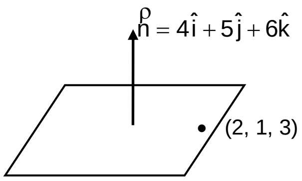
SOLUTIONS & ANSWERS FOR KEAM ENTRANCE -2021
PAPER 2
VERSION- B3
[MATHEMATICS]

1. The equation of the plane through the point -------

Ans: $4x+5y+6z=31$

Sol:



The required vector is 4x+5y+6z=k
which passes through (2,1,3)
$\Rightarrow 4(2)+5(1)+6(3)=k\Rightarrow 31$
$\therefore 4x+5y+6z=31$

2. The angle between the line $\vec{r}=\hat{i}+2\hat{j}+t\left(3\hat{i}+2\hat{j}-\hat{k}\right)$-----

Ans: $\sin^{-1}\left(\frac{1}{14}\right)$

Sol : $\text{Sin}\theta=\left|\frac{\vec{b}.\vec{n}}{|\vec{b}|.|\vec{n}|}\right|=\left|\frac{\left(3\hat{i}+2\hat{j}-\hat{k}\right)\left(2\hat{i}-3\hat{j}-\hat{k}\right)}{\sqrt{9+4+1}.\sqrt{4+9+1}}\right|$

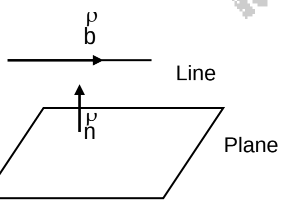
$=\frac{6-6+1}{14}=\frac{1}{14}$

$\therefore \theta=\sin^{-1}\left(\frac{1}{14}\right)$

3. If the line $\vec{r}=2\hat{i}+\hat{j}+t(3\hat{i}+\hat{j}-2\hat{k})$ is parallel to the plane -----

Ans: 5

Sol:



$\vec{b}.\vec{n}=0$
(3i+j-2k).(2i+4j+ak) =0
$\Rightarrow$6+4-2a=0
$\Rightarrow$a=5

4. The angle between the lines $\vec{r}=\hat{i}+4\hat{k}+\lambda(2\hat{i}+\hat{j}-\hat{k})$ -----

Ans: $\cos^{-1}\left(\frac{\sqrt{15}}{6}\right)$

Sol: $\cos\theta=\left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1|.|\vec{b}_2|}\right|=\left|\frac{\left(2\hat{i}+\hat{j}-\hat{k}\right)\left(3\hat{i}+\hat{k}\right)}{\sqrt{4+1+1}.\sqrt{9+1}}\right|$

$=\frac{6-1}{\sqrt{6}.\sqrt{10}}=\frac{5}{\sqrt{6}.\sqrt{10}}$

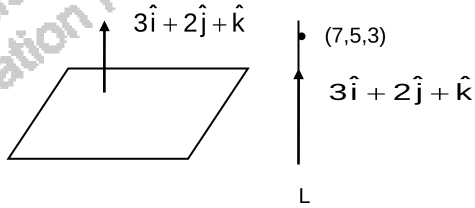
$=\frac{5}{\sqrt{6}.\sqrt{2}\sqrt{5}}=\frac{\sqrt{5}}{\sqrt{12}}=\frac{\sqrt{5}}{2\sqrt{3}}$

$=\frac{\sqrt{15}}{6}\Rightarrow\theta=\cos^{-1}\left(\frac{\sqrt{15}}{6}\right)$

5. The Cartesian equation of the line passing through (7,5,3) -------

Ans: $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$

Sol:



$L:\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$

6. The acute angle between the planes 2x-y-3z=7 -------

Ans: $\pi-\cos^{-1}\left(\frac{-\sqrt{14}}{7}\right)$

Sol: $\cos\theta=\left|\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1|.|\vec{n}_2|}\right|=\left|\frac{\left(2\hat{i}-\hat{j}-3\hat{k}\right)\left(\hat{i}+2\hat{j}+2\hat{k}\right)}{\sqrt{4+1+9}.\sqrt{1+4+4}}\right|$

$=\frac{|2-2-6|}{\sqrt{14}.3}=\frac{2}{\sqrt{14}}=\frac{2\sqrt{14}}{14}=\frac{\sqrt{14}}{7}$

$$\therefore \theta = \cos^{-1}\left(\frac{\sqrt{14}}{7}\right)$$
$$= \pi - \cos^{-1}\left(-\frac{\sqrt{14}}{7}\right)$$
$$\because \cos^{-1}(-x) = \pi - \cos^{-1} x$$

7. The vector equation of the line joining the points (2,1,3) and (-2,4,1) is -------

Ans: $\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda\left(-4\hat{i} + 3\hat{j} - 2\hat{k}\right)$

Sol: $\vec{r} = \vec{a} + \lambda\vec{b}$
$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k},$
$\vec{b} = \left(-2\hat{i} + 4\hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} + 3\hat{k}\right)$
$= -4\hat{i} + 3\hat{j} - 2\hat{k}$
$\vec{r} = \left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \lambda\left(-4\hat{i} + 3\hat{j} - 2\hat{k}\right)$

8. A bag contains 5 yellow, 3 green, 2 blue and 7 white balls. If 4 balls are chosen at random, -------

Ans: $\frac{3}{34}$

Sol: Required probability
$$= \frac{10C_4}{17C_4} = \frac{10 \times 9 \times 8 \times 7}{17 \times 16 \times 15 \times 14} = \frac{3}{34}$$

9. An urn contains 25 marbles which are numbered from 1 to 25 and a marble is chosen at random two times with replacement.-------

Ans: $\frac{1}{25}$

Sol: P=P(1 and 1)+P(2 and 2)+ ....+P(25 and 25)
$$= \left(\frac{1}{25}\right)^2 + \left(\frac{1}{25}\right)^2 + \ldots\ldots + \left(\frac{1}{25}\right)^2$$
$$= 25 \times \frac{1}{25^2} = \frac{1}{25}$$

10. If A and B are two events such hat P(A) = 0, 2, P(B) = 0.55 -----

Ans: 0.45

Sol: $P\left(B \cap A^c\right) = P(B - A) = P(B) - P(A \cap B)$
=0.55-0.1=0.45

11. Two dice are rolled. If A is the event that sum of the numbers is 4 and -------

Ans: $\frac{2}{11}$

Sol: $S = \{(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),$
$(3,1),(3,2)(3,4),(3,5),(3,6)\}$
$\Rightarrow n(S) = 11$
$F = \{(1,3),(3,1)\} \Rightarrow n(F) = 2$
$P = \frac{2}{11}$

12. Assume that n distinct values x_1, x_2, x_3, ….. x_n occur -----

Ans: 45

Sol: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
$$\Rightarrow 7 = \frac{315}{\sum f_i} \Rightarrow \sum f_i = \frac{315}{7} = 45$$

13. The variance of the data x_1, x_2, ……, x_{30} with -----

Ans: 31

Sol: $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$
$$= \frac{10000}{50} - \left(\frac{650}{50}\right)^2$$
=200-169=31

14. If X is a random variable with E(X) = 6 and V(X) = 3 -------

Ans: 39

Sol: $V(x) = \sigma^2 = E\left(x^2\right) - [E(x)]^2$
$\Rightarrow 3 = E\left(x^2\right) - 36 \Rightarrow E\left(x^2\right) = 39$

15. Let $f(x) = \frac{4x+3}{x+2}$. Then the value of -----

Ans: $\frac{-7}{6}$

Sol: $f(x) = \frac{4x+3}{x+2}$
$f^{-1}(-2) = k \Rightarrow f(k) = -2$
$\Rightarrow \frac{4k+3}{k+2} = -2$
$\Rightarrow 4k + 3 = -2k - 4$
$\Rightarrow 6k = -7 \Rightarrow k = \frac{-7}{6}$

16. If $f(x) = \begin{cases} 2x & \text{for } x < 1 \\ 5a - x & \text{for } x \geq 1 \end{cases}$ is continuous -----

Ans: $\frac{3}{5}$

Sol: $\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-}(2x) = 2$

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+}(5a - x) = 5a - 1$

$\therefore 5a - 1 = 2 \Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5}$

17. $\lim\limits_{t\to0} \frac{\sin 2t}{8t^2 + 4t}$ -----

Ans: $\frac{1}{2}$

Sol $\lim\limits_{t\to0}\left(\frac{\sin 2t}{8t^2+4t}\right) = \lim\limits_{t\to0}\left[\frac{\left(\frac{\sin 2t}{t}\right)}{8t+4}\right]$

$= \frac{2}{0+4} = \frac{1}{2}$

18. $\lim\limits_{x\to0} \frac{x}{\sqrt{9-x}-3}$ is equal to-----

Ans: -6

Sol: $\lim\limits_{x\to0} \frac{x}{\left(\sqrt{9-x}-3\right)} = \lim\limits_{x\to0}\left[\frac{1}{\frac{1}{2\sqrt{9-x}}(-1)-0}\right]$

$= \frac{1}{\left(\frac{-1}{2\sqrt{9}}\right)} = -6$

19. Let $f(x)\begin{cases} 3x+2, & \text{if } x < -2 \\ x^2 - 3x - 1, & \text{if } x \ge -2 \end{cases}$ -----

Ans: $-4, 9$

Sol: $\lim\limits_{x\to-2^-} f(x) = \lim\limits_{x\to-2^-}[3x+2]$

$= 3(-2) + 2 = -4$

$\lim\limits_{x\to-2^+} f(x) = \lim\limits_{x\to-2^+}\left(x^2 - 3x - 1\right)$

$= 4 + 6 - 1 = 9$

20. $\lim\limits_{x\to-3} = \left[\frac{x^2+16x+39}{2x^2+7x+3}\right]$ is equal to -----

Ans: -2

Sol: $\lim\limits_{x\to-3} = \left[\frac{x^2+16x+39}{2x^2+7x+3}\right]$

$= \lim\limits_{x\to-3}\left[\frac{2x+16}{4x+7}\right]$

$= \frac{-6+16}{-12+7} = \frac{10}{-5} = -2$

21. Let $f(x) = 6\sqrt[3]{x^5}$. If $f'(x) = ax^p$, where a and p are constants, -----

Ans: $\frac{2}{3}$

Sol: $f(x) = 6.x^{5/3}$

$f'(x) = 6.\frac{5}{3}x^{\frac{5}{3}-1}$

$= 10x^{2/3} = ax^p$

22. Let y = $(\tan x)^{\sin x}$ for $0 < x < \frac{\pi}{2}$. -----

Ans: sec x

Sol: $y = (\tan x)^{\sin x} \Rightarrow \log y = \sin x.\log(\tan x)$

$\frac{1}{y}.\frac{dy}{dx} = \cos x.\log(\tan x) + \sin x.\frac{1}{\tan x}\sec^2 x$

$\Rightarrow \frac{dy}{dx} = (\tan x)^{\sin x}\left[\cos x.\log(\tan x) + \frac{\sin x.\sec^2 x}{\tan x}\right]$

$\Rightarrow g(x) = \sec x$

23. If $f(x) = (x^3 + \sin \pi x)^5$, then $f'(1)$ is equal to

Ans: $5(3-\pi)$

Sol: $f(x) = \left(x^3 + \sin \pi x\right)^5$

$f'(x) = 5\left(x^3 + \sin \pi x\right)^4 \times \left(3x^2 + (\cos \pi x).\pi\right)$

$f'(1) = 5(1+0)^4 \times (3-\pi) = 5(3-\pi)$

24. If $h(x) = 4x^3 - 5x + 7$ is the derivative of f(x), then -----

Ans: 6

Sol: $f'(x) = 4x^3 - 5x + 7$

$\lim\limits_{t\to0}\left[\frac{f(1+t)-f(1)}{t}\right]$

$= \lim\limits_{t\to0}\left[\frac{f'(1+t)-0}{1}\right] = f'(1)$

$= 4(1)^3 - 5(1) + 7 = 6$

25. Let $f(x)=\begin{cases} e^x, & \text{if } x\le 1 \\ mx+6, & \text{if } x>1 \end{cases}$ be differentiable at x = 1-----

Ans: e

Sol: $f'(x)=\begin{cases} e^x & \text{if } x\le 1 \\ m & \text{if } x>1 \end{cases}$

$$f'(1^-)=e^1,\ f'(1^+)=m \Rightarrow m=e$$

26. $\lim\limits_{t\to 0}\dfrac{\tan^2\left(\frac{\pi}{3}+t\right)-3}{t}$ is equal to -----

Ans: $8\sqrt{3}$

Sol: $\lim\limits_{t\to 0}\left[\dfrac{\tan^2\left(\frac{\pi}{3}+t\right)-3}{t}\right]$

$$= \lim_{t\to 0}\left[\frac{2\tan\left(\frac{\pi}{3}+t\right)\sec^2\left(\frac{\pi}{3}+t\right)}{1}\right]$$

$$=2\times\sqrt{3}\times(2)^2=8\sqrt{3}$$

27. If the tangent line to the graph of a function f at the point x=3 has -------

Ans: 6

Sol: Line : $\dfrac{x}{a}+\dfrac{y}{b}=1$

$$\Rightarrow \frac{x}{5/3}+\frac{y}{-10}=1$$

$$\Rightarrow \frac{3x}{5}-\frac{y}{10}=1$$

$$m = \frac{-3/5}{-1/10}=\frac{3}{5}\times 10=6$$

f'(3) = 6

28. The slope of tangent line to the curve -------

Ans: –2

Sol: $4x^2+2xy+y^2=12$

$$8x+2(x'y+xy')+2yy'=0$$

$$8x+2y+2xy'+2yy'=0$$

$$\Rightarrow 2(x+y)y'=-2(4x+y)$$

$$\Rightarrow y'=\frac{-(4x+y)}{(x+y)}=-\frac{(4+2)}{1+2}=-\frac{6}{3}=-2$$

29. Let f(x) $=\sqrt{x}+5$ for $1\le x\le 9$. -----

Ans: 4

Sol: $f(x)=\sqrt{x}+5,\ x\in[1,9]$

$$f'(x)=\frac{1}{2\sqrt{x}}\Rightarrow f'(c)=\frac{1}{2\sqrt{c}}$$

$$\frac{f(b)-f(a)}{b-a}=\frac{f(9)-f(1)}{9-1}=\frac{8-6}{8}=\frac{1}{4}$$

$$\frac{1}{2\sqrt{c}}=\frac{1}{4}\Rightarrow\sqrt{c}=2\Rightarrow c=4\in(1.9)$$

30. The derivative of a function f is given by -----

Ans: $(5,\infty)$

Sol: $f'(x)=\dfrac{x-5}{\sqrt{x^2+4}}>0\Rightarrow x>5$

31. Let f(x) = x^2 log x, x > 0. Then the minimum value of x is

Ans: $\dfrac{-1}{2e}$

Sol: $y=x^2\log x\ ,\ x\ne 0$

$$\frac{dy}{dx}=x^2.\frac{1}{x}+\log x.2x$$

$$=x+2x\log x$$

$$=x(1+2\log x)=0$$

$$\Rightarrow 1+2\log x=0$$

$$\Rightarrow \log x=-\frac{1}{2}$$

$$\Rightarrow x=e^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2}=1+2\left(x.\frac{1}{x}+\log x.1\right)$$

$$=1+2+2\log x$$

$$=3+2\log e^{-1/2}$$

$$=3+2\left(\frac{-1}{2}\right).\log e$$

$$=3-1=2>0 \quad \text{min}$$

$$\therefore y_{min}=\left(e^{-1/2}\right)^2.\log\left(e^{-1/2}\right)$$

$$=e^{-1}\times\frac{-1}{2}=\frac{-1}{2e}$$

32. A cube is expanding in such a way that its edge is increasing at a rate of 2 inches per second.

Ans: $150in^3/sec$

Sol: $V=x^3\Rightarrow\dfrac{dV}{dt}=3x^2.\dfrac{dx}{dt}$

$$=3\times 5^2\times 2=150in^3/sec$$

33. $\int x^5e^{1-x^6}\,dx=$

Ans: $\frac{-1}{6}e^{1-x^6}+C$

Sol: $I=\int x^5 e^{(1-x^6)}dx$

$1-x^6=t \Rightarrow -6x^5dx=dt$

$\Rightarrow x^5dx=-\frac{1}{6}dt$

$\therefore I=-\frac{1}{6}\int e^t+C$

$=-\frac{1}{6}\int e^{(1-x^6)}+C$

34. $\int(5-4x)e^{-x}dx=$

Ans: $e^{-x}(4x-1)+C$

Sol:

$$\int(5-4x)e^{-x}dx=(5-4x)\int e^{-x}dx-\int(-4)\frac{e^{-x}}{(-1)}dx$$

$$=(5-4x)\frac{e^{-x}}{-1}-4\int e^{-x}dx$$

$$=(4x-5)e^{-x}-4\frac{e^{-x}}{-1}+C$$

$$=(4x-5)e^{-x}+4e^{-x}+C$$

$$=(4x-1)e^{-x}+C$$

35. $\int\frac{\cos(\tan x)}{\cos^2 x}dx=$

Ans: $\sin(\tan x)+C$

Sol: $I=\int\frac{\cos(\tan x)}{\cos^2 x}dx=\int\sec^2 x.\cos(\tan x)dx$

$\tan x=t\Rightarrow\sec^2 x dx=dt$

$\therefore I=\int\cos t.dt=\sin t+C=\sin(\tan x)+C$

36. $\int\frac{1}{e^{2x}-1}dx=$

Ans: $\frac{1}{2}\log\left|e^{2x}-1\right|-x+C$

Sol: $I=\int\frac{1}{e^{2x}-1}dx=\int\frac{e^{2x}-(e^{2x}-1)}{(e^{2x}-1)}dx$

$=\int\frac{e^{2x}}{e^{2x}-1}dx-\int 1dx$

$=\frac{1}{2}\int\frac{2e^{2x}}{e^{2x}-1}dx-x+C$

$=\frac{1}{2}\log\left|e^{2x}-1\right|-x+C$

37. $\int\sin 2x\ \cos x\ dx=$

Ans: $\frac{-2}{3}\cos^3 x+C$

Sol: $I=\int 2\sin x.\cos x.\cos x dx$

$=2\int\sin x.\cos^2 x.dx$

$=-2\int t^2dt$ (where t=cosx)

$=-\frac{2t^3}{3}+C=-\frac{2}{3}\cos^3 x+C$

38. $\int\frac{1}{(1+\cot^2 x)\sin^2 x}dx=$

Ans: $x+C$

Sol: $I=\int\frac{1}{\operatorname{cosec}^2 x.\sin^2 x}dx$

$=\int 1\,dx+C=x+C$

39. $\int\frac{4x^9}{x^{10}-10}dx=$

Ans: $\frac{2}{5}\log\left|x^{10}-10\right|+C$

Sol: $I=\int\frac{4x^9}{x^{10}-10}dx=\frac{4}{10}\int\frac{10x^9dx}{x^{10}-10}$

$=\frac{2}{5}\log\left|x^{10}-10\right|+C$

40. The value $\int_0^{\sqrt{3}}\frac{6}{x^2+9}dx$ is equal to

Ans: $\frac{\pi}{3}$

Sol: $I=\int_0^{\sqrt{3}}\frac{6}{x^2+9}dx$

$$=6\int_0^{\sqrt{3}}\frac{dx}{x^2+3^2}=\frac{6}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^{\sqrt{3}}=\frac{\pi}{3}$$

41. The value of $\int_{-5}^{5}(4-|x|)dx$ is equal to -----

Ans: 15

Sol: $I = \int_{-5}^{5}(4-|x|)dx = 2\int_{0}^{5}(4-x)dx$

$= 2\left[4x - \frac{x^2}{2}\right]_0^5$

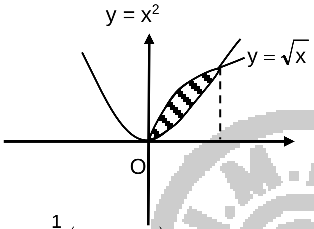
$= 2\left(20 - \frac{25}{2}\right)$

$= 2 \times \frac{15}{2} = 15$

42. The area of he region bounded by the curves-----

Ans: $\frac{1}{3}$

Sol:



$A = \int_0^1 \left(\sqrt{x} - x^2\right)dx$

$= \left[\frac{2}{3}x\sqrt{x} - \frac{x^3}{3}\right]_0^1$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

43. The value of $\int_0^2 \frac{x^2}{\left(x^3+1\right)^2}dx$ is equal to -----

Ans: $\frac{8}{27}$

Sol: $I = \int_0^2 \frac{x^2}{\left(x^3+1\right)^2}dx$

$x^3 + 1 = t \Rightarrow x^2dx = \frac{1}{3}dt$

$x = 0 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 9$

$\therefore I = \frac{1}{3}\int_1^9 \frac{dt}{t^2} = -\frac{1}{3}\left[\frac{1}{t}\right]_1^9$

$= -\frac{1}{3}\left[\frac{1}{9} - 1\right]$

$= -\frac{1}{3} \times -\frac{8}{9} = \frac{8}{27}$

44. The value $\int_{\pi/8}^{3\pi/8} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$ -----

Ans: $\frac{\pi}{8}$

Sol: $I = \int_{\pi/8}^{3\pi/8} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$

, $x \to \frac{\pi}{8} + \frac{3\pi}{8} - x$ ie, $x \to \frac{\pi}{2} - x$

$I = \int_{\pi/8}^{3\pi/8} \frac{\sin^4\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)}dx$

$I = \int_{\pi/8}^{3\pi/8} \frac{\cos^4 x}{\sin^4 x + \cos^4 x}dx$

$2I = \int_{\pi/8}^{3\pi/8}\left(\frac{\sin^4 x}{\sin^4 x + \cos^4 x} + \frac{\cos^4 x}{\sin^4 x + \cos^4 x}\right)dx$

$= \int_{\pi/8}^{3\pi/8} 1dx = [x]_{\pi/8}^{3\pi/8} = \frac{\pi}{4}$

$\Rightarrow I = \frac{\pi}{8}$

45. The area of the region bounded by y=5x, -------

Ans: 40

Sol: Area $= \int_0^4 5xdx = 5\left[\frac{x^2}{2}\right]_0^4 = \frac{5}{2}(16-0) = 40$

46. The general solution of the different equation $y - xy' = x^2 + y^{2+}$ is

Ans: $y = x\tan(C - x)$

Sol: $y - x\frac{dy}{dx} = x^2 + y^2$

$\Rightarrow \frac{xdy - ydx}{x^2} = -(\frac{x^2+y^2}{x})$

$\Rightarrow d\left(\frac{y}{x}\right) = -(1 + \left(\frac{y}{x}\right)^2)dx$

Let $\frac{y}{x}$= u

$\Rightarrow \tan^{-1}u = -x + C$

$\Rightarrow u = \tan(x + C)$

$\Rightarrow \frac{y}{x} = \tan(-x + C)$

$\Rightarrow y = x\tan(C - x)$

47. The integrating factor of the differential equation -----

Ans: x^2

Sol: $x\frac{dy}{dx} + 2y = 7x^3$

$$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = 7x^2$$

$$\text{I.F} = e^{\int P dx} = e^{\int \frac{2}{x} dx}$$

$$= e^{2\log x} = \left(e^{\log x}\right)^2 = x^2$$

48. The general solution of the differential equation -----

Ans: $(y + 3)(2x^2 + 3) = C$

Sol: $\left(2x^2+3\right)\frac{dy}{dx} = -(4xy + 12x)$

$$\frac{dy}{dx} = \frac{-4xy}{2x^2+3} - \frac{12x}{2x^2+3}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{4x}{2x^2+3}\right) y = \frac{-12x}{2x^2+3}$$

$$\text{I.F} = e^{\int \frac{4x}{2x^2+3} dx} = e^{\log\left|2x^2+3\right|} = 2x^2+3$$

$$y.\left(2x^2+3\right) = \int \frac{-12x}{\left(2x^2+3\right)}.\left(2x^2+3\right)dx$$

$$= -12.\frac{x^2}{2} + k = -6x^2 + k$$

$$\Rightarrow 2x^2y + 3y + 6x^2 = k$$

$$\Rightarrow (y+3)\left(2x^2+3\right) = C \ \ (\text{where } C = k+9)$$

49. The constrains of linear programming problem are $x + 2y \le 10$ -----

Ans: (1, 3)

Sol: (1 , 3) satisfy the inequalities $x+2y \le 10$ and $6x+3y \le 18$

50. Let $f : [-4, 2] \to R$ -----

Ans: [0, 4]

Sol: $y = \sqrt{16 - x^2}$

$$\Rightarrow 16 - x^2 = y^2$$

$$\Rightarrow x^2 = 16 - y^2$$

$$\Rightarrow x = \pm\sqrt{16 - y^2}$$

$$\Rightarrow 16 - y^2 \ge 0$$

$$\Rightarrow y^2 \le 16$$

$$\Rightarrow 0 \le y \le 4$$

($\because$ "$\sqrt{\ }$" represents non –negative roots)

51. Let $f(x) = x^2$ and $g(x) = \sqrt{9+x}$. -----

Ans: 8

Sol: fog(4) = $f[g(4)] = f\left(\sqrt{13}\right) = 13$

gof(4)= $g[f(4)] = g(16) = \sqrt{9+16} = 5$

$\therefore \text{Ans} = 13 - 5 = 8$

52. Let A and B be subsets of the universal set U.----

Ans: 55

Sol: $n(A' \cup B') = n[(A \cap B)']$

$= n(U) - n(A \cap B)$

=63-8=55

53. Let f(x) = [x], $x \in R$, where [x] -----

Ans: –5, 2

Sol: f(x)=[x]

f(-4.6)=[-4.6]=-5

f[2.7]=[2.7]=2

54. For any two positive rational numbers m and n, -------

Ans: 2

Sol: $m * n = \frac{m+n}{3}$

$$\frac{7}{2} * \frac{5}{2} = \frac{\frac{7}{2}+\frac{5}{2}}{3} = \frac{6}{3} = 2$$

55. The function $f : R \to R$ given by f(x) = 7 – 3x is

Ans: one-one and onto

Sol: f(x)-ax+b,

$f : R \to R$ is always bijective function

(where a, b $\in$ R)

56. A relation R on {0, 1, 2} is given by -----

Ans: reflexive

Sol: Clearly R is reflexive

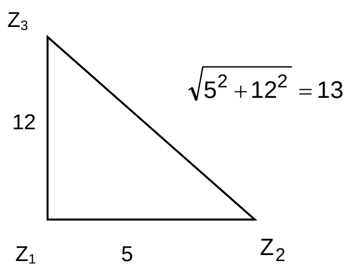
since $(a, a) \in R$

for all $a \in A$

57. Let z_1, z_2 and z_3 be three distinct points in the complex -----

Ans: 13

Sol:



58. If $\frac{z}{i} = 11 - 13i$, then -----

Ans: 26

Sol: $z = i(11 - 13i) = 11i - 13i^2$
$= 13 + 11i$
$z + \bar{z} = (13 + 11i) + (13 - 11i) = 26$

59. Let $\alpha = 2 - 3i$ be a root of he equation ----

Ans: -10

Sol: $\alpha = 2 - 3i, \beta = 2 + 3i$
$\alpha^2 + \beta^2 = (2-3i)^2 + (2+3i)^2$
$= 2(2^2 + (3i)^2)$
$=2(4-9) = -10$

60. If $z = 2 - i\sqrt{3}$, then ----

Ans: 49

Sol: $z = 2 - \sqrt{3}i$
$|z^4| = |z|^4 = (\sqrt{4+3})^4$
$= (\sqrt{7})^4 = 49$

61. The imaginary part of ----

Ans: $\frac{1}{2}$

Sol: $z = \frac{2+i}{3-i} = \frac{(2+i)(3+i)}{9-i^2}$
$= \frac{6+2i+3i-1}{10} = \frac{5+5i}{10}$
$= \frac{1}{2} + \frac{1}{2}i$
$\therefore Im(z) = \frac{1}{2}$

62. The area of a triangle on the complex plane -------

Ans: 16

Sol: The required area $= \frac{1}{2}|z|^2 = 128$
$\Rightarrow |z|^2 = 256 \Rightarrow |z| = 16$

63. If the real part of the complex number z=-------

Ans: $\sqrt{5}$

Sol:

$$z = \frac{p+2i}{p-i} = \frac{(p+2i)(p+i)}{p^2 - i^2} = \frac{p^2 + pi + 2pi + 2i^2}{p^2+1}$$
$$= \frac{(p^2-2) + 3pi}{p^2+1} = \frac{p^2-2}{p^2+1} + \frac{3p}{p^2+1}$$

$\frac{p^2-2}{p^2+1} = \frac{1}{2} \Rightarrow 2p^2 - 4$
$= p^2 + 1 \Rightarrow p^2 = 5 \Rightarrow p = \sqrt{5}$

64. The value of $\sqrt{(-25)} + 3$ ----

Ans: 17!

Sol: $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} = 5i + (3)2i + 2(3)i$
$= 5i + 6i + 6i = 17i$

65. The value $\sum_{k=5}^{36} \frac{1}{k^2 - k}$ is ----

Ans: $\frac{2}{9}$

Sol: $\frac{1}{k^2-k} = \frac{1}{k(k-1)}$
$= \frac{k-(k-1)}{k(k-1)} = \frac{1}{k-1} - \frac{1}{k}$
1^{st} term $= \frac{1}{4} - \frac{1}{5}$
2^{nd} term $= \frac{1}{5} - \frac{1}{6}$
3^{rd} term $= \frac{1}{6} - \frac{1}{7}$

Last term $= \frac{1}{35} - \frac{1}{36}$
sum $= \frac{1}{4} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$

66. If a_1, a_2, a_3, ……., a_n are in A.P. ----

Ans: 10

Sol:

$a_1 + a_2 + \ldots\ldots + a_n = \frac{n}{2}(3+39) = 21n = 210$
$\Rightarrow n = 10$

67. Let t_n, n = 1, 2, 3, …… be the ----

Ans: 101

Sol: $t_n = a + (n-1)d$
$\Rightarrow 5 + (n-1)3 = 305$
$\Rightarrow (n-1)3 = 300$
$\Rightarrow n - 1 = 100$
$\Rightarrow n = 101$

68. If the first term of a G.P is 1 and the sum of -------

Ans: 3

Sol: a=1

$$ar^2 + ar^4 = r^2 + r^4 = 90$$
$$\Rightarrow r = 3 \text{(by trial and error)}$$

69. In an A.P. the difference between the last and the first terms is 632 -------

Ans: 159

Sol: $a_n - a_1 = 632$

$\Rightarrow (n-1)d = 632$

$\Rightarrow (n-1) = \frac{632}{4} = 158$

$\Rightarrow n = 159$

70. If the 10th and 12th terms of an A. -----

Ans: 3

Sol: $a + 9d = 15$ ....(1)

$a + 11d = 21$ ....(2)

(2)-(1) $\Rightarrow 2d = 6 \Rightarrow d = 3$

71. The first term of a G.P. is 3 and the common ratio -------

Ans: 765

Sol: $a = 3$, $r = 2$

$$S_8 = \frac{a(r^8 - 1)}{(r-1)} = \frac{3(2^8 - 1)}{2-1} = 765$$

72. A covid-19 vaccination reduces the probability of getting -------

Ans: 0.22

Sol: P(non vaccinated)=0.55
Required probability= 0.55 ×0.4=0.22

73. The number of ways a committee of 3 women and 5 men -------

Ans: 560

Sol: Number of ways =

$^8C_5 \times ^5C_3 = ^8C_3 \times ^5C_2$

$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{5 \times 4}{1 \times 2} = 560$

74. A set contain 9 elements. Then the number of subsets -------

Ans: 256

Sol: Number of the subjects which contains maximum 4 elements=

$^9C_0 + ^9C_1 + ^9C_2 + ^9C_3 + ^9C_4$

$= 1 + 9 + \frac{9.8}{1.2} + \frac{9.8.7}{1.2.3} + \frac{9.8.7.6}{1.2.3.4} = 256$

75. If p and q are positive integers such that -------

Ans: 6, 1

Sol: $= {}^{(p+q)}P_2 = (p+q)(p+q-1) = 42$

$\Rightarrow p + q = 7$.........(1)

${}^{(p+q)}P_2 = (p-q)(p-q-1) = 20$

$\Rightarrow p - q = 5$.........(2)

(1) + (2) $\Rightarrow 2p = 12$

$\Rightarrow p = 6$ and $q = 1$

76. The number of 3-digit numbers that can be formed from the digits 0,2,3,5,7 -------

Ans: 100

Sol:

□	□	□
4	5	5

$= 4 \times 5 \times 5 = 100$

77. If x^{22} is in the (r + 1)th term of he binomial -------

Ans: 5

Sol: $(3x^3 + -x^2)^9$

$T_{r+1} = {}^9C_r (3x^3)^{9-r} (-x^2)^r$

$= {}^9C_2 . 3^{9-r} . (-1)^r . x^{27-3r} . x^{2r}$

$= {}^9C_2 . 3^{9-r} . (-1)^r x^{27-r} . x^{2r}$

$27 - r = 22 \Rightarrow r = 5$

78. The term independent of x in the binomial expansion of -------

Ans: $\binom{20}{5} 2^5$

Sol: $(x + 2x^{-3})^{20}$

$T_{r+1} = {}^{20}C_r x^{20-r} (2x^{-3})^r$

$= {}^{20}C_r 2^r . x^{20-r} . x^{-3r}$

$= {}^{20}C_r 2^r . x^{20-4r}$

$20 - 4r = 0 \Rightarrow r = 5$

$\therefore$ The term independent of x is ${}^{20}C_5 2^5$

79. Let $A + B = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 4 \end{bmatrix}$ ------

Ans: $\begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 4 \end{bmatrix}$

Sol: A=(A+B)-B $=\begin{bmatrix}3 & 1 & 6\\ 2 & 1 & 4\end{bmatrix}$

80. The value of the determinant $\begin{vmatrix}4 & 4^2 & 4^3\\ 3 & 3^2 & 3^3\\ 2 & 2^2 & 2^3\end{vmatrix}$ is

Ans: –48

Sol: $\begin{vmatrix}4 & 4^2 & 4^3\\ 3 & 3^2 & 3^3\\ 2 & 2^2 & 2^3\end{vmatrix}$

$= 4\times3\times2\begin{vmatrix}1 & 4 & 4^2\\ 1 & 3 & 3^2\\ 1 & 2 & 2^2\end{vmatrix}$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$= 24\times\begin{vmatrix}0 & 1 & 7\\ 0 & 1 & 5\\ 1 & 2 & 4\end{vmatrix}$

$= 24\times(5-7) = -48$

81. If $\begin{vmatrix}1 & 2 & 1\\ 0 & x & -3\\ 2 & -1 & x\end{vmatrix}$ = 0, then the values of x are

Ans: 5, –3

Sol: Expanding ,

$1(x^2-3)+2(-6-x)$

$x^2-3-12-2x = x^2-2x-15$

$=(x-5)(x+3)-0 \Rightarrow x=5$ or -3

82. If $AB=\begin{bmatrix}4 & 3\\ 5 & 4\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}$,-----

Ans: $\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$

Sol: $B=(A^{-1}.A)B = A^{-1}(AB)$

$=\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\begin{bmatrix}4 & 3\\ 5 & 4\end{bmatrix}$

$=\begin{bmatrix}12-10 & 9-8\\ -4+5 & -3+4\end{bmatrix}=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$

83. The matrix $\begin{bmatrix}-2 & 1 & 0\\ 3 & 4 & 1\\ -4 & \lambda & 0\end{bmatrix}$ is non-significant -----

Ans: 2

Sol: |A|=0

$-2(0-\lambda)-1(0+4)=0$

$\Rightarrow 2\lambda-4=0$

$\Rightarrow \lambda=2$

84. Let $\begin{vmatrix}x-1 & 2 & 1\\ 2 & x-1 & 2\\ 1 & x+2 & x-1\end{vmatrix}$-----

Ans: 16

Sol: Put x=0 both sides,

$d=\begin{vmatrix}-1 & 2 & 1\\ 2 & -1 & 2\\ 1 & 2 & -1\end{vmatrix}$

$=-(1-4)-2(-2-2)+1(4+1)$

$=3+8+5=16$

85. If the inequality $-13 \le x \le 5$ is ----

Ans: –4, 9

Sol: $|x-a| \le b$

$\Rightarrow -b \le x-a \le b$

$\Rightarrow -b+a \le x \le b+a$

$\therefore a-b=-13$ ....(1)

$a+b=5$ . ...(2)

(1) + (2) $\Rightarrow 2a=-8 \Rightarrow a=-4, b=9$

86. The solution set of inequality 5(4x + 6) ----

Ans: $(4, \infty)$

Sol: $20x+30<25x+10$

$\Rightarrow 20<5x \Rightarrow 5x>20 \Rightarrow x>4$

87. The set of all integer values of x that satisfy the inequality -------

Ans: {–9, –8, –7}

Sol: $19 \le -3x \le 27$

$\frac{19}{-3} \ge x \ge \frac{27}{-3}$

$\Rightarrow -9 \le x \le -\frac{19}{3}$

$\Rightarrow x=-9,-8,-7$

88. Let X be the set $\{4 2\sqrt{2}\}\{1 3\}$ ------

Ans: P and R only

Sol: P and R only

89. The value of θ in the range ------

Ans: $\frac{\pi}{6}$

Sol: $\sin\left(\theta+\frac{\pi}{6}\right)=\cos\theta$

$\Rightarrow \sin\left(\theta+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{2}-\theta\right)$

$\Rightarrow \theta+\frac{\pi}{6}=\frac{\pi}{2}-\theta$

$2\theta=90^o-30^o=60^o \Rightarrow \theta=30^o=\frac{\pi}{6}$

90. If cosec θ + cot θ = 5, -----

Ans: $\frac{5}{12}$

Sol: $\text{cosec}\,\theta+\cot\theta=5 \quad ----(1)$

$\text{cosec}\,\theta-\cot\theta=\frac{\text{cosec}^2\theta-\cot^2\theta}{\text{cosec}\,\theta+\cot\theta}$

$=\frac{1}{5} \quad ----(2)$

$(1)-(2) \Rightarrow 2\cot\theta=5-\frac{1}{5}=\frac{24}{5}$

$\Rightarrow \cot\theta=\frac{12}{5}$

$\therefore \tan\theta=\frac{5}{12}$

91. The value of $\tan^{-1}\left(\frac{7}{4}\right)$ -----

Ans: $\frac{\pi}{4}$

Sol: $\tan^{-1}\left(\frac{7}{4}\right)-\tan^{-1}\left(\frac{3}{11}\right)$

$\tan^{-1}\left(\frac{\frac{7}{4}-\frac{3}{11}}{1+\frac{7}{4}.\frac{3}{11}}\right)=\tan^{-1}\left(\frac{77-12}{44+21}\right)$

$=\tan^{-1}\left(\frac{65}{65}\right)=\tan^{-1}(1)=\frac{\pi}{4}$

92. If $0<\theta<\frac{\pi}{2}$ and θ -----

Ans: $\frac{2}{3}$

Sol: $\cos\theta=\frac{1}{\sec\theta}=\frac{1}{\sqrt{1+\tan^2\theta}}$

$=\frac{1}{\sqrt{1+\frac{5}{4}}}=\frac{1}{\sqrt{\frac{9}{4}}}=\frac{1}{3/2}=\frac{2}{3}$

93. The value of $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ is equal to

Ans: $\frac{16}{25}$

Sol: $\sin(\cos^{-1}x)=\cos(\sin^{-1}x)=\sqrt{1-x^2}$

$\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)=\sqrt{1-\left(\frac{3}{5}\right)^2}=\frac{4}{5}$

$\therefore \sin^2\left[\cos^{-1}\left(\frac{3}{5}\right)\right]=\frac{16}{25}$

94. $\cos^4\frac{\pi}{12}-\sin^4\frac{\pi}{12}$ ------

Ans: $\frac{\sqrt{3}}{2}$

Sol: $\cos^4\frac{\pi}{12}-\sin^4\frac{\pi}{12}$

$=\left(\cos^2\frac{\pi}{12}+\sin^2\frac{\pi}{12}\right)\left(\cos^2\frac{\pi}{12}-\sin^2\frac{\pi}{12}\right)$

$=1\times\cos 2\left(\frac{\pi}{12}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

95. $\tan\left[2\tan^{-1}\left(\frac{2}{5}\right)\right]$ is equal to

Ans: $\frac{20}{21}$

Sol: $\tan\left[2\tan^{-1}\left(\frac{2}{5}\right)\right]=\frac{2\tan^{-1}\left(\tan\left(\frac{2}{5}\right)\right)}{1-\tan^2\left(\tan^{-1}\frac{2}{5}\right)}$

$=\frac{2\times\frac{2}{5}}{1-\left(\frac{2}{5}\right)^2}=\frac{4/5}{21/25}=\frac{4}{5}\times\frac{25}{21}=\frac{20}{21}$

96. The values of x in the interval [0, π] such that ----

Ans: $\frac{\pi}{6},\frac{\pi}{3}$

Sol: $\sin 2x=\frac{\sqrt{3}}{2}$

$\Rightarrow 2x=0,60^\circ,180-60^\circ,360^\circ+60^\circ etc$

$\Rightarrow x=0^\circ,30^\circ,90-30^\circ,180^\circ+30^\circ etc$

$\Rightarrow x=0^\circ,\frac{\pi}{6},\frac{\pi}{3}$

97. If sin α + sin β = -----

Ans: 0

Sol: Squaring and adding both equations, we get

$$cos(\alpha-\beta)=0$$

98. If ay = x + b is the equation of the line -----

Ans: 5

Sol: $A(-5,-2),\ B(4,7)$

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$$

$$\Rightarrow y+2=\frac{9}{9}(x+5)\Rightarrow y=x+3$$

$$a=1\ ,\ b=3\Rightarrow 2a+b=5$$

99. The y-intercept of the line passing through-----

Ans: 4

Sol: Equation of the line:

$$y-5=\frac{1}{2}(x-2)$$

$$\Rightarrow 2y-10=x-2$$

$$\Rightarrow 2y=x+8$$

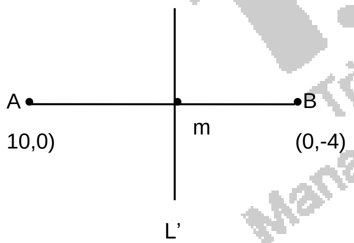
$$\Rightarrow y=\frac{1}{2}x+4$$

$\therefore$ y intercept = 4

100. The equation of perpendicular bisector of the line segment joining the points -------

Ans: 5x + 2y = 21

Sol:



$$m\left(\frac{10+0}{2},\frac{0+-4}{2}\right)=(5,-2)$$

Slope of AB= $\frac{-4-0}{0-10}=\frac{2}{5}$

Slope of L= $-\frac{5}{2}$

$$\therefore L: y+2=-\frac{5}{2}(x-5)$$

$$\Rightarrow 2y+4=-5x+25$$

$$\Rightarrow 5x+2y=21$$

101. The equation of the line which is parallel to -------

Ans: 2x + y = 5

Sol: The required line is = $x+\frac{1}{2}y=k$

Which passes through (1 , 3)

$$\Rightarrow 1+\frac{3}{2}=k\Rightarrow k=\frac{5}{2}$$

$$\therefore x+\frac{y}{2}=\frac{5}{2}$$

$$\Rightarrow 2x+y=5$$

102. If x-intercept of the straight line ax+2ay=30 is 10, then the y-intercept is -------

Ans: 5

Sol: ax+2ay-30=0

x intercept = $-\frac{c}{a}=-\frac{(-30)}{a}=\frac{30}{a}=10$

$$\Rightarrow a=3$$

y intercept = $-\frac{c}{b}=-\frac{(-30)}{2a}=\frac{15}{3}=5$

103. A straight line makes an angle α with -------

Ans: $\sqrt{3}y-x+2\sqrt{3}=0$

Sol: $\alpha=30^{o}\Rightarrow m=\tan\alpha=\frac{1}{\sqrt{3}}$

Equation: y-(-2)= $\frac{1}{\sqrt{3}}(x-0)$

$$\Rightarrow y+2=\frac{1}{\sqrt{3}}x$$

$$\Rightarrow \sqrt{3}y+2\sqrt{3}=x$$

104. The equation of the circle is $3x^2+3y^2+6x-4y-1=0$ -----

Ans: $\frac{4}{3}$

Sol: $x^2+y^2+2x-\frac{4}{3}y-\frac{1}{3}=0$

$$2g=2, 2f=-\frac{4}{3},\ c=-\frac{1}{3}$$

$$r=\sqrt{g^2+f^2-c}$$

$$=\sqrt{1^2+\left(-\frac{2}{3}\right)^2-\left(-\frac{1}{3}\right)}$$

$$=\sqrt{1+\frac{4}{9}+\frac{1}{3}}$$

$$=\sqrt{\frac{13}{9}+\frac{1}{3}}=\sqrt{\frac{16}{9}}=\frac{4}{3}$$

105. The end-points of a diameter of a circle are -------

Ans: $(x-2)^2+(y-4)^2=9$

Sol: Centre : $\left(\frac{-1+5}{2},\frac{4+4}{2}\right)=(2,4)$

$r=\sqrt{(5-2)^2+(4-4)^2}=3$

$\therefore (x-2)^2+(y-2)^2=3^2$

106. The two diameters of a circle are segments of the straight lines x-y=5 -------

Ans: $x^2+y^2-6x+4y=12$

Sol: $x-y=5$ ...(1)

$2x+y=4$ ...(2)

(1) + (2) $\Rightarrow 3x=9 \Rightarrow x=3$

$\therefore 3-y=5 \Rightarrow y=-2$

Centre : (3 , -2) , radius = 5

$\therefore (x-3)^2+(y+2)^2=5^2$

$\Rightarrow x^2+y^2-6x+4y-12=0$

107. The equation of the parabola with vertex (–6, 2) -------

Ans: $(y-2)^2=3x+18$

Sol: $[y-2]^2=4a[x-(-6)]$

$\Rightarrow (y-2)^2=4a(x+6)$

Passes through (-3 , 5)

$\Rightarrow (5-2)^2=4a(-3+6)$

$\Rightarrow 9=(4a)(3)$

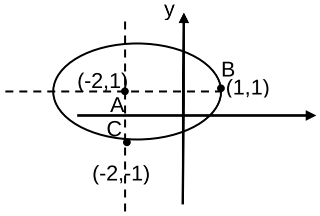
$\Rightarrow 4a=3$

$\Rightarrow (y-2)^2=3(x+6)$

108. One of the vertices of the major axis of an ellipse is -------

Ans: $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1$

Sol:



a=AB=3

b=AC=2

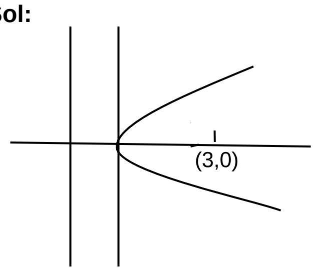
$\Rightarrow \frac{[x-(-2)]^2}{3^2}+\frac{(y-1)^2}{2^2}=1$

$\Rightarrow \frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1$

109. The equation of the parabola with focus (3, 0) ----

Ans: $y^2=12x$

Sol:



Focus=(3 , 0)

$\Rightarrow a=3$

Directrix x+3=0

$\Rightarrow$ Required parabola is

$y^2=4.3.x$

$\Rightarrow y^2=12x$

110. The eccentricity of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ is

Ans: $\frac{\sqrt{5}}{3}$

Sol: $\frac{x^2}{36}+\frac{y^2}{16}=1$

$a^2=36 \quad b^2=16$

$e^2=\frac{a^2-b^2}{a^2}=\frac{36-16}{36}=\frac{20}{36}$

$\Rightarrow e=\sqrt{\frac{20}{36}}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

111. The foci of a hyperbola are (8, 3) and (0, 3) ----

Ans: 6

Sol: foci (8 , 3) and (0 , 3)

2ae=8 $\quad$ e= $\frac{4}{3}$

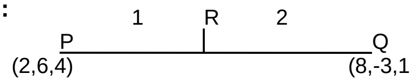
$\Rightarrow a=3$

2a=6

112. The co-ordinates of the points P and Q are -----------

Ans: (4, 3, 3)

Sol:



$$R = \left(\frac{8\times1+2\times2}{1+2}, \frac{-3\times1+6\times2}{1+2}, \frac{1\times1+4\times2}{3}\right)$$
$= (4, 3, 3)$

113. If $|\vec{a}| = 2$, $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$ and the angle between ----

Ans: $2\sqrt{7}$

Sol: $|a| = 2$

$\bar{b} = 2i - j - 3k \Rightarrow |b| = \sqrt{14}$

Angle between $\bar{a}$ and $\bar{b}$, $\theta = \frac{\pi}{4}$

a.b=|a||b| $\cos\theta$

$= 2 \times \sqrt{14} \times \frac{1}{\sqrt{2}}$

$= 2\sqrt{7}$

114. If α is the angle made by the vector -------

Ans: $\frac{\sqrt{2}}{2}$

Sol: Let α be the angle

$\cos\alpha = \frac{5}{\sqrt{25+9+16}}$

$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

115. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and ----

Ans: 9

Sol: $|\vec{a}| = 3 \quad |\vec{b}| = 4 \quad |\vec{a} - \vec{b}| = \sqrt{7}$

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$7 = 9 + 16 - 2a.b$

2a.b=18

a.b=9

116. If $\vec{a} = \hat{i} + \lambda\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ ----

Ans: 4

Sol: $\vec{a} = i + \lambda j - 2k \quad \vec{b} = 2i - 3j + 5k$

$\vec{a}.\vec{b} = 2 - 3\lambda - 10 = -20$

$\Rightarrow 3\lambda = 12$

$\Rightarrow \lambda = 4$

117. If $\vec{a} = \hat{i} - 3\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 4\hat{k}$ ----

Ans: 1

Sol: $\vec{a} = i - 3j + \alpha k$

$\vec{b} = i - 2j + 4k$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -3 & \alpha \\ 1 & -2 & 4 \end{vmatrix}$$

$= i(-12 + 2\alpha) - j(4 - \alpha) + k(1)$

Comparing with -2i+j+βk , β =1

118. The value of α so that the vectors ----

Ans: $\frac{3}{2}, -2$

Sol: $\alpha i + (\alpha - 1)j + 3k$ and $(\alpha + 2)i + \alpha j$ -2k are perpendicular

$\Rightarrow \alpha^2 + 2\alpha + \alpha^2 - \alpha - 6 = 0$

$\Rightarrow 2\alpha^2 + \alpha = 6 = 0$

$\Rightarrow \alpha = -2, \frac{3}{2}$

119. If $|\vec{u}| = 5, |\vec{v}| = 4$ and the angle -----

Ans: 10

Sol: |u|=5 |v|=4

Angle between u and v= $\frac{\pi}{6}$

$|u \times v| = 5 \times 4 \times \sin\frac{\pi}{6}$

$= 5 \times 4 \times \frac{1}{2} = 10$

120. If the point P(x, 1, 4) lies on he line ----

Ans: 5

Sol: Given line is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{0}$

Since (x , 1 , 4) lie on the Line , it satisfies the line

$\Rightarrow \frac{x-1}{2} = \frac{1-3}{-1}$

$\Rightarrow x - 1 = 4$

$\Rightarrow x = 5$