T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2016

Paper I – Code 0

PART I

1	2	3	4	5
A	**B**	**B**	**D**	**D**

6	7	8	9	10	11	12	13
A, D	**A, C**	**A, B, D**	**B, D**	**B, C, D**	**A, C, D**	**A, B, D**	**C, D**

14	15	16	17	18
6	**9**	**3**	**8**	**9**

Section I

1. For $\alpha = 45° = i$
   $r = 30°$. When ray graces PR after separation
   $c = 45°$
   ext. $L = 45 + 30 = 75°$
   $\theta = 180 - (75 + 90) = 15°$

2. $eV_0 = \frac{hc}{\lambda} - \phi$

   $e(2) = \frac{hc}{0.3\times10^{-6}} - \phi$

   $e(1) = \frac{hc}{0.4\times10^{-6}} - \phi$

   $e = hc\left[\frac{1}{0.3} - \frac{1}{0.4}\right]\times10^{6}$

   $e = hc\times\frac{0.1}{0.3\times0.4}\times10^{6}$

   $h = \frac{1.6\times10^{-19}\times0.3\times0.4}{3\times10^{8}\times0.1\times10^{6}}$
   $= 0.64 \times 10^{-33}$
   $= 6.4 \times 10^{-34}$

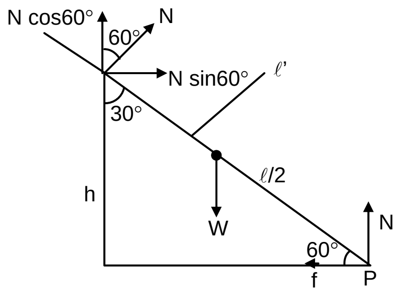
3. Rate of heat generated $\frac{dQ}{dt} = 3$ kW
   Let any time `t', temperature of cooler = T
   Rate of cooling:
   $ms\frac{dT}{dt} = 3\text{ kW} - P$

   $\int_{10}^{30} dT = \frac{(3\text{ kW} - P)}{ms}\int_{0}^{3} dt$

   $30 - 10 = \frac{(3\text{ kW} - P)\times3\times3600}{120\times4.2\times10^{3}}$

   $3\text{ kW} - P = \frac{20\times120\times42}{3\times36} = \frac{2800}{3}$

   $P = 3000 - 933 = 2067$

4. 

   $\cos30° = \frac{h}{\ell'} = \frac{\sqrt{3}}{2}$

   $\ell' = \frac{2h}{\sqrt{3}}$

   $N\cos60° + N = 16 = \frac{3N}{2} \Rightarrow N = \frac{32}{3}$

   $N\sin60° = f$

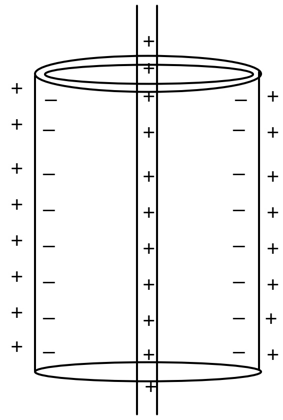
   $16\times\frac{2}{3}\times\frac{\sqrt{3}}{2} = f \Rightarrow f = \frac{16\sqrt{3}}{3}$ newton

   Taking torque about P,

$N \times \ell' = W \times \frac{\ell}{2}\cos 60°$

$\frac{32}{3} \times \frac{2h}{\sqrt{3}} = 16 \times \frac{\ell}{2} \times \frac{1}{2} \Rightarrow \frac{h}{\ell} = \frac{3\sqrt{3}}{16}$

5.



It is a case of discharging of a capacitor in the resistor.

$Q = Q_0 e^{-t/RC}$

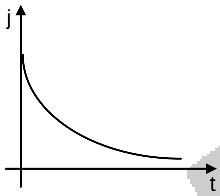
Dividing by ℓ,

$\frac{Q}{\ell} = \frac{Q_0}{\ell} e^{-t/RC}$

$\lambda' = \lambda e^{-t/RC}$

$j = \sigma E = \frac{\sigma \times \lambda'}{2\pi\varepsilon r} = \frac{\sigma\lambda e^{-t/RC}}{2\pi\varepsilon r}$ (at a point)

Hence



Section II

6. $\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{1}{f}$

$\frac{1}{60} - \frac{-1}{-30} = \frac{1}{f}$ $\qquad \frac{1}{f} = \frac{60+30}{60 \times 30}$

f = 20 cm

(D) correct

$\frac{1}{+10} + \frac{1}{-30} = \frac{1}{f'} \Rightarrow f' = 15$ cm

Since convex mirror, (c) wrong

Radius of curvature $R_1 = 2f' = 30$ cm

$\Rightarrow$ (B) wrong

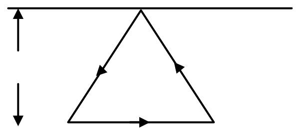
$(\mu - 1) \times \left(\frac{1}{R_1}\right) = \frac{1}{f}$

$(\mu - 1) \times \frac{1}{30} = \frac{1}{20}$

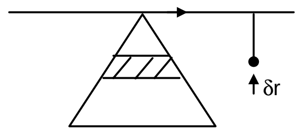
$\Rightarrow \mu - 1 = \frac{3}{2} = 1.5 \Rightarrow \mu = 2.5$

$\Rightarrow$ (A) correct

7.



The induced current will be parallel to the current in the Hypotenuse. Consider the reciprocal case.



Suppose the current I flows in the infinite wire

Flux $\delta\phi$ through the shaded area is

$\delta\phi = \frac{\mu_0 I}{2\pi r} 2r.\delta r = \frac{\mu_0 I}{\pi}.dr$

The total flux = $\frac{\mu_0 I}{\pi}\int_0^{\ell} dr = \frac{\mu_0 I \ell}{\pi}$

$\therefore$ Mutual inductance $M = \frac{\phi}{I} = \frac{\mu_0 \ell}{\pi}$

When the current flows through the triangle, the flux associated with the wire $\phi = \frac{\mu_0 \ell}{\pi} I$

$\therefore E = -\frac{d\phi}{dt}$; we have $|E| = \frac{\mu_0 \ell}{\pi}\frac{dI}{dt}$

Putting $\ell = 10$ cm & $\frac{dI}{dt} = 10$ A/ s

$\therefore$ Induced emf = $\left(\frac{\mu_0}{\pi}\right)$ volt

Force is repulsive follows from Lenz law.

Any element of the rotating wire will only have its velocity parallel or anti-parallel to the magnetic field (in the reciprocal case) of the straight wire.

8. $\bar{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$\bar{v} = 3t^2\alpha\hat{i} + 2t\beta\hat{j}$

$\bar{a} = 6t\alpha\hat{i} + 2\beta\hat{j}$

$\bar{v} = 3 \times 1^2 \times \frac{10}{3}\hat{i} + 2 \times 1 \times 5\hat{j}$

$= 10\hat{i} + 10\hat{j}$ (A) correct

$\bar{F} = m\bar{a} = 0.1 \times \left[6 \times 1 \times \frac{10}{3}\hat{i} + 2 \times 5\hat{j}\right]$

$= \left(2\hat{i} + \hat{j}\right)$ (C) Wrong

$\bar{L} = \bar{r} \times \bar{p} = \left(\alpha\hat{i} + \beta\hat{j}\right) \times 0.1\left(3\alpha\hat{i} + 2\beta\hat{j}\right)$

$= 0.1\left[2\alpha\beta\hat{k} - 3\alpha\beta\hat{k}\right]$

$= 0.1\alpha\beta\left(-\hat{k}\right)$

$= 0.1 \times \frac{10}{3} \times 5\left(-\hat{k}\right) = -\frac{5}{3}\hat{k}$

(B) correct

$\bar{\tau} = \bar{r} \times \bar{F} = \left(\alpha\hat{i} + \beta\hat{j}\right)\left(2\hat{i} + \hat{j}\right)$

$= \alpha\hat{k} - 2\beta\hat{k} = \left(\frac{10}{3} - 2\times 5\right)\hat{k}$

$= -\frac{20}{3}\hat{k}$

(D) correct

9. E = farad/m = $\frac{\text{coulomb}}{\text{volt m}} = \frac{\text{coulomb}^2}{\text{J m}}$

k_BT = energy = joule

A. $\sqrt{\frac{\text{coloumb}^2}{m^3 \times \frac{\text{coulomb}^2}{\text{J m}} \times J}} = \frac{1}{m} \Rightarrow$ WRONG

B. $\sqrt{\frac{\text{coloumb}^2}{\text{J m}}} = \frac{J}{\frac{1}{m^3}\times \text{coulomb}^2} = m$

$\Rightarrow$ CORRECT

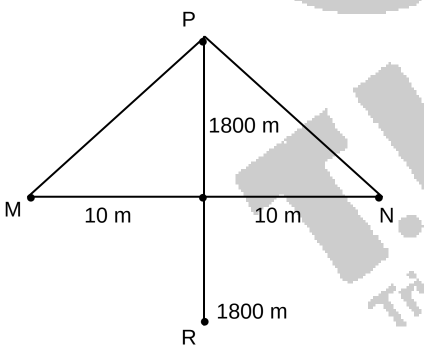
C. $\sqrt{\frac{\text{coloumb}^2}{\frac{\text{coulomb}^2}{J\times m}\times\left(\frac{1}{m^3}\right)^{2/3}\times J}} = m^{3/2}$

$\Rightarrow$ WRONG

D. $\sqrt{\frac{\text{coloumb}^2}{\frac{\text{coulomb}^2}{\text{J m}}\times\left(\frac{1}{m^3}\right)^{1/3}\times J}} = m$

$\Rightarrow$ CORRECT

10.



The component of the car's velocity along PM and PN are to be considered.

$f_1' = f_1\left[\frac{c+v_0'}{c}\right]$ $f_2' = f_2\left[\frac{c+v_0'}{c}\right]$

Since the distances PQ and RQ are large, the component velocity is nearly equal to the car's velocity.

$v_P = f_1' - f_2' = (f_1 - f_2)\left(\frac{c+v_0'}{c}\right)$ (approaching)

$v_R = f_1'' - f_2'' = (f_1 - f_2)\left(\frac{c-v_0'}{c}\right)$ (Receding)

$v_Q = (f_1 - f_2)$ since motion is perpendicular to MN

$v_P + V_R = (f_1 - f_2)\left[\frac{c+v_0}{c} + \frac{c-v_0'}{c}\right]$

$= v_Q \times 2 \Rightarrow$ (c) correct

(D) Correct since component velocity is nearly the same as car's velocity. Hence beat frequency is nearly constant.

(B) correct since approaching car suddenly becomes receding car.

11. The gradient in `n' is along z-direction.
n sinθ remains invariant when θ is measured from z-direction.
Hence option (C) is correct
It is also clear that ℓ is independent of n_2 and depends only on n(z).

12. $r_n = r_0\frac{n^2}{Z}$

$\frac{r_{n+1} - r_n}{r_n} = \frac{(n+1)^2 - n^2}{n^2}$ independent of Z

$\Rightarrow$ (A) correct

$= \frac{2n+1}{n^2}$

$\approx \frac{2n}{n^2} \approx \frac{1}{n} \Rightarrow$ (B) correct

$E_n = -\text{Rhc}\frac{Z^2}{n^2}$

$\frac{E_{n+1} - E_n}{E_n} = \frac{\frac{-1}{(n+1)^2} + \frac{1}{n^2}}{\frac{1}{n^2}} = \frac{-n^2 + (n+1)^2}{n^2(n+1)^2}n^2$

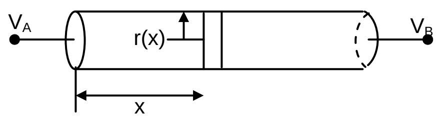
$= \frac{2n+1}{(n+1)^2} \approx \frac{2n}{n^2} \approx \frac{1}{n}$ (C) wrong

$L_n = \frac{nh}{2\pi}\ \frac{L_{n+1} - L_n}{L_n} = \frac{(n+1)-n}{n} = \frac{1}{n}$

(D) correct

13. If the temperature distribution was uniform (assuming a uniform cross section for the filament initially) the rate of evaporation from the surface would be same everywhere. But because the filaments break at random locations; it follows that the cross-sections of various filaments are non-uniform.

$\delta R(x) = \rho\frac{\delta x}{\pi r^2(x)}$



The temperature of points A and B are decided by ambient temperature are identical. Then the average heat flow through the section S is O. After sufficiently long time, this condition implies that the temperature across the filament will be uniform.

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

If the instantaneous current is i(t) through the filament then by conservation of energy:

$$\frac{(V_B - V_A)^2}{R(t)^2} \times \frac{dx}{k\pi r^2(x)}$$

$= e\sigma 2\pi r(x).\delta(x)T^4 + \rho\pi r^2(x).dxL_v$

in above κ = material conductivity

R(t) = Resistance of whole filament as a function of time

ρ = material density

L_V = Latent heat of vapourisation for the material at temperature T.

Since R(t) increases with time

$P(t) = \dfrac{(V_B - V_A)^2}{R(t)}$ decreases.

Answer C, D.

Section III

14. $E = \dfrac{hc}{\lambda} = \dfrac{6.6\times10^{-34}\times3\times10^{8}}{970\times10^{-10}\times1.6\times10^{-19}}$

$= 12$ eV

$\Delta E = 13.6 - 12 = 1.6$

$\dfrac{13.6}{n^2} = 1.6 \Rightarrow n = 3$

Transitions = 6

15. ${}^{12}_{5}B \rightarrow {}^{12}_{6}C + e^-$

12.014u 12u

Energy released due to mass defect of

0.014u = 931.5 × 0.014 = 13.041 MeV

$\Rightarrow$ Energy of e^-

= 13.041 MeV – excitation energy of 4.041 MeV

= 9 MeV

16. $v = \dfrac{2}{9}\dfrac{[\rho-\sigma]ga^2}{\eta}$

$v_P = \dfrac{2}{9}\dfrac{[8-0.8]g\times1}{3}$

$v_Q = \dfrac{2}{9}\dfrac{[8-1.6]g\times(0.5)^2}{2}$

$\dfrac{V_P}{V_Q} = \dfrac{7.2\times1\times2}{6.4\times(0.5)^2\times3}$

$= \dfrac{14.4}{4.8} = 3$

17. $I_{min} = \dfrac{5}{12}$

$\dfrac{1}{R_{eff}} = \dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{12}$

$\therefore R_{eff} = \dfrac{12}{8} = 1.5$

$\therefore I_{max} = \dfrac{5}{1.5}$

So $\dfrac{I_{max}}{I_{min}} = \dfrac{12}{1.5} = 8$

18. $P \propto T^4$

$\log_2\left(\dfrac{P_1}{P_0}\right) = 1 = \log_2\left(\dfrac{760}{T_0}\right)^4$

$\log_2\left(\dfrac{P_2}{P_0}\right) = x = \log_2\left(\dfrac{3040}{T_0}\right)^4$

$\Rightarrow 2^1 = \left(\dfrac{760}{T_0}\right)^4$

$2^x = \left(\dfrac{3040}{T_0}\right)^4$

$\dfrac{2}{2^x} = \left(\dfrac{760}{3040}\right)^4 = \left(\dfrac{1}{4}\right)^4 = \left(\dfrac{1}{2^2}\right)^4$

$\dfrac{2}{2^x} = \dfrac{2}{2.2^{x-1}} = \dfrac{1}{2^8}$

$\Rightarrow 2^{x-1} = 2^8$

$x - 1 = 8$

$x = 9$

PART II

19	20	21	22	23
B	**B**	**A**	**A**	**C**

24	25	26	27	28	29	30	31
B, C	**B, C**	**B**	**B, C, D**	**A, C, D**	**A**	**A, B, C**	**B, D**

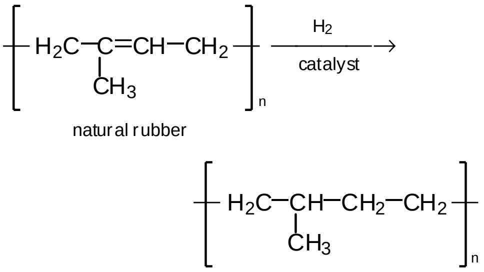
32	33	34	35	36
4	**5**	**9**	**5**	**6**

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

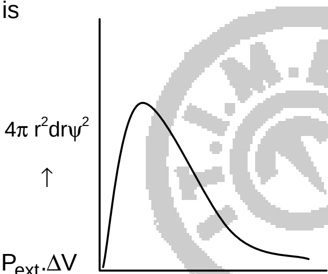
Section I

19. Atomic radii increases in the order
Ga < Al < In < Tl

20. $[NiCl_4]^{2-}$, $Na_3[CoF_6]$ and CsO_2 are paramagnetic Compounds.

21. 

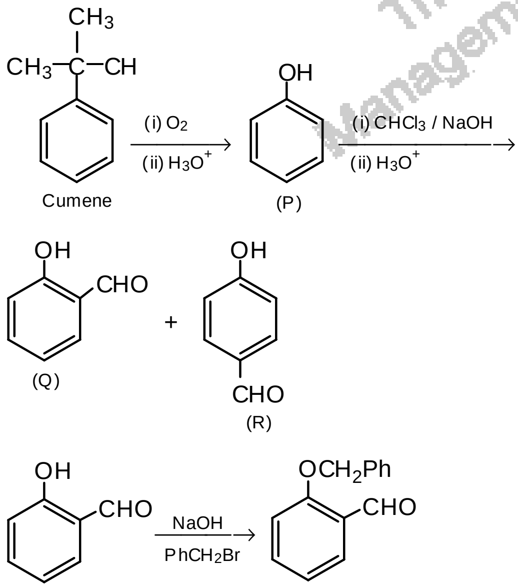
22. The plot of radial probability function ($4\pi r^2 dr\psi^2$) against distance from the nucleus (r) for 1s orbital is



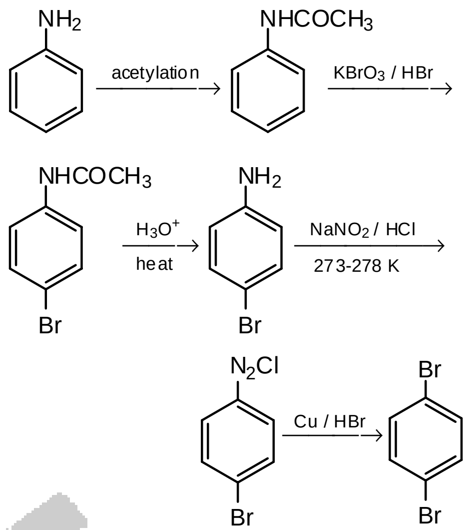
23. $q_{rev} = P_{ext}.\Delta V$
$= 3 \times 1$ L atm
$= 3 \times 101.3$ J

$$\Delta S_{surr} = \frac{-q_{rev}}{T}$$
$$= \frac{-3\times 101.3}{300}$$
$= -1.013$ J K^{-1}

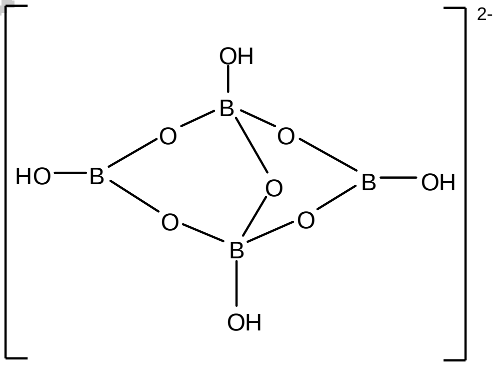
Section II

24. 

25. BrF_5 – sp^3d^2 hybridisation
– 5 bp and 1 lp
ClF_3 – sp^3d hybridisation
– 3 bp and 2 lp
XeF_4 – sp^3d^2 hybridisation
– 4 bp and 2 lp
SF_4 – sp^3d hybridisation
– 4 bp and 1 lp

26. 

27. Higher the activation energy, slower the reaction.
Rate constant increases with increase in temperature as the rate of collision increases which in turn increases the number of activated molecules.
Larger the activation energy, greater the influence of change in temperature on rate constant.
The pre-exponentitial factor (A) is a measure of the number of collisions.

28. 

29. Only CuS is precipitated selectively.

30. Ketones containing –OH group on α-carbon and all aldehydes answer Tollens' test.

31. Nuclides that lie below the band of stability have too few neutrons for stability and decay by positron-emission or electron capture. In either

case the number of neutrons increases and decreases the number of protons.

Section III

32. $D \propto \lambda.\bar{c}$

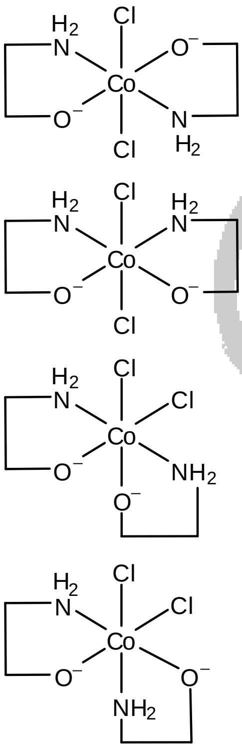
But $\lambda \propto \dfrac{T}{P}$

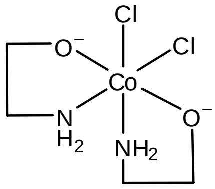
and $\bar{c} \propto \sqrt{T}$

$\therefore D \propto \dfrac{T^{3/2}}{P}$

$\dfrac{D_2}{D_1} = \dfrac{(2^2)^{3/2}}{2} = 4$

33. The geometrical isomers are





34. $\dfrac{n_2}{n_1} = \dfrac{1}{9}$

$m = \dfrac{1\times1000}{9\times M_1}$

$M = \dfrac{1\times2\times1000}{1\times M_2 + 9\times M_1}$

Given, $\dfrac{1000}{9M_1} = \dfrac{2000}{M_2 + 9M_1}$

$\therefore \dfrac{M_2}{M_1} = 9$

35. Possible chiral products are

$BrCH_2-\overset{H}{\underset{Br}{C^*}}-CH_2-CH_2-CH_3$

$CH_3-\overset{H}{\underset{Br}{C^*}}-\overset{H}{\underset{Br}{C^*}}-CH_2-CH_3$

(R and S forms)

$CH_3-\overset{H}{\underset{Br}{C^*}}-CH_2-\overset{H}{\underset{Br}{C^*}}-CH_3$

$CH_3-\overset{H}{\underset{Br}{C^*}}-CH_2-CH_2-CH_2Br$

36. $\underset{(n=3)}{8MnO_4^-} + \underset{(n=8)}{3S_2O_3^{2-}} \rightarrow MnO_2 + 6SO_4^{2-}$

PART III

37	38	39	40	41
C	**A**	**C**	**C**	**C**

42	43	44	45	46	47	48	49
A, D	**B, C, D**	**A, B, C**	**B, C**	**B, C**	**A, C**	**A**	**A, C, D**

50	51	52	53	54
5	**7**	**1**	**2**	**1**

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

Section I

37. $P(T_1)=\frac{20}{100}$ $P(T_2)=\frac{80}{100}$ D – Defective

Let $P\left(D/T_2\right)=x$ $P\left(D/T_1\right)=10x$

P(D) = 0.07

$P(D) = P(T_1)\ P\left(D/T_1\right)+P(T_2)\times P\left(D/T_2\right)$

$.07 = \frac{20}{100}\times 10x+\frac{80}{100}\times x$

$\Rightarrow 7 = 200x + 80x$

$\Rightarrow x = \frac{7}{280}=\frac{1}{40}$

$\Rightarrow P\left(D/T_2\right)=\frac{1}{40}\Rightarrow P\left(D'/T_2\right)=\frac{39}{40}$

$P\left(D/T_1\right)=\frac{1}{4}\Rightarrow P\left(D'/T_1\right)=\frac{3}{4}$

Required Probability = $P\left(T_2/D'\right)$

$$=\frac{P(T_2)\ \ P\left(D'/T_2\right)}{P(T_1)\ P\left(D'/T_1\right)+P(T_2)\ P\left(D'/T_1\right)}$$

$$=\frac{\frac{80}{100}\times\frac{39}{40}}{\frac{20}{100}\times\frac{3}{4}+\frac{80}{100}\times\frac{39}{40}}$$

$$=\frac{78}{93}$$

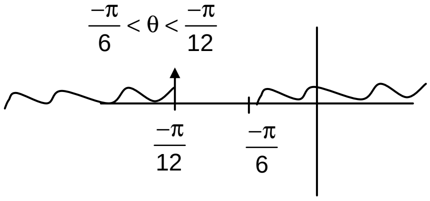
38. 6G 4B $\Rightarrow$ atmost one boy
(i) Boy + 3G $\Rightarrow {}^4C_1\times{}^6C_3$
(ii) 4G $\Rightarrow {}^6C_4$
Total ways of selection of team
= 4 × 20 + 15 = 95
From each team the captain may be chosen in ${}^4C_1$ ways
$\therefore$ (Team + Captain) = 95 × 4 = 380

39. $x^2 - 2x\sec\theta + 1 = 0$

$$x=\frac{(2\sec\theta)\pm\sqrt{4\sec^2\theta-4}}{2}$$

$= \sec\theta \pm \tan\theta$

$\frac{-\pi}{6}<\theta<\frac{-\pi}{12}$



$\alpha_1 = \sec\theta - \tan\theta$
$\beta_1 = \sec\theta + \tan\theta$
$x^2 + 2x\tan\theta - 1 = 0$

$$x=\frac{-2\tan\theta\pm\sqrt{4\tan^2\theta+4}}{2}$$

$= \tan\theta \pm \sec\theta$

$\alpha_2 = -\tan\theta + \sec\theta$
$\beta_2 = -\tan\theta - \sec\theta$
$\alpha_1 + \beta_2 = \sec\theta - \tan\theta - \tan\theta - \sec\theta$
$= -2\tan\theta$

40. $\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}+2\left[\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}\right]=0$

$\Rightarrow 2\cos\left(\frac{\pi}{3}-x\right)+2\cos 2x=0$

$\cos 2x = \cos\left(x-\frac{\pi}{3}\right)$

$\therefore x = (6n-1)\frac{\pi}{3}$ or $(6n+1)\frac{\pi}{9}$

$\therefore x = \frac{-\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}\in(-\pi, \pi)$

$\therefore$ sum = 0

41. $4\alpha x^2+\frac{1}{x}\geq 1$

Since x >0, $4\alpha+\frac{1}{x^3}\geq\frac{1}{x^2}$

$\Rightarrow \alpha\geq\frac{1}{4}\left(\frac{1}{x^2}-\frac{1}{x^3}\right)$

Minimum value of α is the maximum value of

$f(x) = \frac{1}{4}\left(\frac{1}{x^2}-\frac{1}{x^3}\right)$

$\frac{1}{4}\left(y^2-y^3\right)$, where $y=\frac{1}{x}$

$f' = \frac{1}{4}\left(2y-3y^2\right)$

$f'' = \frac{1}{4}(2-6y)$

$f' = 0 \Rightarrow y = 0$ or $\frac{2}{3}$

But $y = \frac{1}{x}\neq 0$

$\therefore y = \frac{2}{3}$

$\Rightarrow$ Also, $f'' < 0$ at $y = \frac{2}{3}$

$\therefore$ Maximum of f = $\frac{1}{27}$

Section II

42. $(x^2 + xy + 4x + 2y + 4)\ \frac{dy}{dx}-y^2=0$

$[(x + 2)^2 + y(x + 2)]\ \frac{dy}{dx}-y^2=0$

X = x + 2
Y = y

$\frac{dY}{dX}=\frac{dy}{dx}$

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

$(X^2+YX)\,\frac{dY}{dX}-Y^2=0$

Y = VX

$\frac{dY}{dX}=V+X\frac{dV}{dX}$

$(X^2+X^2V)\left(V+X\frac{dV}{dX}\right)-V^2X^2=0$

$(1+V)\left(V+X\frac{dV}{dX}\right)-V^2=0$

$V+X\frac{dV}{dX}+V^2+V\times\frac{dV}{dV}-V^2=0$

$X(1+V)\,\frac{dV}{dX}+V=0$

$\frac{(1+V)dV}{V}+\frac{dX}{X}=0$

Integrating logV + V +logX = C

logY +V = C

$\Rightarrow$ logy + $\frac{y}{x+2}=C$

Curve passes through (1, 3)

$\log 3+\frac{3}{3}=C$

C = 1+ log3

Solution curve is

logy + $\frac{y}{x+2}=1+\log 3$

y = x +2 intersects at (1, 3)

(A) true

2logx + 2 = 1 + log3 – (x + 2)

= log3 – x – 1

x + logx^2 = –3 + log3

x + 2logx = log3 – 3

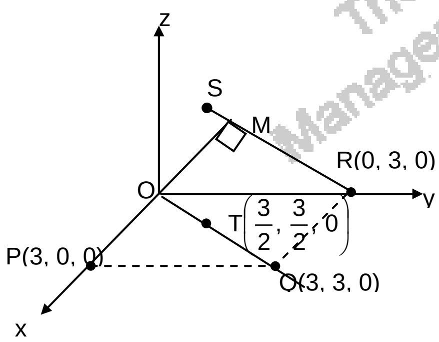
2log(y + 3) + $\frac{(x+3)^2}{x+2}=1+\log 3$

$\therefore$ y = (x + 3)^2 do not intersect

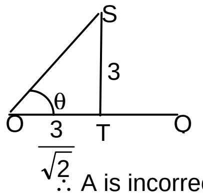
$\therefore$ D is correct

$\therefore$ A, D correct

43.



(A)



OT = $\frac{3}{\sqrt{2}}$

ST = 3

tanθ = $\sqrt{2}$

$\therefore$ A is incorrect

(B) Equation of line OQ is x = y

As S lies directly above T, equation of the plane containing ΔOQS is x – y = 0

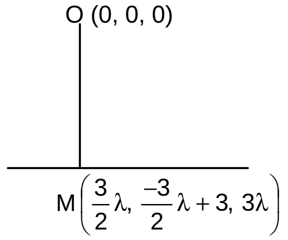
(B) correct

(C) Length of the $\perp$ar from P(3, 0, 0) to x – y = 0

Is $\frac{3}{\sqrt{2}}$

$\therefore$ (C) correct

(D)



Equation of the line joining R and S is

$$\frac{x-0}{\frac{3}{2}}=\frac{y-3}{\frac{-3}{2}}=\frac{z-0}{3}$$

M lies on this line

$\Rightarrow$ M is $\left(\frac{3}{2}\lambda,\ \frac{-3}{2}\lambda+3,\ 3\lambda\right)$

D. r' s of OM are $\frac{3}{2}\lambda,\ \frac{-3}{2}\lambda+3,\ 3\lambda$

D. r's of RS are $\frac{3}{2},\ \frac{-3}{2},\ 3$

Since OM $\perp$ RS,

$$\frac{3}{2}\lambda,\times\frac{3}{2}+\left(\frac{-3}{2}\lambda+3\right)\times\frac{-3}{2}+3\lambda\times 3=0$$

$\Rightarrow \lambda=\frac{1}{3}$

$\Rightarrow$ M is $\left(\frac{1}{2},\frac{5}{2},1\right)$

$\therefore$ OM = $\sqrt{\frac{15}{2}}$

(D) correct

44. x^2 + y^2 = 3

x^2 = 2y

y^2 + 2y – 3 = 0

(y + 3) (y – 1) = 0

y = –3, 1

Since P is in the first quadrant, y = 1

x^2 = 2 $\Rightarrow$ x = $\sqrt{2}$

P is $\left(\sqrt{2},1\right)$

Circle C_2 : x^2 + (y – p)^2 = 12 Q_2 (0, p)

Circle C_3 : x^2 + (y – q)^2 = 12 Q_3 (0, q)

Tangent at P to $C_1 \Rightarrow x\sqrt{2}+y=3 \rightarrow \cancel{(x)}$

$\cancel{(x)}$ touches C_2

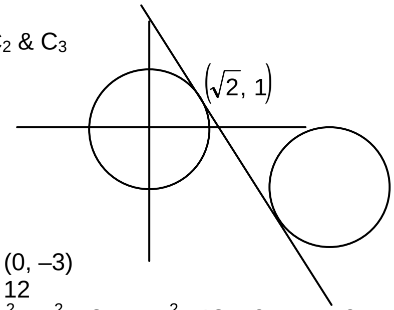
$\frac{p-3}{\sqrt{2+1}}=\pm 2\sqrt{3}$

p = 3 = ± 6

p = 9, –3

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

$p = 9$
$q = -3$
centres of C_2 & C_3



$(0, 9)$ $(0, -3)$
$Q_2Q_3 = 12$
Circle C_2 : $x^2 + y^2 - 2py + p^2 - 12 = 0 \rightarrow p = 9$
$x^2 + y^2 - 18y + 69 = 0$
$xx_1 + yy_1 - 9(y + y_1) + 69 = 0$
$xx_1 + (y_1 - 9)y + 69 - 9y_1 = 0$
$x\sqrt{2} + y = 3$

$$\frac{x_1}{\sqrt{2}} = \frac{y_1 - 9}{1} = \frac{9y_1 - 69}{3}$$

$3y_1 - 27 = 9y_1 - 69$
$6y_1 = 69 - 27$
$= 42$
$y_1 = 7$

$$\frac{x_1}{\sqrt{2}} = \frac{7-9}{1} = -2$$

$x_1 = -2\sqrt{2}$
R_2 is $\left(-2\sqrt{2}, 7\right)$
Circle C_3
$x^2 + y^2 + 6y - 3 = 0$
$xx_1 + yy_1 + 3(y + y_1) - 3 = 0$
$xx_1 + (y_1 + 3)y + 3y_1 - 3 = 0$
$x\sqrt{2} + y = 3$

$$\frac{x_1}{\sqrt{2}} = \frac{y_1 + 3}{1} = \frac{3 - 3y_1}{3}$$

$$\frac{x_1}{\sqrt{2}} = +2$$

$x_1 = +2\sqrt{2}$
R_3 is $\left(2\sqrt{2}, -1\right)$
$3y_1 + 9 = 3 - 3y_1$
$6y_1 = -6$
$y_1 = -1$
$=$ R_1 is $\left(-2\sqrt{2}, 7\right)$
R_2 is $\left(2\sqrt{2}, -1\right)$
$R_1R_2^2 = 32 + 64 = 96$
$R_1R_2 = \sqrt{16 \times 6} = 4\sqrt{6}$
ΔOR_2R_3
$(0, 0)$ $\left(-2\sqrt{2}, 7\right)$ $\left(2\sqrt{2}, -1\right)$

$$\text{Area of } \Delta OR_2R_3 = \frac{1}{2}\begin{vmatrix} -2\sqrt{2} \times (-1) \\ -7 \times 2\sqrt{2} \end{vmatrix}$$

$$= \frac{1}{2}\left|-12\sqrt{2}\right| = 6\sqrt{2}$$

Q_2 $(0, 9)$
Q_3 $(0, -3)$
$P\left(\sqrt{2}, 1\right)$ $(0, 9)$ $(0, -3)$

Area of ΔPQ_2Q_3

$$= \frac{1}{2}\left\{\sqrt{2}(12)\right\} = 6\sqrt{2}$$

45. $g(f(x) = x \Rightarrow f(x) = g^{-1}(x)$ ——— (1)
Also, $g'(f(x))\ f'(x) = 1$
$\therefore g'(x^3 + 3x + 2) \times (3x^2 + 3) = 1$
Put $x = 0$
Then $g'(2) = \dfrac{1}{3}$
$\therefore$ (A) incorrect
Now, $h(g(g(x))) = x$
$\Rightarrow h(x) = [g(g(x))]^{-1} = g^{-1}(g^{-1}(x)) = f(f(x))$, by (1)
$\therefore h'(x) = f'(f(x)) \times f'(x)$
$\therefore h'(1) = f'(6) \times f'(1)$
$= 111 \times 6 = 666$
(B) correct
Also, $h(x) = f(f(x)) \Rightarrow h(0) = f(f(0))$
$f(2) = 16$
$\therefore$ (C) correct
$h(x) = f(f(x)) \Rightarrow h(g(3)) = f[f(g(3))]$
$= f(3)$ [since $f = g^{-1}$]
$= 38$
(D) incorrect

46. $PQ = kI \Rightarrow Q = kP^{-1}$
Equate the (2, 3) element on both sides

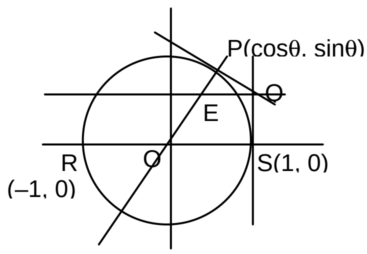
$$\therefore \frac{-k}{8} = k \times \frac{1}{12\alpha + 20} \times -(3\alpha + 4)$$

$\Rightarrow \alpha = -1$. Then $|P| = 8$
Also, $PQ = kI \Rightarrow |P|\ |Q| = k^3 \times 1$

$$\Rightarrow 8 \times \frac{k^2}{2} = k^3$$

$\Rightarrow k = 4$ $\therefore |Q| = 8$
$\therefore$ (B) correct and (A) incorrect
$|P\ adj\ Q| = |P|\ |Q|^2 = 8 \times 8^2 = 2^9$
$\therefore$ (C) correct
$|Q\ adj\ P| = |Q|\ |P|^2 = 8 \times 8^2 = 2^9$
$\therefore$ (D) incorrect

47.



Tangent at P
$x\cos\theta + y\sin\theta = 1$
Tangent at $S \rightarrow x = 1$
$y\sin\theta = 1 - x\cos\theta$
$= 1 - \cos\theta$

$$y = \frac{1 - \cos\theta}{\sin\theta}$$

Q is $\left(1, \dfrac{1 - \cos\theta}{\sin\theta}\right)$
Equation of the line through Q parallel to RS is

$$y = \frac{1 - \cos\theta}{\sin\theta} \text{ ——— (1)}$$

Equation of the normal at P is

$y = (\tan\theta)x$ ——— (2)

$x\tan\theta = \dfrac{1-\cos\theta}{\sin\theta}$

$x = \dfrac{1-\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\cos\theta - \cos^2\theta}{\sin^2\theta}$

$x = \dfrac{\cos\theta-\cos^2\theta}{\sin^2\theta},\ \ y = \dfrac{1-\cos\theta}{\sin\theta}$

$= \dfrac{(\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}, = \dfrac{\cos\theta}{1+\cos\theta}$

$x\cos\theta + x = \cos\theta$

$\cos\theta = \dfrac{x}{1-x}$ ——— (1)

$y\sin\theta = 1 - \cos\theta = 1-\dfrac{x}{1-x}$

$= \dfrac{1-2x}{1-x}$

$\sin\theta = \dfrac{1-2x}{(1-x)y}$ ——— (2)

Locus of E is

$\dfrac{(1-2x)^2}{y^2(1-x)^2} + \dfrac{x^2}{(1-x)^2} = 1$

$(1 - 2x)^2 + y^2x^2 = y^2(1 - x)^2$

$(1 - 2x)^2 \pm y^2\,[1+ x^2 - 2x - x^2]$

$= y^2(1 - 2x)$

$x \neq \dfrac{1}{2}$

$y^2 = 1 - 2x$ is the locus

$x = \dfrac{1}{3} \rightarrow y^2 = \dfrac{1}{3}$, $y = \pm\dfrac{1}{\sqrt{3}}$

48. $f'(x) + \dfrac{f(x)}{x} = 2$

General solution is

$xf(x) = C + x^2$

$f(x) = \dfrac{C}{x} + x$

$\lim\limits_{x\to0^+} xf\left(\dfrac{1}{x}\right) = \lim\limits_{x\to0^+}\{Cx^2+1\}$

$= 1$ (B false)

$\lim\limits_{x\to0^+} f'\left(\dfrac{1}{x}\right) = \lim\limits_{x\to0^+}\left(-Cx^2+1\right)$

$= 1 \rightarrow$ (A) is true)

$\lim\limits_{x\to0^+} x^2f'(x) = \lim\limits_{x\to0}\left(-C+x^2\right)$

$= -C \neq 0 \quad \begin{cases} f(1)\neq 1 \\ C+1\neq 1 \\ C\neq 0\end{cases}$

$\Rightarrow$ (C) is false

We have

$f(x) = \dfrac{C}{x} + x$

C is arbitrary

$|f(x)| \leq 2$ for $x\in (0, 2)$ not true

$\therefore$ only (A)

49. $\dfrac{s-x}{4} = \dfrac{s-y}{3} = \dfrac{s-z}{2}$

$= \dfrac{3s-2s}{9} = \dfrac{s}{9}$

$9(s - x) = 4s$

$9x = 5s \Rightarrow x = \dfrac{5s}{9}$

$\dfrac{s-y}{3} = \dfrac{s}{9}$

$3s - 3y = s$

$3y = 2s \Rightarrow y = \dfrac{2s}{3}$

$\dfrac{s-z}{2} = \dfrac{s}{9}$

$9s - 9z = 2s$

$9z = 7s$

$z = \dfrac{7s}{9}$

Area of XYZ = $\sqrt{s(s-x)(s-y)(s-z)}$

$= \sqrt{s\times\dfrac{4s}{9}\times\dfrac{s}{3}\times\dfrac{2s}{9}}$

$= \dfrac{s^2\times2\sqrt{2}}{9\sqrt{3}}$

$r = \dfrac{\Delta}{s}$ $\quad \pi\left(\dfrac{\Delta^2}{s^2}\right) = \dfrac{8\pi}{3}$ (given)

$\dfrac{\Delta^2}{s^2} = \dfrac{8}{3}$

$\dfrac{8s^4}{81\times3s^2} = \dfrac{8}{3} \Rightarrow s^2 = 81$

$s = 9$

$x = \dfrac{5}{9}\times9 = 5$

$y = \dfrac{2s}{3} = \dfrac{2\times9}{3} = 6$

$z = \dfrac{7s}{9} = \dfrac{7\times9}{9} = 7$

Area of $\Delta XYZ = \dfrac{2\sqrt{2}}{9\sqrt{3}}\times81$

$= \left(2\sqrt{2}\right)3\sqrt{3} = 6\sqrt{6}$

$\dfrac{abc}{4R} = \Delta = 6\sqrt{6}$

$R = \dfrac{abc}{4\times6\sqrt{6}} = \dfrac{5\times6\times7}{4\times6\sqrt{6}}$

$= \dfrac{35}{4\sqrt{6}}$ (B) is wrong

$4R\sin\dfrac{x}{2}\ \sin\dfrac{y}{2}\ \sin\dfrac{z}{2} = \dfrac{\Delta}{s} = \dfrac{6\sqrt{6}}{9} = \dfrac{2\sqrt{6}}{3}$

$\sin\frac{x}{2}\ \ \sin\frac{y}{2}\ \ \sin\frac{z}{2}=\frac{2\sqrt{6}}{3\times 4R}$

$=\frac{2\sqrt{6}\times 4\sqrt{6}}{12\times 35}$

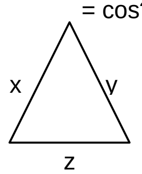
$=\frac{8\times 6}{12\times 35}=\frac{4}{35}$

$\text{Sin}^2\left(\frac{x+y}{2}\right)$

$\sin^2\left(\frac{180^\circ - z}{2}\right)$

$=\sin^2\left(90^\circ-\frac{z}{2}\right)$

$=\cos^2\frac{z}{2}=\frac{1}{2}(1+\cos z)$



$z^2 = x^2 + y^2 - 2xy\cos Z$

$49 = 25 + 36 - 2\times 5\times 6\cos Z$

$-60\cos Z = 49 - 61 = -12$

$\cos Z = \frac{1}{5}$

$\cos^2\frac{Z}{2}=\frac{1}{2}\left(1+\frac{1}{5}\right)=\frac{1}{2}\times\frac{6}{5}=\frac{3}{5}$

$\therefore$ A, C, D are correct

Section III

50. Coefficient of x^2

$= 1 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^{49}C_2 + {}^{50}C_2(m^2)$

$= {}^{50}C_3 + (m^2)\,{}^{50}C_2,$

As ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

${}^{50}C_3 + m^2\,{}^{50}C_2 = (3n+1)\,{}^{51}C_3$

${}^{51}C_3 + (m^2-1)\,{}^{50}C_2 = (3n+1)\,{}^{51}C_3$

$(m^2-1)\,{}^{50}C_2 = 3n\ {}^{51}C_3$

$\frac{50\times 49}{2}(m^2-1)=3n\ \frac{51\times 50\times 49}{3\times 2}$

$m^2 - 1 = 51n$

$m^2 = 51n + 1 \quad 51\times 5 = 255$

$m = 16 \quad 255 + 1 = 256$

$n = 5$

51. $\lim\limits_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}\Rightarrow\lim\limits_{x\to 0}\frac{x\sin\beta x}{\frac{\alpha x-\sin x}{x}}=0$ if $\alpha\neq 1$

If $\alpha = 1$

$$\frac{\beta x^3\,\frac{\sin\beta x}{\beta x}}{x^3\left(\frac{x-\sin x}{x^3}\right)}=\frac{\beta}{\frac{1}{6}}$$

$6\beta = 1 \quad \therefore \beta = \frac{1}{6}$

$\therefore 6(\alpha+\beta) = 7$

52. $z=\frac{-1+\sqrt{3}i}{2}$

$P^2=\begin{bmatrix}(-\omega)^r & \omega^{2s}\\ \omega^{2s} & \omega^r\end{bmatrix}\begin{bmatrix}(-\omega)^2 & \omega^{2s}\\ \omega^{2s} & \omega^r\end{bmatrix}$

$=\begin{bmatrix}(-\omega)^{2r}+\omega^{45} & (-\omega)^r\omega^{2s}+\omega^{2s}\omega^r\\ \omega^{2s}(-\omega)^r+\omega^r\omega^{2s} & \omega^{4s}+\omega^{2r}\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$(-\omega)^{2r} + \omega^{4s} = -1$

$\omega^{2s}((-\omega)^r + \omega^r) = 0 \qquad \omega^{2s} = 0$(not possible)

$\therefore (-\omega)^r + \omega^r = 0 \Rightarrow$ r should be odd

r = 1 or 3

$-\omega + \omega = 0 \quad -\omega^2 + \omega^2 = 0$

$r = 1 \quad \omega^2 + \omega^{4S} = -1 \qquad \therefore 4s = 4 \Rightarrow s = 1$

$r = 3 \quad 1 + \omega^{4s} = -1$ Not possible

$\therefore$ Only ordered pair is (1, 1)

53. $\begin{vmatrix}x & x^2 & 1\\ 2x & (2x)^2 & 1\\ 3x & (3x)^2 & 1\end{vmatrix}+\begin{vmatrix}x & x^2 & x^3\\ 2x & (2x)^2 & (2x)^3\\ 3x & (3x)^2 & (3x)^3\end{vmatrix}=10$

$\begin{vmatrix}x & x^2 & 1\\ x & 3x^2 & 0\\ 2x & 8x^2 & 0\end{vmatrix}+6x^3\begin{vmatrix}1 & x & x^2\\ 1 & 2x & (2x)^2\\ 1 & 3x & (3x)^2\end{vmatrix}=10$

$\Rightarrow (8x^3 - 6x^3) + 6x^3 \times$

$x^3\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 3 & 9\end{vmatrix}=10$

$\Rightarrow 2x^3 + 6x^6(-1)(-1)(2) = 10$

$\Rightarrow 2x^3 + 12x^6 = 10$

Let $x^3 = y$

$12y^2 + 2y - 10 = 0$

$(y + 12)(y - 10) = 0$

$y = -12$; $y = 10$

$x^3 = -12$ or $x^3 = 10$

$\therefore$ 2 real roots

54. Given

$f(x)=\int_0^x\frac{t^2}{1+t^4}\,dt=2x-1$

Differentiating both sides with respect to x

$f'(x)=\frac{x^2}{1+x^4}-2$

$\frac{1+x^4}{x^2}=x^2+\frac{1}{x^2}\leq 2$

$\therefore\ \frac{x^2}{1+x^4}\leq\frac{1}{2}$

$\therefore f'(x)\leq\frac{-3}{2}$

$\therefore$ f(x) is continuous and decreasing in [0, 1]
and f(0) = 1

$\therefore$ By intermediate value theorem f(x) has exactly one root.

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.