SOLUTIONS & ANSWERS FOR JEE MAINS-2016
VERSION – E

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

1. A student measures the time period of 100 oscillations ----

Ans: 92 ± 2 s

Sol: $\bar{x} = \frac{\sum xi}{N} = 92$

$\Delta\bar{x} = \frac{\sum \Delta x}{N} = 1.5$ s

The least count is 1 s, $\Delta x = 2$ s

Reported mean time = 92 ± 2 s

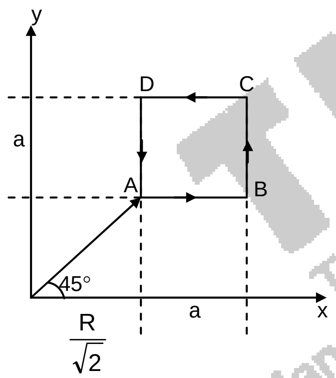
2. A particle of mass m is moving along the side of a square of side ----

Ans: $\bar{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from, C to D.

AND

$\bar{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from, D to A.

Sol:



$\bar{L}_{AB} = mv\frac{R}{\sqrt{2}}\left(-\hat{k}\right)$ [at A]

$L_{CD} = mv\left(\frac{R}{\sqrt{2}} + a\right)\left(+\hat{k}\right)$ [at C]

$L_{BC} = mv\left(\frac{R}{\sqrt{2}} + a\right)\left(+\hat{k}\right)$ [at C]

$L_{DA} = mv\frac{R}{\sqrt{2}}\left(-\hat{k}\right)$ [at A]

Option (2 and 4) can be the answer.

3. A point particle o f mass m, moves along the uniformly ----

Ans: 0.29 and 3.5 m

Sol: $\mu mg\cos\theta \times PQ = \mu mg\,QR$ (data)

$\frac{x}{PQ} \times PQ = QR \Rightarrow x = QR$

$\tan 30^\circ = \frac{2}{x} = \frac{1}{\sqrt{3}} \Rightarrow QR = x = 2\sqrt{3}$

= 3.46 m

= 3.5 m

Work energy theorem

$-\mu mgx - \mu mg(QR) = 0 - mgh$

$2\mu x = h$

$\mu = \frac{h}{2x} = \frac{2}{2 \times 2\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

$= \frac{1.732}{6} = 0.3$

4. A person trying to lose weight by burning fat lifts a mass of 10 kg ----

Ans: 12.89×10^{-3}

Sol: $m \times 3.8 \times 10^7 \times 20\%$

$= 10 \times 9.8 \times 1 \times 1000$

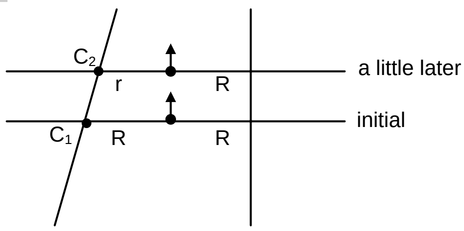
$m = \frac{9.8 \times 10^4}{3.8 \times 0.2 \times 10^7}$

$= 12.89 \times 10^{-3}$ kg

5. A roller is made by joining together two cones at their vertices O. It is kept ----

Ans: turn left

Sol:



At point of contact C_1 $v = \omega R$ (pure roll)

At point of contact C_2 $v > \omega r \Rightarrow$

forward slipping, ∴ friction will act backwards towards C_1.

Roller will turn anti-clockwise since torque of friction about CM is anti-clockwise.

6. A satellite is revolving in a circular orbit at a height `h' from the earth's ----

Ans: $\sqrt{gR}\left(\sqrt{2} - 1\right)$

Sol: $v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

$v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

Increase = $\sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\left[\sqrt{2} - 1\right]$

7. A pendulum clock loses 12 s a day if the temperature is 40 °C and gains 4 s ----

Ans: 25 °C; $\alpha = 1.85 \times 10^{-5}$ /°C

Sol: $12 = \frac{1}{2}\alpha(40 - T_0)$

$4 = \frac{1}{2}\alpha(T_0 - 20)$

If temperature increases, clock loses time and temperature decreases, clock gains time.

Solving T_0 = 25 °C, $\alpha = 1.85 \times 10^{-5}$ /°C

8. An ideal gas undergoes a quasi static, reversible process in which its ----

Ans: $n = \frac{C - C_p}{C - C_V}$

Sol: pV^n = constant, μ = no. of moles

work = $\int pdV = \frac{\mu R\Delta T}{-n+1}$

According to I law of thermodynamics

$\mu C\Delta T = \mu C_V\Delta T + \frac{\mu R\Delta T}{-n+1}$

$C - C_V + \frac{C_p - C_V}{-n+1}$

$-n + 1 = \frac{C_p - C_V}{C - C_V}$

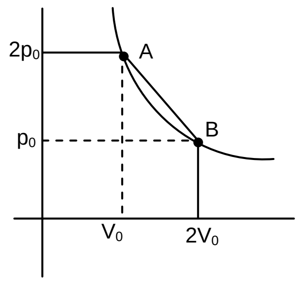
$n = 1 - \frac{C_p - C_V}{C - C_V} = \frac{C - C_V - C_p + C_V}{C - C_V}$

$= \frac{C - C_p}{C - C_V}$

9. `n' moles of an ideal gas undergoes a process $A \rightarrow B$ as ----

Ans: $\frac{9}{4}\frac{p_0V_0}{nR}$

Sol: Draw the isotherm through A & B ($2p_0V_0 = nRT$)



Maximum temperature will be at mid point of AB; that is an isotherm tangent to the straight line AB at its mid point.

$p = 1.5p_0$ $V = 1.5\ V_0$

$pV = nRT \Rightarrow T = \frac{1.5p_0 \times 1.5V_0}{nR}$

$= \frac{9}{4}\frac{p_0V_0}{nR}$

10. A particle performs simple harmonic motion with amplitude A. Its speed is ----

Ans: $\frac{7}{3}$ A

Sol: Potential energy $U = \frac{1}{2}K\left(\frac{2}{3}A\right)^2$

Kinetic energy = $\frac{1}{2}K\left[A^2 - \left(\frac{2}{3}A\right)^2\right]$

New K.E =9K (K.E $\propto v^2$)

New total energy = 9K + U = $\frac{1}{2}KA_1^2$

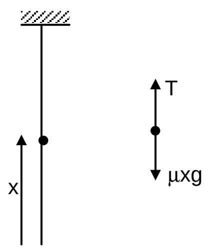
$\frac{9}{2}K\left[A^2 - \frac{4}{9}A^2\right] + \frac{1}{2}K\frac{4}{9}A^2$

$= \frac{1}{2}KA_1^2 \Rightarrow A_1 = \frac{7}{3}A$

11. A uniform string of length 20 m is suspended from a rigid support. A short ----

Ans: $2\sqrt{2}$ s

Sol:



μ = mass per unit length

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu xg}{\mu}}$

$\Rightarrow v = \sqrt{xg} = \frac{dx}{dt}$

$\int \sqrt{g}dt = \int_0^{20} x^{-\frac{1}{2}}dx$

$\sqrt{g} \times t = \frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}} = 2\sqrt{x}\Big|_0^{20}$

$\sqrt{10} \times t = 2 \times \sqrt{20}$

$\Rightarrow t = 2\sqrt{2}$ s

12. The region between two concentric spheres of radii `a' and `b', respectively ----

Ans: $\frac{Q}{2\pi a^2}$

Sol: $\oint \overline{E}_r \bullet d\overline{A} = \frac{(q_{enclosed})}{\varepsilon_0} r$ -----(1)

$q_{enclosed}$ (r) = $\int_a^r 4\pi r^2 dr \times \rho$

$= \int_a^r 4\pi r^2 dr \times \frac{A}{r}$

$= 4\pi A \frac{r^2}{2}\Big|_a^r$

$= 2\pi A(r^2 - a^2)$

(1) $\Rightarrow E_r \times 4\pi r^2 = \frac{2\pi A(r^2 - a^2)}{\varepsilon_0}$

$E_r = \frac{A}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right)$

Total E at r = E_r due to volume charge + E due to charge at centre

$E_{total} = \frac{A}{2\varepsilon_0}\left(1 - \frac{a^2}{r^2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

For r^2 term to vanish $A = \frac{Q}{2\pi a^2}$

13. A combination of capacitors is set up as shown in the figure. The magnitude ----

Ans: 420 N/C

Sol: Charge on 4 μF is 24 μC
Charge on 9 μF is 18 μC
∴ Q = 42 μC

$E = 9 \times 10^9 \times \frac{42 \times 10^{-6}}{(30)^2} = 420$ N/C

14. The temperature dependence of resistances of Cu and undoped Si in the ----

Ans: Linear increase for Cu, exponential decreases of Si

Sol: Intrinsic carrier concentration

$n = n_0 e^{-Eg/2k_BT}$

n increases exponentially with increase in T. Hence conductivity increases exponentially or resistivity decreases exponentially.
For Cu, $R = R_0(1 + \alpha t)$, it is a linear increase.

15. Two identical wires A and B, each of length `ℓ' carry the same current I. Wire A is bent ----

Ans: $\frac{\pi^2}{8\sqrt{2}}$

Sol: $B_A = \frac{\mu_0 I}{2R}$ (For circle)

$B_B = \frac{\mu_0 I}{4\pi \frac{a}{2}}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] \times 4$ sides

$= \frac{\mu_0 I}{\pi a} 2\sqrt{2}$

[Using $\frac{\mu_0 I}{4\pi d}$ ($\sin\theta_1 + \sin\theta_2$)]

Also $\ell = 2\pi R = 4a$

$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$

16. Hysteresis loops for two magnetic materials A and B ----

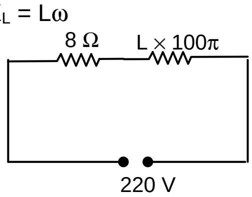
Ans: B for electromagnets and transformers

Sol: For transformers to reduce hysteresis loss, area of the loop should be small and for electromagnets the material should have low retentivity and co-ercivity.

17. An arc lamp requires a direct current of 10 A at 80 V to function. If it is ----

Ans: 0.065 H

Sol: I = 10 A
V = 220 V, f = 50 Hz
R = 0.8 Ω
$X_L = L\omega$



$I = 10 = \frac{220}{\sqrt{64 + L^2 10^5}}$

$64 + 10^5 L^2 = 484$

$10^5 L^2 = 420$

$L^2 = 420 \times 10^{-5}$

$= 42 \times 10^{-4}$

$L = 6 \times 10^{-2}$
$= 0.06$

18. Arrange the following electromagnetic radiations per quantum----

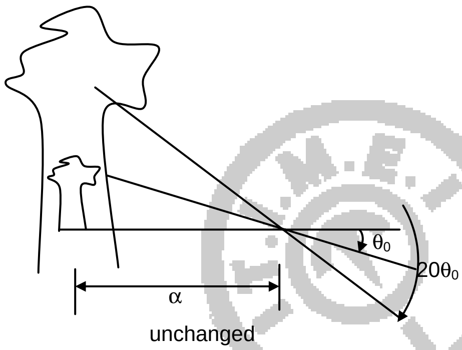
Ans: D, B, A, C

Sol: Least : Radiowave
: yellow (500 nm)
: Blue (400 nm)
Highest : X-ray
$D \rightarrow B \rightarrow A \rightarrow C$

19. An observer looks at a distant tree of height 10 m with a ----

Ans: 20 times taller

Sol:



θ_0 = visual image

$\tan\theta_0 = \frac{h}{d}$ $\tan 20\theta_0 = \frac{h'}{d}$

($\tan\theta = \theta$ for small θ)

20. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that ----

Ans: $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

Sol: Fresnel distance

$= \frac{a^2}{\lambda} = L$

$a = \sqrt{\lambda L}$

$\beta = b_{min} = \frac{2\lambda L}{\sqrt{\lambda L}} = \sqrt{4\lambda L}$

21. Radiation of wavelength λ, is incident on a photocell. The fastest emited----

Ans: $v' > v\left(\frac{4}{3}\right)^{1/2}$

Sol: $v = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi_0\right)}$

$v' = \sqrt{\frac{2}{m}\left(\frac{4}{3}\frac{hc}{\lambda} - \phi_0\right)}$

$\therefore\ v' > v\left(\frac{4}{3}\right)^{1/2}$

22. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes ----

Ans: 5 : 4

Sol: $\frac{N_1}{N_0} = \left(\frac{1}{2}\right)^{\frac{80}{20}} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\frac{N_2}{N_0} = \left(\frac{1}{2}\right)\frac{80}{40} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow$ decayed $N_1 = \frac{15}{16} N_0$

$N_2 = \frac{3}{4} N_0$

$\therefore$ ratio is $\frac{5}{4}$

23. If a, b, c, d are inputs to a gate and x is its output, then, as per ----

Ans: OR

Sol: Theoretical

24. Choose the correct statement:
(1) In amplitude modulation the ----

Ans: In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

Sol: Theoretical

25. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to ----

Ans: 0.80 mm

Sol: $LC = \frac{1}{100}$ mm ZE = 0.05
TR = 0.5 + (0.25 + 0.05)
= 0.80 mm

26. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped ----

Ans: $\frac{f}{2}$

Sol: $f_0 = \frac{v}{2\ell}$

$f_c = \frac{v}{4\ell}$

$f_c = \frac{f_0}{2} = \frac{f}{2}$

27. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when ----

Ans: 0.01

Sol: $S = \frac{I_g G}{I - I_g} \approx \frac{1\times10^{-3}\times100}{10}$

$= 0.01\ \Omega$

28. In an experiment for determination of refractive index of glass of aprism ----

Ans: 1.5

Sol: $i_1 + i_2 = 114$

$D = i_1 + i_2 - A \Rightarrow A = 74$

$\mu = \frac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}} = \frac{\sin 57}{\sin 37}$

≈ 1.5

29. Identify the semiconductor devices whose characteristics are given ----

Ans: Simple diode, zener diode, solar cell, light dependent resistance

Sol: Theoretical

30. For a common emitter configuration, if α and β have their usual meanings, ----

Ans: $\alpha = \frac{\beta}{1-\beta}$

Sol: $I_C = I_C + I_B$

$\alpha = \frac{I_C}{I_E}$ $\quad\quad \beta = \frac{I_C}{I_B}$

PART – B – CHEMISTRY

31. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires ----

Ans: C_3H_8

Sol: $C_3H_{8(g)} + 5O_{2(g)} \rightarrow 3CO_{2(g)} + 4H_2O_{(\ell)}$

15 mL 75 mL 45 mL

After combustion, the gases occupy 345 mL

32. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure ----

Ans: $2P_i\left(\frac{T_1}{T_1+T_2}\right)$

Sol: $P_i \times 2T_1 = P_f(T_1+T_2)$

$P_f = 2P_i\left(\frac{T_1}{T_1+T_2}\right)$

33. A stream of electrons from a heated filament was passed between two charged----

Ans: $\sqrt{2m\ eV}$

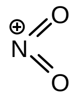
Sol: $\lambda = \frac{h}{\sqrt{2m\ eV}}$

$\frac{h}{\lambda} = \sqrt{2m\ eV}$

34. The species in which the N atoms is in a state of sp hybridisation is: ----

Ans: NO_2^+

Sol:



bp = 2

lp = 0

Hybridisation = sp

35. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}----

Ans: –110.5

Sol: $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$

ΔH = (–393.5) – (–283.5)

= –110 kJ

36. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapour pressure of water (in torr) ----

Ans: 752.4

Sol: $\frac{P^\circ - P_s}{P^\circ} = \frac{n_2}{n_1+n_2}$

$\frac{760 - p_s}{760} = \frac{0.1}{0.1+9.9}$

$P_s = 760 - 7.6$

$= 752.4$ Torr

37. The equilibrium constant at 298 K for a reaction A + B ⇌ C + D is 100. ----

Ans: 1.818

Sol:

	A	+	B ⇌ C	+	D
Initial	1		1	1	1
Eqbm	1-x		1-x	1+x	1+x

$K_c = \frac{(1+x)^2}{(1-x)^2}$

$10 = \frac{1+x}{1-x}$

$x = \frac{9}{11}$

$$[D] = 1 + x = 1 + \frac{9}{11}$$
$$= \frac{20}{11}$$
$$= 1.818$$

38. Galvanization is applying a coating of: ----

Ans: Zn

Sol: Coating of Zn on metal surface is galvanization.

39. Decomposition of H_2O_2 follows a first order reaction. ----

Ans: 6.93×10^{-4} mol min^{-1}

Sol: $H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$

0.5 M $\rightarrow$ 0.25 M $\rightarrow$ 0.125 M

$t_{1/2} = 25$ minutes

Rate of disappearance of $H_2O_2 = k[H_2O_2]$

$$= \frac{0.693 \times 0.05}{25}$$

Rate of formation of oxygen

$$= \frac{0.693}{25} \times 0.05 \times \frac{1}{2}$$
$$= 6.93 \times 10^{-4} \text{ mol min}^{-1}$$

40. For a linear plot of log (x/m) versus log p in a Freundlich adsorption isotherm, ----

Ans: Only 1/n appears as the slope

Sol: $x/m = k\ P^{1/n}$

$\log x/m = \log k + 1/n \log P$

$1/n$ is the slope

41. Which of the following atoms has the highest first ionization energy? ----

Ans: Sc

Sol: Sc has the highest first ionisation enthalpy
Sc – 631 kJ mol^{-1}
Na – 496 kJ mol^{-1}
K – 419 kJ mol^{-1}
Rb – 403 kJ mol^{-1}

42. Which one of the following ores best concentrated by froth floatation method? ----

Ans: Galena

Sol: Galena is PbS.
Sulphide ores are concentrated by froth floatation method.

43. Which one of the following statements about water is **FALSE**? ----

Ans: There is extensive intramolecular hydrogen bonding in the condensed phase

Sol: There is no intramolecular hydrogen bonding in water.

44. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively----

Ans: Li_2O, Na_2O_2 and KO_2

Sol: Li mainly forms monoxide, Li_2O, Na forms peroxide, Na_2O_2 and K forms superoxide, KO_2.

45. The reaction of Zinc with dilute and concentrated nitric acid, respectively ----

Ans: N_2O and NO_2

Sol: $4Zn + 10HNO_3$ (dilute) $\rightarrow$
$$4Zn(NO_3)_2 + 5H_2O + N_2O$$
$Zn + 4HNO_3$(conc) $\rightarrow$
$$Zn(NO_3)_2 + 2H_2O + 2NO_2$$

46. The pair in which phosphorus atoms have a formal oxidation state of +3 is:----

Ans: Orthophosphorus and pyrophosphorus acids

Sol: Orthophosphorus acid is $H_3\overset{+3}{P}O_3$ and pyrophosphorus acid is $H_4P_2O_5$.

47. Which of the following compounds is metallic and ferromagnetic? ----

Ans: CrO_2

Sol: CrO_2 is metallic and ferromagnetic

48. The pair having the same magnetic moment is: ----

Ans: $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Sol: $Cr^{2+} \rightarrow [Ar]3d^4$

$Fe^{2+} \rightarrow [Ar]3d^6$

Both contains 4 unpaired electrons.

49. Which one of the following complexes shows optical isomerism? ----

Ans: cis $[Co(en)_2Cl_2]Cl$

Sol: cis $[Co(en)_2Cl_2]Cl$ is optically active.

50. The concentration of fluorides, lead, nitrate and iron in a water sample ----

Ans: Nitrate

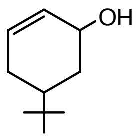
Sol: The maximum permissible concentration of NO_3^- in drinking water is only 50 ppm.

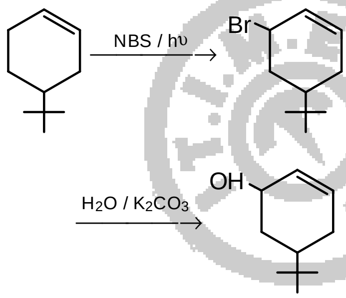
51. The distillation technique most suited for separating glycerol from spent-lye----

Ans: Distillation under reduced pressure

Sol: Glycerol is purified by distillation under reduced pressure.

52. The product of the reaction given below is:----

Ans: 

Sol: 

53. The absolute configuration of ----

Ans: (2S, 3R)

Sol: According to CIP rules.

54. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields: ----

Ans: (c) only

Sol: Tertiary alkyl halide will undergo elimination with $NaOCH_3$. No ether will be formed.

55. The reaction of propene with HOCl (Cl_2 + H_2O) proceeds through the intermediate ----

Ans: $CH_3–CH^+–CH_2–Cl$

Sol: $CH_3 - CH = CH_2 \xrightarrow{Cl^+} CH_3 - \overset{\oplus}{C}H - CH_2Cl$

56. In the Hoffmann bromamide degradation reaction, the number of moles of ----

Ans: Four moles of NaOH and one mole of Br_2

Sol: $RCONH_2 + Br_2 + 4NaOH \rightarrow RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O$

57. Which of the following statements about low density polytthene is **FALSE**?----

Ans: It is used in the manufacture of buckets, dust-bins etc

Sol: HDP is used for the manufacture of buckets, dust-bins, etc

58. Thiol group is present in ----

Ans: Cysteine

Sol: $H_2N-\underset{\substack{|\\CH_2-SH}}{CH}-COOH$

Cysteine

59. Which of the following is the anionic detergent? ----

Ans: Sodium lauryl sulphate

Sol: Sodium lauryl sulphate is an anionic detergent

60. The hottest region of Bunsen flame ----

Ans: region 4

Sol: Outer layer of the flame is the hottest region.

PART – C -MATHEMATICS

61. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0,$ and S = {x∈ **R**: f(x)----

Ans: contains exactly two elements

Sol: Let f(x) = A and $f\left(\frac{1}{x}\right) = B$

Given : A + 2B = 3x, replacing x by $\frac{1}{x}$

$\Rightarrow 2B + 4A = \frac{6}{x}$

$\therefore 3A = \frac{6}{x} - 3x$

$\Rightarrow A = \frac{2}{x} - x$

$\frac{2}{x} - x = \frac{-2}{x} + x$

$2x = \frac{4}{x}$

$x^2 = 2$

$\therefore\ x = \pm\sqrt{2} \Rightarrow$ Two elements

62. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ ----

Ans: $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol: $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ purely imaginary

$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta}=0$

$\Rightarrow \sin^2\theta = \frac{1}{3}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{3}}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

63. The sum of all real values of x satisfying the equation----

Ans: 3

Sol: $x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60 \in R$
Or $x^2 - 5x + 5 \in R$ and non zero and $x^2 + 4x - 60 = 0$
or $x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ is even
$\therefore$ x = 1, 4 or x = –10, 6 or x = 2 , 3
But x = 3 $\Rightarrow x^2 + 4x - 60$ is odd
$\therefore$ x = 1, 4, –10, 6, 2
Sum of the roots = 3

64. If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T, then----

Ans: 5

Sol: A adj A = AA^T
$|A|I = AA^T$

$\Rightarrow \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a^2-b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$

$\Rightarrow$ 10a + 3b = 13
15a – 2b = 0
Solving $a = \frac{2}{5}$ b = 3
$\therefore$ 5a + b = 2 + 3 = 5

65. The system of linear equations
$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$ ----

Ans: exactly three values of λ

Sol: $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$

$\lambda^3 - \lambda = 0 \Rightarrow \lambda(\lambda^2 - 1) = 0$
$\lambda = 0, \lambda^2 = 1 \Rightarrow \lambda = \pm 1$
Exactly three values of λ

66. If all the words (with or without meaning) having five letters----

Ans: 58^{th}

Sol: Arranging SMALL is ascending order ALLMS

Starting with A $-\ \frac{4!}{2!} = \frac{24}{2} = 12$
Starting with L –4! = 24
Starting with M $-\ \frac{4!}{2!} = \frac{24}{2} = 12$
Starting with SA $-\ \frac{3!}{2!} = \frac{6}{2} = 3$
Starting with SL –3! = 6
SMALL $\Rightarrow$ 1
Total = 12 + 24 + 12 + 3 + 6 + 1
= 58^{th}

67. If the number of terms in the expansion of ----

Ans: 729

Sol: Number of terms = 28

$\frac{(n+1)(n+2)}{2} = 28$

$\Rightarrow$ n + 1 = 7 $\Rightarrow$ n = 6
Put x = 1
$(1 - 2 + 4)^6 = 3^6 = 729$

68. If the 2^{nd}, 5^{th}, and 9^{th} terms of a non – constant A.P are in G.P----

Ans: $\frac{4}{3}$

Sol: a + d, a + 4d, a + 8d are in GP.
$\therefore (a + 4d)^2 = (a + d)(a + 8d)$
$a^2 + 16d + 8ad = a^2 + 8d^2 + 9ad$
$\Rightarrow 8d^2 - ad = 0$
d(8d – a) = 0 $d \neq 0$
$\therefore$ a = 8d
$\therefore$ Terms are 9d, 12d, 16d are in G.P
$\therefore\ r = \frac{12d}{9d} = \frac{4}{3}$

69. If the sum of the first ten terms of the series ----

Ans: 101

Sol: The sequence may be rearranged as

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2$ ...... up to ten terms

$= \frac{16}{25}\left[2^2 + 3^2 + 4^2 + ... + 11^2\right]$

$= \frac{16}{25}\left[\frac{11\times12\times23}{6} - 1\right] = \frac{16}{25}\times505$

$= \frac{16}{5} m(\text{given}) \Rightarrow m = 101$

70. Let $p = \lim_{x\to 0^+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p ----

Ans: $\frac{1}{2}$

Sol: $p = e^{\lim_{x\to 0}\left(\frac{\tan^2\sqrt{x}}{2x}\right)}$

$= e^{\lim_{x\to 0}\frac{2\tan\sqrt{x}\ \sec^2\sqrt{x}\ \frac{1}{2\sqrt{x}}}{2}}$

$= e^{\lim_{x\to 0}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)\sec^2\sqrt{x}\ \frac{1}{2}}$

$= e^{\frac{1}{2}}$

$\therefore$ log p = $\frac{1}{2}$

71. For x ∈ **R,** f(x) = |log2 – sinx| and g(x) = ----

Ans: g is not differential at x = 0

Sol: f(x) = |log2 –sinx|
g(x) = f(f(x))
g ‘ (0) = f ‘(f (0)) f ‘(0)
f(0) = log2
and f ‘(log2) does not exist
$\therefore$ the derivative does not exist

72. Consider $f(x) = \tan^{-1}$----

Ans: $\left(0, \frac{2\pi}{3}\right)$

Sol: $f(x) = \tan^{-1}\sqrt{\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}}$

$= \tan^{-1}\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$= \frac{\pi}{4}+\frac{x}{2}$

$f'(x) = \frac{1}{2}$

$\therefore$ Slope of normal = –2

$\therefore$ Equation $y-\frac{\pi}{3} = -2\left(x-\frac{\pi}{6}\right)$

Satisfying $\left(0, \frac{2\pi}{3}\right)$

73. A wire of length 2 units is cut into two parts which are bent respectively----

Ans: x = 2r

Sol: Given $4x + 2\pi r = 2$

$\Rightarrow x = \frac{1-\pi r}{2}$

Area = $x^2 + \pi r^2$

$= \frac{1+\pi^2 r^2 - 2\pi r}{4} + \pi r^2$

$\frac{dA}{dr} = 0 \Rightarrow r = \frac{1}{\pi+4}$

$x = \frac{2}{\pi+4}$

$\frac{d^2A}{dr^2} > 0$

$\therefore$ x = 2r

74. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to----

Ans: $= \frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

Sol: $\int \frac{\left(2x^{12}+5x^9\right)dx}{\left(x^5+x^3+1\right)} = \int \frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}$

$= \int \frac{-du}{u^3}$ if $u = 1+\frac{1}{x^2}+\frac{1}{x^5}$

$= \frac{1}{2u^2}+C$

$= \frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$

$= \frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

75. $\lim_{n\to\infty}\left(\frac{(n+1)\ (n+2)......3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to----

Ans: $\frac{27}{e^2}$

Sol: $L = \lim_{n\to\infty}\left[\frac{(n+1)(n+2).....(n+2n)}{n^{2n}}\right]^{\frac{1}{n}}$

$\log L = \lim_{n\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+...\right.$

$\left......+\log\left(1+\frac{2n}{n}\right)\right]$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\frac{r}{n}\right)$

$$= \int_0^2 \log(1+x)dx$$

$$= \left\{ [\log(1+x) \times x]_0^2 - \int_0^2 \frac{1}{1+x} x dx \right\}$$

= 2log3 – (2 – log3)

= 3log3 – 2

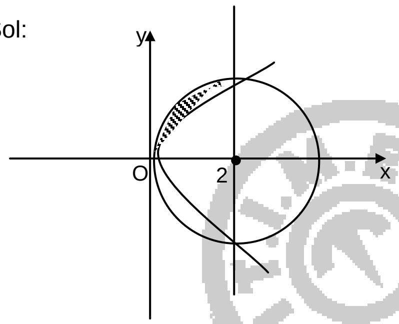
$$= \log\left(\frac{27}{e^2}\right)$$

$$L = \frac{27}{e^2}$$

76. The area (in sq. units) of the region ----

Ans: $\pi - \frac{8}{3}$

Sol:



$y^2 = 2x$ ——— (1)

$x^2 - y^2 - 4x = 0$ ——— (2)

To find the x- coordinates of their points of intersection, solve (1) and (2)

$x^2 + 2x - 4x = 0$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0$ or 2

$$\therefore \text{Area} = \int_0^2 (y_1 - y_2)dx$$

$$= \int_0^2 \left[\sqrt{4-(x-2)^2} - \sqrt{2}\ \sqrt{x}\right]dx$$

$$= \left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-2}{2}\right) - \sqrt{2}\ \frac{2}{3}x^{\frac{3}{2}}\right]_0^2$$

$$= \pi - \frac{8}{3}$$

77. If a curve y = f(x) passes through the point (1, –1) and satisfies----

Ans: $\frac{4}{5}$

Sol: y(1 + xy)dx = xdy

$$x\frac{dy}{dx} = y + xy^2$$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{-1}{xy} - 1$$

$$-\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = -1$$

i.e. $\frac{dY}{dx} + \frac{Y}{x} = -1$ where $Y = \frac{1}{y}$

Solution is $Yx = \frac{-x^2}{2} + C$

$$-1 = -\frac{1}{2} + C \Rightarrow C = \frac{-1}{2}$$

$$\frac{x}{y} = -\frac{x^2+1}{2}$$

$$y = \frac{-2x}{1+x^2};\quad y\left(-\frac{1}{2}\right) = \frac{1}{1+\frac{1}{4}}$$

$$= \frac{4}{5}$$

78. Two sides of a rhombus are along the lines, x – y + 1 = 0----

Ans: $\left(\frac{1}{3}, \frac{-8}{3}\right)$

Sol: Point of intersection of x – y + 1 = 0 and 7x – y – 5 = 0 is (1, 2) is one of the vertex. The image point is also not the required point. Equation of the diagonal containing (1, 2) and (–1, –2) is 2 x – y + 1 = 0

∴ The other diagonal is x + 2y + k = 0

i.e. –1 – 4 + k = 0

x + 2y + 5 = 0

Solving x + 2y + 5 = 0 and

7x – y – 5 = 0

We get $x = \frac{1}{3}$ and $x = \frac{-8}{3}$

∴ required vertex is $\left(\frac{1}{3}, \frac{-8}{3}\right)$

79. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$----

Ans: a parabola

Sol: If (a, b) is the centre then radius is b(if b > 0) & –b (if b < 0)

We have

$(a-4)^2 + (b-4)^2 = (6+b)^2$ if b > 0

$(a-4)^2 + (b-4)^2 = (6-b)^2$ if b < 0

∴ locus is

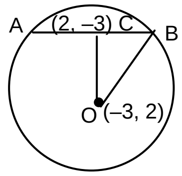
$(x-4)^2 + (y-4)^2 = (6+y)^2$ or

$(x-4)^2 + (y-4)^2 = (6-y)^2$, each of which is a parabola

80. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$ ----

Ans: $5\sqrt{3}$

Sol:



AB is the diameter of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ ——— (1)
Midpoint of AB = centre of (1) = (2, –3)
CB = radius of (1)

$= \sqrt{4+9+12} = 5$

$OC = \sqrt{5^2+5^2} = 5\sqrt{2}$

$\therefore$ Radius of S = OB

$= \sqrt{OC^2 + CB^2}$

$= \sqrt{25+50}$

$= \sqrt{75}$

$= 5\sqrt{3}$

81. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle----

Ans: $x^2 + y^2 - 4x + 8y + 12 = 0$

Sol: Let P be the point $(2t^2, 4t)$
C is (0, –6)
$D = PC^2 = (2t^2)^2 + (4t+6)^2$
$= 4t^4 + 16t^2 + 48t + 36$

$\frac{d}{dt}(D) = 16t^3 + 32t + +48$

$\frac{d^2}{dt^2}(D) = 48t^2 + 32$

$\frac{d}{dt}(D) = 0 \;\; \Rightarrow t = -1$

$\therefore$ P is (2, –4)
$\therefore$ Circle with centre at P is
$(x-2)^2 + (y+4)^2 = r^2$
Since, passes through (0, –6)
$r^2 = 8$
$\therefore$ Required circle is
$(x-2)^2 + (y+4)^2 = 8$
or $x^2 + y^2 - 4x + 8y + 12 = 0$

82. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 ----

Ans: $\frac{2}{\sqrt{3}}$

Sol: $\frac{2b^2}{a} = 8$ —— (1)
$2b = ae$ —— (2)
(2) $\Rightarrow 4b^2 = a^2e^2$
$4a^2(e^2 - 1) = a^2e^2$
$\Rightarrow 4e^2 - 4 = e^2$

$\Rightarrow e^2 = \frac{4}{3}$

$\Rightarrow e = \frac{2}{\sqrt{3}}$

83. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the ----

Ans: $10\sqrt{3}$

Sol: (1, –5, 9)
$x - y + z = 5$
$x = y = z$

$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda \Rightarrow$

Point is $(\lambda + 1, \lambda - 5, \lambda + 9)$
Where the two meet the plane
$\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$
$\lambda = -10$
$\therefore$ point is (–9, –15, 1)
distance $= \sqrt{100+100+100} = 10\sqrt{3}$

84. If the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in----

Ans: 2

Sol: $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$

lies on $\ell x + my - z = 9$
$3\ell - 2m + 4 = 9$
$2\ell - m - 3 = 0$
$\Rightarrow 3\ell - 2m = 5$
$2\ell - m = 3$
$\Rightarrow \ell = 1, m = -1$
$\ell^2 + m^2 = 2$

85. Let $\bar{a}$, $\bar{b}$ and $\bar{c}$ be three ----

Ans: $\frac{2\pi}{3}$

Sol: $\frac{\sqrt{3}}{2}(\bar{b} + \bar{c}) = \bar{a} \times (\bar{b} \times \bar{c})$

$= (\bar{a}\,.\bar{c})\bar{b} - (\bar{a}.\bar{b})\,\bar{c}$

$\therefore\; (\bar{a}\,.\,\bar{b}) = -\frac{\sqrt{3}}{2}$

$\therefore$ angle between $\bar{a}$ & $\bar{b}$ is $\frac{2\pi}{3}$

$(\bar{a}\,,\,\bar{b}$ are unit vectors$)$

86. If the standard deviation of the numbers 2, 3, a and 11 is 3.5,----

Ans: $3a^2 - 32a + 84 = 0$

Sol: Variance $\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$$12.25 = \frac{4+9+121+a^2}{4} - \frac{(16+a)^2}{16}$$

$$= \frac{134\times4+4a^2-256-32a-a^2}{16}$$

$\Rightarrow 196 = 3a^2 - 32a + 536 - 256$

$\Rightarrow 3a^2 - 32a + 84 = 0$

87. Let two fair six – faced dice A and B be thrown simultaneously.----

Ans: E_1, E_2, E_3 are independent

Sol: $E_1 = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$

$E_2 = \{(1, 2), (2, 2), (3, 2), (4,2), (5, 2), (6, 2)\}$

$E_3 = \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)\}$

$P(E_1).\ P(E_2) = \frac{1}{6}.\ \frac{1}{6} = \frac{1}{36} = P(E_1 \cap E_2)$

$\therefore E_1, E_2$ independent

$P(E_1)\ P(E_3) = \frac{1}{6}.\ \frac{1}{2} = \frac{1}{12} = P(E_1 \cap E_3)$

E_1, E_3 independent

$P(E_2)\ P(E_3) = \frac{1}{6}\ \frac{1}{2} = \frac{1}{12} = P(E_2 \cap E_3)$

$P(E_1)\ P(E_2)\ P(E_3)$

$= \frac{1}{6}.\ \frac{1}{6}\ \frac{1}{12} = \frac{1}{72} \neq P(E_1 \cap E_2 \cap E_3)$

$\therefore E_1, E_2, E_3$ not independent

88. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation----

Ans: 7

Sol: The equation $\cos x(\cos 2x + \cos 3x) = 0$

$\therefore \cos x = 0$ or $\cos 3x = -\cos 2x$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $3x = (2n + 1)\pi \pm 2x$

$\Rightarrow x = (2n + 1)\pi$

or $x = (2n+1)\frac{\pi}{5}$

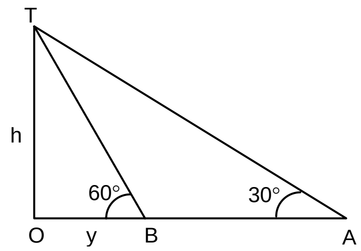
Solution are $\frac{\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{3\pi}{2}$

Total 7 nos

89. A man is walking towards a vertical pillar in a straight path, at a uniform speed----

Ans: 5

Sol:



OT is tower – height h(say)

AB = x

OB = y

$$x + y = \frac{h}{\tan 30^\circ} = h\sqrt{3}$$

$$x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$$

$$\therefore y = \frac{2h}{\sqrt{3}}$$

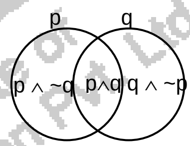
$$\frac{y}{x} = \frac{1}{2}$$

x takes 10 m, $\therefore$ true for y = 5 m

90. The Boolean Expression $(p\wedge \sim q) \vee q \vee (\sim p \wedge q)$ ----

Ans: $p \vee q$

Sol:



$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= p \vee q$