$$= \text{Coefficient of } x^3 \text{ in } \frac{\left(1-x^3\right)^7}{(1-x)^7}$$

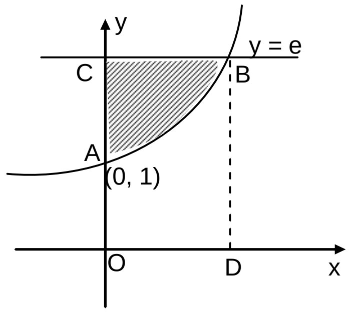
$= \text{Coefficient of } x^3 \text{ in } (1 - x^3)^7 (1 - x)^{-7}$

$= \frac{7.8.9}{1.2.3} - 7 \times 1$

$= 84 - 7 = 77$

Section II

29.



Required area = area of the region ABC

= Area OCBD – Area OABD

$= e \times 1 - \int_0^1 e^x\,dx$

$= e - \int_0^1 e^x\,dx$

$= e - (e - 1) = 1$

$\int_1^e \ell n\,y dy = [y \log y - y]_1^e$

$= (e - e) - (0 - 1)$

$= 1$

$\int_1^e \ell n\,y dy = \int_1^e \ell n(1 + e - y)dy$

30. $L = \lim_{x\to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}\ (a > 0)\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to 0} \dfrac{\left(-\frac{1}{2}\right)\frac{(-2x)}{\sqrt{a^2 - x^2}} - \frac{x}{2}}{4x^3}$

$= \lim_{x\to 0} \dfrac{\frac{1}{\sqrt{a^2 - x^2}} - \frac{1}{2}}{4x^2}$

It is given that L is finite $\Rightarrow \frac{1}{a} = \frac{1}{2}$

$\Rightarrow a = 2$

When a = 2

$L = \lim_{x\to 0} \dfrac{2 - \sqrt{4 - x^2} - \frac{x^2}{4}}{x^4}$

$= \lim_{x\to 0} \dfrac{\left(2 - \frac{x^2}{4}\right)^2 - (4 - x^2)}{x^4\left(2 - \frac{x^2}{4} + \sqrt{4 - x^2}\right)} = \lim_{x\to 0} \frac{1}{16}\,\frac{1}{4} = \frac{1}{64}$

31. $2\cos\frac{B+C}{2}\cos\frac{B-C}{2} = 4\sin^2\frac{A}{2}$

$2\sin\frac{A}{2}.\cos\frac{B-C}{2} = 4\sin^2\frac{A}{2}$

$\cos\left(\frac{B-C}{2}\right) = 2\sin\frac{A}{2}$

$= 2\cos\frac{B+C}{2}$

$\cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2}$

$= 2\left\{\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right\}$

$\cos\frac{B}{2}\cos\frac{C}{2} = 3\sin\frac{B}{2}\sin\frac{C}{2}$

$\tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{3}$

$\Rightarrow \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}\ \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{3}$

$\Rightarrow \frac{s-a}{s} = \frac{1}{3}$

3s – 3a = s

2s – 3a = 0

a+ b + c – 3a = 0

b + c = 2a

b + c = 2a means

CA + BA = 2a, a constant

$\Rightarrow$ Locus of A is an ellipse

32. Given $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ — (1)

Dividing by $\cos^4 x$

$\frac{\tan^4 x}{2} + \frac{1}{3} = \frac{\sec^4 x}{5}$

$= \frac{(1 + \tan^2 x)^2}{5}$

$\Rightarrow \tan^4 x\left(\frac{1}{2} - \frac{1}{5}\right) - \frac{2}{5}\tan^2 x + \frac{1}{3} - \frac{1}{5} = 0$

$\Rightarrow \frac{3}{10}\tan^4 x - \frac{2}{5}\tan^2 x + \frac{2}{15} = 0$

$\Rightarrow 9\tan^4 x - 12\tan^2 x + 4 = 0$

$\Rightarrow (3\tan^2 x - 2)^2 = 0$

$\Rightarrow \tan^2 x = \frac{2}{3}$ — (2)

$\therefore$ (A) is true

$$\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}$$

$$= \cos^8 x\left\{\frac{\tan^8 x}{8}+\frac{1}{27}\right\}$$

$$= (\cos^2 x)^4 \left\{\frac{\left(\frac{2}{3}\right)^4}{8}+\frac{1}{27}\right\}$$

$$= \left(\frac{1}{1+\tan^2 x}\right)^4\left\{\frac{16}{81\times 8}+\frac{1}{27}\right\}$$

$$= \left(\frac{3}{5}\right)^4\left\{\frac{2}{81}+\frac{1}{27}\right\}$$

$$= \frac{81}{625}\times\frac{5}{81}=\frac{1}{125}$$

Equation (2) $\Rightarrow \frac{\sin^2 x}{2} = \frac{\cos^2 x}{2} = k$

$\Rightarrow \sin^2 x = 2k$ and $\cos^2 x = 3k$

$\therefore$ 2k + 3k = 1

$\Rightarrow k = \frac{1}{5}$

$$\therefore \frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{(2k)^4}{8}+\frac{(3k)^4}{27}$$

$$= k^4[2+3] = 5.k^4 = \frac{1}{125}$$

Section III

33. A symmetric matrix can be written as $\begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}$

But we have five 1s and four 0s.
The three symmetrical pairs can be filled as per the following.
Case 1
2 pairs of 1s and 1 pair of 0s. This is done in 3 ways. The main diagonal is filled using the remaining 1, 0, 0 in 3 ways.
$\therefore$ 9 ways.
Case 2
1 pair of 1s and 2 pairs of 0s. This is done in 3 ways. The main diagonal is filled using the remaining 1, 1, 1
$\therefore$ Total 3 ways
$\therefore$ 9 + 3 = 12 matrices

34. The matrices are

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ (1) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (3)

$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (5) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (6)

$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (7) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (8) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (9)

$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ (10) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ (11) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (12)

Determinants of the matrices 1, 2, 3, 6, 9 and 12 are zeros and all the other 6 matrices are non – singular. Each of these six matrices provide a unique solution to the given system.

35. When we observe matrices 1and 9, since right hand side is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, they vanish for all Δ_i and thus give infinite number of solutions. Matrices 2, 3, 6 and 12 give inconsistent systems.

36. $P(X = 3) = \frac{5}{6}.\frac{5}{6}.\frac{1}{6}=\frac{25}{216}$

37. $P(X \geq 3) = 1 - P(X = 1 \text{ or } X = 2)$

$$= 1 - \left[\frac{1}{6}+\frac{5}{6}.\frac{1}{6}\right] = 1-\frac{11}{36}=\frac{25}{36}$$

38. $P(X \geq 6 / X > 3) = P\left(\frac{(X\geq 6)\cap(X>3)}{P(X>3)}\right)$

$$= \frac{P(X\geq 6)}{P(X\geq 4)}$$

$$= \frac{\left(\frac{5}{6}\right)^5.\frac{1}{6}+\left(\frac{5}{6}\right)^6.\frac{1}{6}+......}{\left(\frac{5}{6}\right)^3.\frac{1}{6}+\left(\frac{5}{6}\right)^4.\frac{1}{6}+.....}$$

$$= \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$$

Section IV

39. (A) $\frac{dy}{dx} = \frac{-y}{(x-3)^2}$

$\frac{dy}{y} = -\frac{dx}{(x-3)^2}$

$\ell ny = \frac{1}{x-3}$

$y = e^{\frac{1}{x-3}}$

Domain of non zero solution is D : R – {3}

Intervals contained in the domain D are

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{8}\right)$

$\therefore$ A $\rightarrow$ p, q, s

(B) $I = \int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\ dx$

$= \int_{-2}^{2} (t+2)(t+1)\,t\,(t-1)\,(t-2)\,dt$

= 0

$(\because \int_{-a}^{a} f(x)dx = 0,\ \text{if}\ f(-x) = -f(x))$

Intervals containing the value I = 0 are

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $(-\pi, \pi)$

B $\rightarrow$ (p, t)

(C) $y = \cos^2 x + \sin x$

$y' = -\ 2\cos x \sin x + \cos x$

$= \cos x\ (-\ 2\sin x + 1) = -\ \sin 2x + \cos x$

For extremum, y' = 0

$\Rightarrow \cos x = 0$ or $\sin x = \frac{1}{2}$

$y'' = -\ 2 \cos 2x - \sin x$

When $\cos x = 0$, $y'' = 2(1) - 1 > 0$

$\therefore$ cos 2 = 0 gives a local minimum

When $\sin x = \frac{1}{2}$,

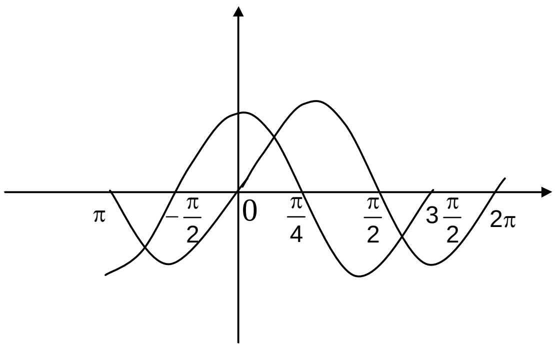
$y'' = -\ 2\left(1 - \frac{2}{4}\right) - \frac{1}{2} < 0$

$\Rightarrow \sin x = \frac{1}{2}$ gives a local maximum

$\Rightarrow\ x = n\pi + (-1)^n \frac{\pi}{6}$

$\therefore$ C $\rightarrow$ p, q, r, t

(D)



$y = \tan^{-1}(\sin x + \cos x)$

$y' = \frac{1}{(\sin x + \cos x)^2 + 1}(\cos x - \sin x)$

y = f(x) is increasing if y' > 0

$\Rightarrow \cos x > \sin x$ since denominator > 0

$\Rightarrow x \in \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$

Interval in which y is increasing is $\left(0, \frac{\pi}{8}\right)$

D $\rightarrow$ s

40. (p) $m = \frac{-h}{k},\ a = 2, c = \frac{1}{k}$

$\frac{1}{k^2} = 4\left(1 + \frac{h^2}{k^2}\right)$

$\Rightarrow h^2 + k^2 = \frac{1}{4}$

$\Rightarrow$ Locus of (h, k) is a circle

$\Rightarrow$ (A)

(q) Difference = a constant 3.

$\Rightarrow$ Locus of z is a hyperbola $\quad \Rightarrow$ (D)

(r) $x = \sqrt{3}\ \cos 2\theta, y = \sin 2\theta$

$\frac{x^2}{3} + \frac{y^2}{1} = 1$

$\Rightarrow$ Ellipse $\quad \Rightarrow$(C)

(s) Eccentricity = 1 $\rightarrow$ Parabola

Eccentricity > 1 $\rightarrow$ hyperbola

$\Rightarrow$ (B), (D)

(t) $\text{Re}\{(x + 1 + i\,y)^2\} = x^2 + y^2 + 1$

$\Rightarrow (x + 1)^2 - y^2 = x^2 + y^2 + 1$

$\Rightarrow 2y^2 = 2x$

$\Rightarrow y^2 = x$

$\Rightarrow$ Parabola $\Rightarrow$ (B)

PART III

41	42	43	44	45	46	47	48
B	**B**	**C**	**A**	**C**	**D**	**A**	**D**

49	50	51	52
A	**C, D**	**A, D**	**B, D**

53	54	55	56	57	58
D	**A**	**B**	**A**	**B**	**D**

59	60
A – p, r, s	**A – p, t**
B – r, s	**B – q, s, t**
C – p, q, t	**C – p, r, t**
D – r, s	**D – q**

Section I

41. $\frac{Q_1}{R_1^2} = \frac{Q_1 + Q_2}{R_2^2} = \frac{Q_1 + Q_2 + Q_3}{R_3^2}$

$\Rightarrow \frac{Q_2}{Q_1} = 3;\ \frac{Q_3}{Q_1} = 5;$

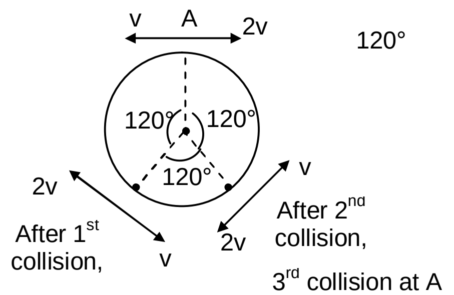
42. At 60°, $mg\sin\theta \frac{h}{2} > mg\cos\theta \frac{a}{2}$

$\therefore$ it will topple at $\theta < 60°$

43. $v^2 = 2gs = 2 \times 10 \times (20 - 12.8) \Rightarrow$
$v = 12\ m\ s^{-1}$

$v' = \mu \times v = \frac{4}{3} \times 12 = 16\ m\ s^{-1}$

44. $y_{CM} = \frac{ma + ma + m.0 + m(-a) + 6m.0}{10m} = \frac{a}{10}$

45. 

46. ϕ = AB, increases. By Lenz's law, induced current in direction dc and ab

47. Charged enclosed = $\frac{1}{2}$ that on disc + $\frac{1}{4}$ that on rod + point charge –7c

$\therefore \phi = {-2C}/{\varepsilon_0}$

48. T = 8s, phase = $\frac{2\pi}{T}.t = \frac{\pi}{3}$

$\omega = \frac{2\pi}{T}\ \therefore\ a = -\omega^2 A.\sin\frac{\pi}{3}$ (A = 1 cm)

$= \frac{-\sqrt{3}}{32}\pi^2.cm\ s^{-2}$

Section II

49. Internal forces can convert K.E to P.E (eg. Spring masses system). Since Newton's third law. A couple exerts no force but a torque.

50.

Reading	f	Error	Calculation
(42, 56)	24	0	$0.2 \times \left(24/56\right)^2$
(48, 48)	24	0	$0.2 \times \left(24/48\right)^2$
(60, 40)	24	0	$0.2 \times \left(24/40\right)^2$
(66, 33)	22	–2	$0.2 \times \left(24/33\right)^2$
(78, 39)	26	+2	$0.2 \times \left(24/39\right)^2$

51. $R_{eq} = 3.2\ K\Omega \Rightarrow I = \dfrac{24v}{3.2\,K\Omega} = 7.5$ mA

$V_{RL} = 7.5\ mA \times 1.2\ K\Omega = 9V$

Effective emf formula = $\dfrac{E/R_1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$ and

$\dfrac{E/R_2}{1/R_2 + 1/R_1} \Rightarrow$ ratio = 3

$\therefore$ Ratio of power = 9

52. $C_p - C_v = R$ for all gases

$C_v = \frac{3}{2}R$ for monoatomic

$\frac{5}{2}R$ for diatomic

Section III

53. High temperature ionizes the gas

54. Total KE = 3KT = P.E = $\dfrac{e^2}{4\pi\varepsilon_0}.\dfrac{1}{r}$

$\therefore T \simeq 1.4 \times 10^9$ K

55. Multiply and check nt with Lawson Number

56. $n\dfrac{\lambda}{2} = a$

$p = \dfrac{h}{\lambda}$

$E = \dfrac{p^2}{2m} \Rightarrow E \propto \dfrac{1}{\lambda^2} \propto \dfrac{1}{a^2}$

57. $E = \left.\dfrac{h^2}{8ma^2}\right|_{\text{for } n=1} = 8 \times 10^{-3}$ eV

$$\left(E = \frac{p^2}{2m} = \left(\frac{h}{\lambda}\right)^2\Big/2m = \left(\frac{h}{2a}\right)^2\Big/2m = \frac{h^2}{8ma^2}\right)$$

58. $v \propto p,\ p = \dfrac{h}{\lambda} \Rightarrow \lambda \propto \dfrac{1}{n}$

$\Rightarrow p \propto h \Rightarrow v \propto n$

Section IV

59. Unlike charges moving along a circle $\Rightarrow$ no current (say reason 1)

(p) +, − charges are symmetric
$\therefore$ E = 0
Same reason, V = 0
Due to reason 1, B = 0 and $\mu = 0$

(q) Unsymmetric distribution or charges about M. Hence $E \neq 0$ and V = 0
Due to reason (1), B = 0 and $\mu = 0$

(r) Due to symmetry E = 0, $V \neq 0$
Clearly $B \neq 0$, $\mu \neq 0$

(s) By symmetry, E = 0, distances being not commensurate, $V \neq 0$, negative currents reinforce B plus charges oppose but of different magnitude.

(t) Due to lack of symmetry $E \neq 0$. But V can be zero. Due to reason (1) $B = 0 \Rightarrow \mu = 0$

60. (p) Y has constant velocity. Therefore, reaction force is equal to weight.
PE is continuously decreasing. Mechanical energy decreasing due to frictional loss. Torque is variable

(q) Magnetic force between Z and Y is Mg
$\therefore$ Normal reaction is 2 Mg. Since it is moving up gravitational P.E is increasing and thus mechanical energy is increasing. By symmetry, torque is zero

(r) Pulley supports the mass M. So reaction force = $\left(m_0 + \sqrt{2}\,M\right)g$. Since it is moving down gravitational P.E is decreasing and so the mechanical energy is decreasing. Torque is a non-zero constant

(s) Sphere moving down with uniform acceleration. Therefore force < Mg. Gravitational P.E of x is increasing and Mechanical energy is conserved. Torque is a non-zero constant

(t) Terminal velocity $\Rightarrow$ net force zero. Gravitational P.E of x is increasing, but mechanical energy is decreasing because of frictional forces. Torque is a non-zero constant.